Production Planning with Uncertain Power Supply

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Setting

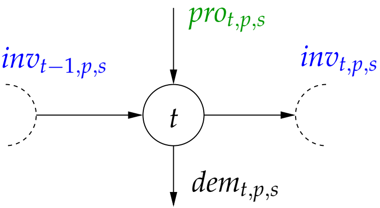
- Two plants, two products
- Each has an inventory with given maximum capacity
- Customer demand known for every *time bucket*

⇒ Plan production at each time bucket

... while minimizing total prod./inv. cost

Classical (non-robust) production planning

- Variables: production $pro_{t,p,s}$, inventory $inv_{t,p,s}$.
- Constraints: for all $t \in T, p \in \{P_1, P_2\}, s \in \{S_1, S_2\}$:
  - Production capacity: $pro_{t,p,s} \leq C_{pro}$
  - Inventory capacity: $inv_{t,p,s} \leq C_{inv}$
  - Conservation: $pro_{t,p,s} + inv_{t-1,p,s} = dem_{t,p,s} + inv_{t,p,s}$



- Obj. function: $\sum_{t,p,s}(c_{\text{prod}} \cdot pro_{t,p,s} + c_{\text{inv}} \cdot inv_{t,p,s})$

Disruptions

Both plants are subject to power interruptions.

Part of a “flexible supply” contract with the power company. Clauses might specify

- How many interruptions there might be
- Duration (# time buckets)
- What plant(s) are affected

However,

- Short notice ($\approx$ hours, even minutes)
- Unknown duration (apart from per-contract limits)

Uncertainty in power interruptions

Optimization problems under uncertainty may be dealt with

- Robust Optimization[1]: ideal when uncertainty is **limited** but the scenarios seem all equally likely.
- Stochastic Programming[2]: more suited when a probability distribution is known.

Our case is prototypical for Robust Optimization: there are a few interruptions which, however, are all equally likely.

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[1] A. Ben-Tal, L. El Ghaoui, A. Nemirovski, Robust Optimization. Princeton Univ. Press, 2009. http://sites.google.com/site/robustoptimization

[2] J.R. Birge, F. Louveaux, Introduction to Stochastic Programming, Springer, 1997.

Robust model

Production $pro_{t,p,s}$ is influenced by an opponent that can shut off a plant under certain conditions.

$$\begin{array}{ll} \min & \sum_{t,p,s}(c_{\text{prod}} \cdot pro_{t,p,s} + c_{\text{inv}} \cdot inv_{t,p,s}) \\ & \text{[production constraints]} \\ & \text{[inventory constraints]} \\ & \min_U \{\text{prod. of } S \text{ at time } T\} \geq dem_{TS} \end{array}$$

Opponent's problem

The **inner** problem is a worst-case estimate of the production plan at every time bucket. Its objective function value

$$z(pro) = \min_{U}\{\text{prod. of } S \text{ at time } T\}$$

depends on the production levels, but has to be computed by *implicitly* solving the optimization problem

Robust model

Some remarks:

- The "opponent" decides when to shut off a plant,

... while respecting some sort of "terms" of uncertainty.

- In general, the opponent will try to give us a hard time

$\Rightarrow$ we need a plan so that customer demands are met,

... no matter what the opponent decides

Duality trick

Soyster 1973[3]: “solve” the inner problem by taking its dual.

$$\begin{array}{ll} \min & \sum_{t,p,s}(c_{\text{prod}} \cdot pro_{t,p,s} + c_{\text{inv}} \cdot inv_{t,p,s}) \\ & \text{[prod. constraints], [inv. constraints]} \\ & \min\{c^\top x : Ax \geq b, x \in \mathbb{R}^n_+\} \geq d_{TS} \end{array}$$

$$\Downarrow$$

$$\begin{array}{ll} \min & \sum_{t,p,s}(c_{\text{prod}} \cdot pro_{t,p,s} + c_{\text{inv}} \cdot inv_{t,p,s}) \\ & \text{[prod. constraints], [inv. constraints]} \\ & \max\{u^\top b : u^\top A \leq c, u \in \mathbb{R}^m_+\} \geq d_{TS} \end{array}$$

[3] A.L. Soyster, “Convex Programming with set-inclusive constraints and applications to inexact LP,” *Operations Research* 21(5), 1973, pp. 1154-1157.

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Inner optimization model(s)

The opponent acts at every time bucket and for both products by “solving” a minimization problem for every T and S.

(Opponent’s) variables are $x_{tp} \in \{0, 1\}$: one if plant p is shut down at time bucket t, zero otherwise.

$$
\begin{aligned}
P_{TS}: \ \min \quad & f(pro, x) \\
& \sum_{t=1}^{n} (x_{t1} + x_{t2}) \leq K \\
& x_{t1} + x_{t2} \leq 1 \\
& x_{tp} \in \{0, 1\}
\end{aligned}
$$

Production levels *pro* are parameters in the inner problem, while x are the variables

Solving the robust model

- LP Duality allows to implicitly solve the above problem
- We know what the opponent wants, and we can model it
- The inner problem is a Mixed Integer Linear Program

$\Rightarrow$ It needs to be strengthened

This can be done by

- cut separation in the inner problem's primal, or
- column generation in its dual

Can significantly improve the total cost

Test #1: production levels

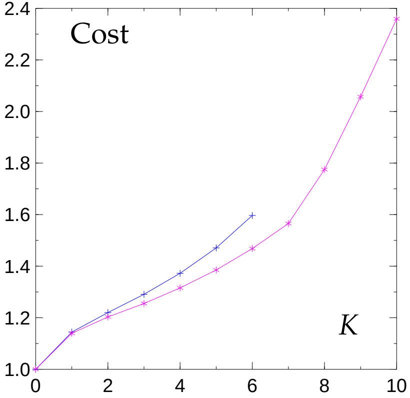
- Two plants, two products, 14 time buckets.
- K: max. number of interruptions ($\approx$ uncertainty level).
- More interruptions $\rightarrow$ production "spreads" over time

t	$K=0$	1	2	3	4	5	6
1	100	100	100	100	100	100	100
2	106	106	106	106	106	106	106
3	130	130	130	130	130	130	130
4	99	142	138	141	131	118	124
5	210	168	172	168	178	192	185
6	149	149	149	149	149	149	185
7	111	111	111	111	111	192	185
8	130	130	130	130	178	192	185
9	153	153	153	153	178	192	185
10	92	92	92	168	178	192	185
11	96	96	165	168	178	192	185
12	134	134	172	168	178	192	198
13	112	168	172	168	178	198	198
14	112	168	172	198	198	198	198

Test #2: policy vs. production cost

Two plants, two products, 14 time buckets. Cost is normalized to 1 (for $K = 0$, i.e., no interruptions).

- K: max. number of interruptions ($\approx$ uncertainty level).
- Blue (upper curve): cost with less accurate model, purple (lower): more accurate
- Real cost: between purple line and Cost=1



Current steps

- Integrating simulation with optimization
- Try several uncertainty sets (contract policies), e.g.
  1. interruptions allowed at one plant only
  2. not more than k interruption over n time buckets