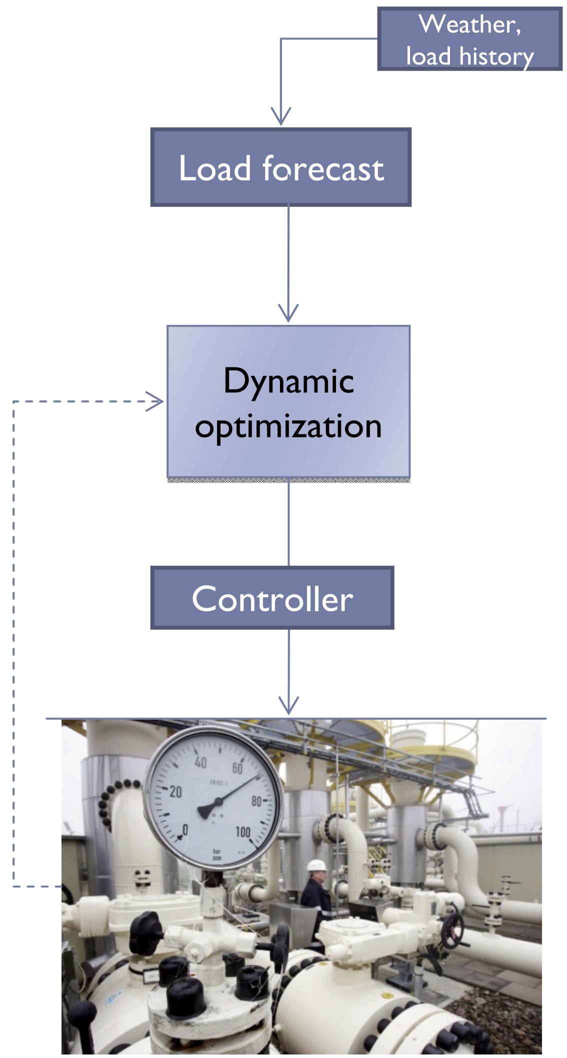
A moving horizon solution to the gas pipeline optimization problem

EWO MEETING, Fall 2010

Ajit Gopalakrishnan
Advisor: L. T. Biegler

Background: Gas pipeline optimization

- Gas pipeline networks – Branched network consisting of compressor stations, suppliers and consumers (chemical industries, power plants, residential/commercial heating)
- Modeling & control challenge – Highly coupled nonlinear model describing flow of gas through a pipe.
- Optimization scope – Compression energy for the supplier to be minimized, while satisfying gas demands, contract pressures and physical constraints.
  - Moving Horizon Optimization – Repeated solution of dynamic optimization over a finite horizon.



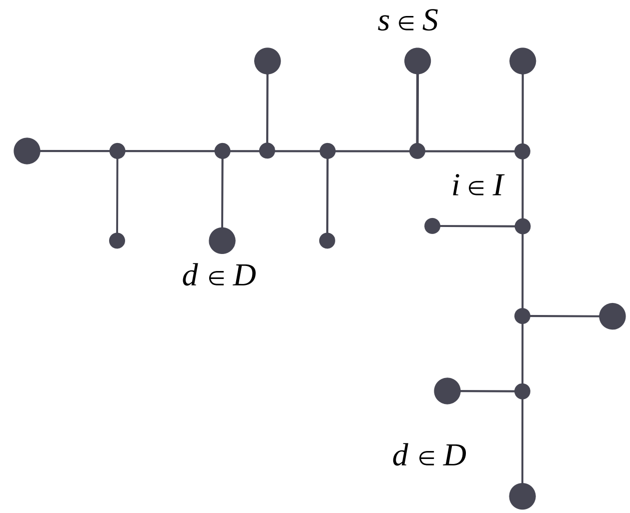
Pipeline modeling [Baumrucker & Biegler, 09]

- Pipe segment equations

  Material balance $\frac{M_W A_i L_i}{RT_{ref}}(\overline{P}_{i,t+1} - \overline{P}_{i,t}) = \int_t^{t+1} (q_i^{in} - q_i^{out})dt \quad \forall i \in I, t \in T$

  Momentum balance $\frac{dP}{dz}_{i,k,t} = \frac{-f_{i,k,t} RT_{ref} q_{i,k,t} |q_{i,k,t}|}{2D_i A_i^2 M_W P_{i,k,t}} \quad \forall i \in I, t \in T, k \in K$

  Network inventory $mass_{i,t} = \frac{\overline{P}_{i,t} M_W A_i L_i}{RT_{ref}} \quad \forall i \in I, t \in T$



- Node equations
  - Flow balance: no accumulation at nodes.
  - Pressure balance: pressure at a node is equal to the pressure either into or out of the connected arc.
- Compressor equation: $Power_{s,t} = q_{s,t}^{Supply} \frac{C_p T_{ref}}{\eta} \left[\left(\frac{P_{s,t}}{P_{inlet}} \right)^{(\gamma-1)/\gamma} - 1 \right] \quad \forall s \in S, t \in T$
- Laminar friction factor/Colebrook-White turbulent friction factor
- Switches for permitting flow reversals, and non-smooth flow transitions (laminar to turbulent)

SET DEFINITIONS

I	Arcs (pipe segments)
J	Nodes (intersection)
$K = \{in, out\}$	End-points of the pipe
S	Suppliers in network
D	Demands in network
$T = \{0, \ldots, t_f\}$	Time points

B.T. Baumrucker and L.T. Biegler. MPEC strategies for cost optimization of pipeline operations.
doi:10.1016/j.compchemeng.2009.07.012

NMPC/Moving horizon formulation

- Formulation (NLP):

$$\begin{aligned} &\min \quad \text{Objective fn. } (\textit{states, inputs, parameters}) \\ &s.t. \quad \text{Discretized dynamic pipeline model} \\ &\qquad\;\; \text{Constraints} \end{aligned}$$

- Objective function (Energy minimization):

$$J(t_0) = \underbrace{\sum_{s \in S} \int_{t=t_0}^{t=t_0+T_P} Power_{s,t} dt}_{\text{Total Energy}} + \underbrace{\rho \sum_{s \in S} \sum_{t \in T \setminus \{t_0\}} (Power_{s,t} - Power_{s,t-1})^2}_{\text{Smoothing term}}$$

Term	Meaning
States	Pressures & flow rates at various points
Inputs (MV)	Supplier discharge pressures and supplier flow rates for $t \in T$
Parameters	Forecast of future demand loads for $t \in T$

Constraints:

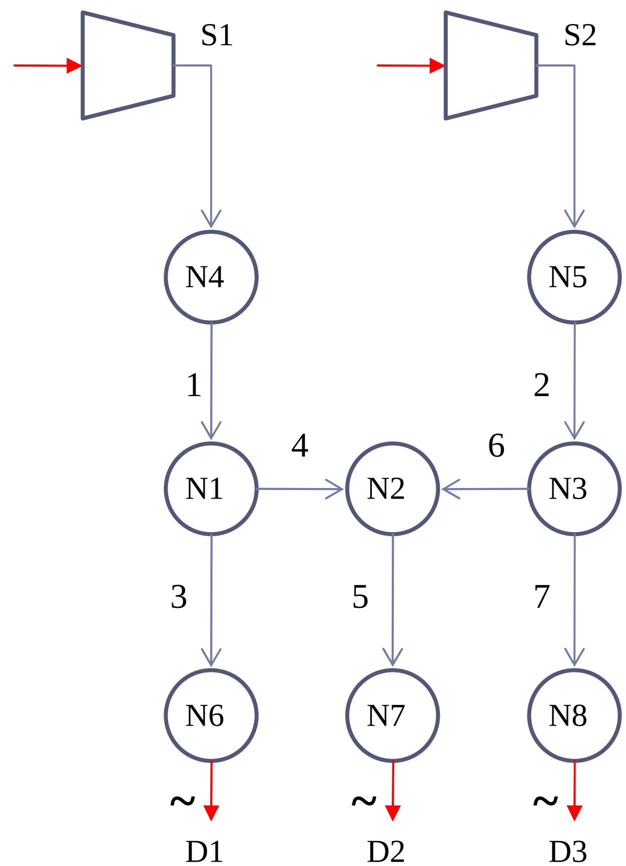
- **Delivery points:** Contract pressures should be satisfied.
- **Supplier points**: Compressor limits on minimum & maximum discharge, pressures and work.
- **Linepack/Inventory targets:** Restoring sustainable gas pipeline inventory and pressure.

$$\sum_i mass_{i,t_f} \geq \sum_i mass_{i,0}$$

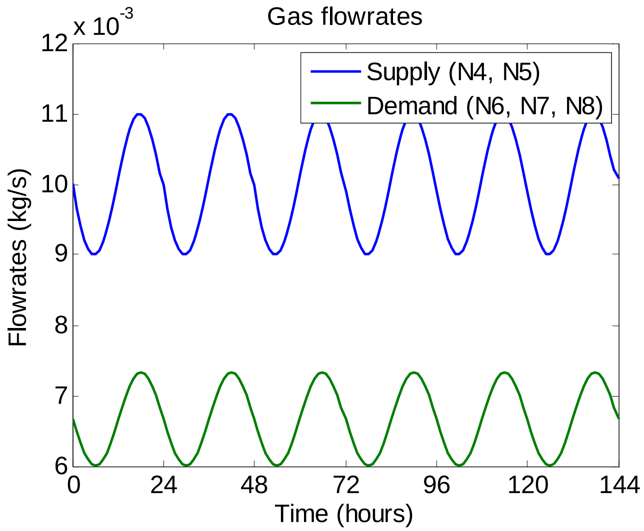
- **Cyclic steady states (Terminal Constraint):** Gas demand is diurnal. Ensuring the states reach a prescribed optimal SS target state at the end of each day. Implemented as a PF in the objective function.

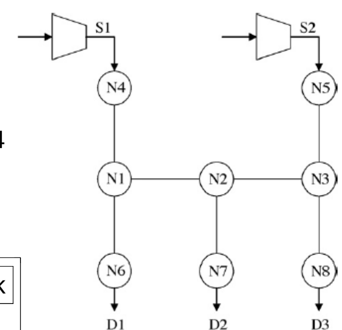
Scenarios & Simulation details

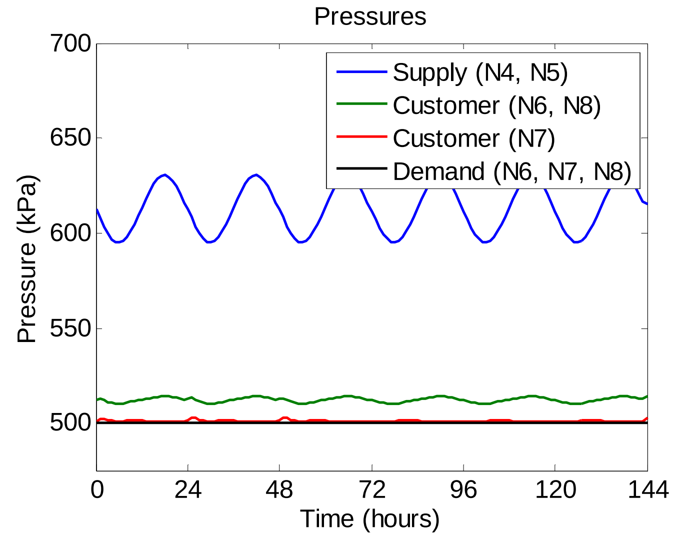
- Gas demand: Average of 24 kg/h. Sinusoidal with a period of 24-h and 5% amplitude of oscillations.
- Contract pressure: 500 kPa.
- Length of horizon (T_P) : 2 days.
- Time discretization: 1-h.
- NLP size:
  - Equations: 12,066
  - Variables: 12,302
- Solver used: IPOPT (Interior point Optimizer for large scale NLPs).

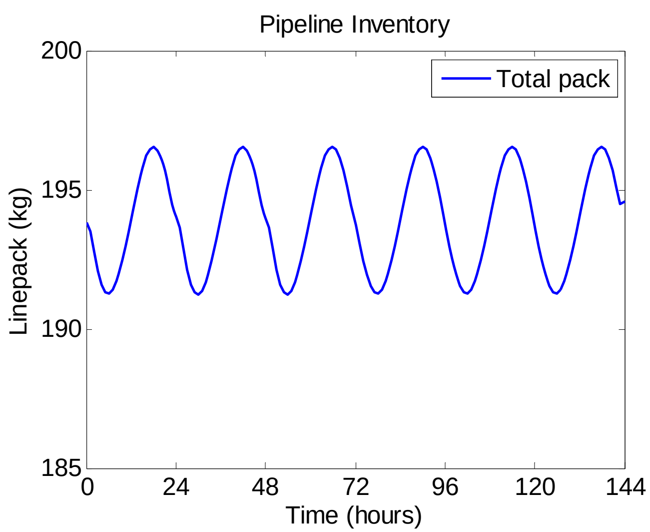


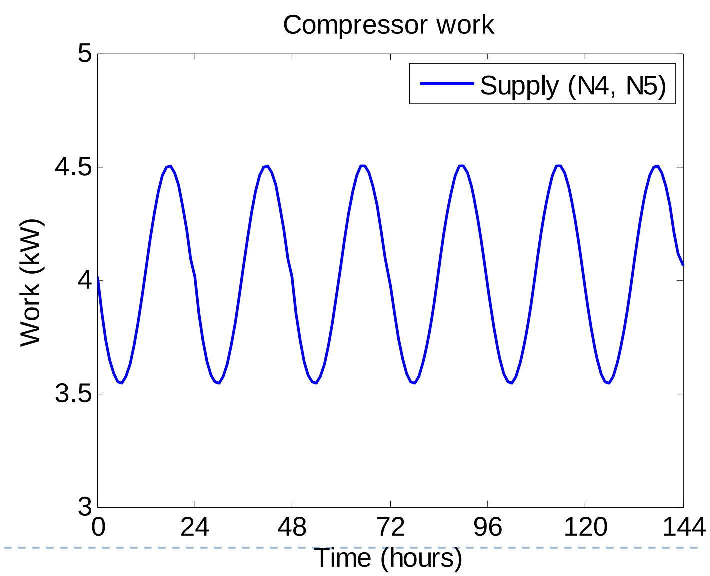
Case study 1 – Energy minimization



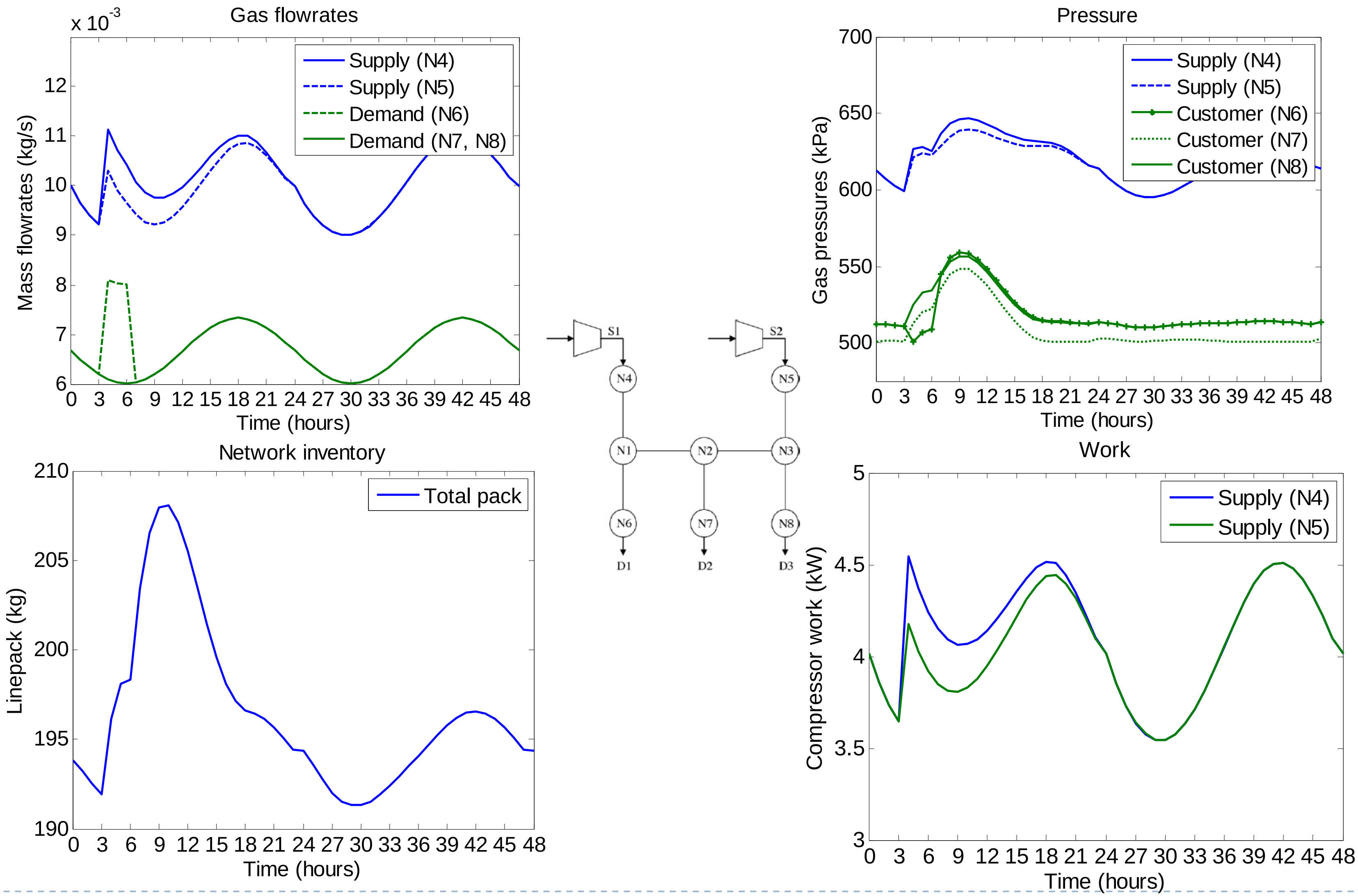








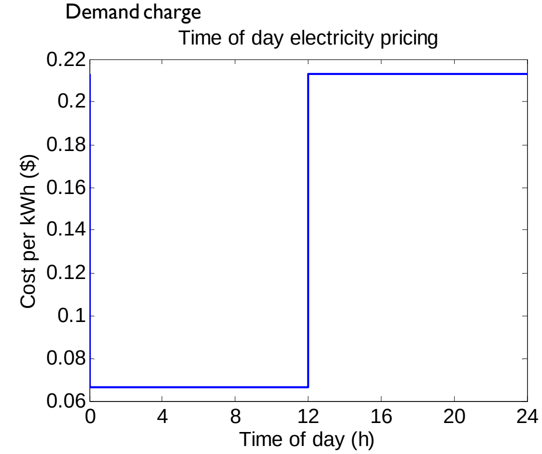
Case study 2 – Energy minimization w/ Disturbance



Electricity pricing

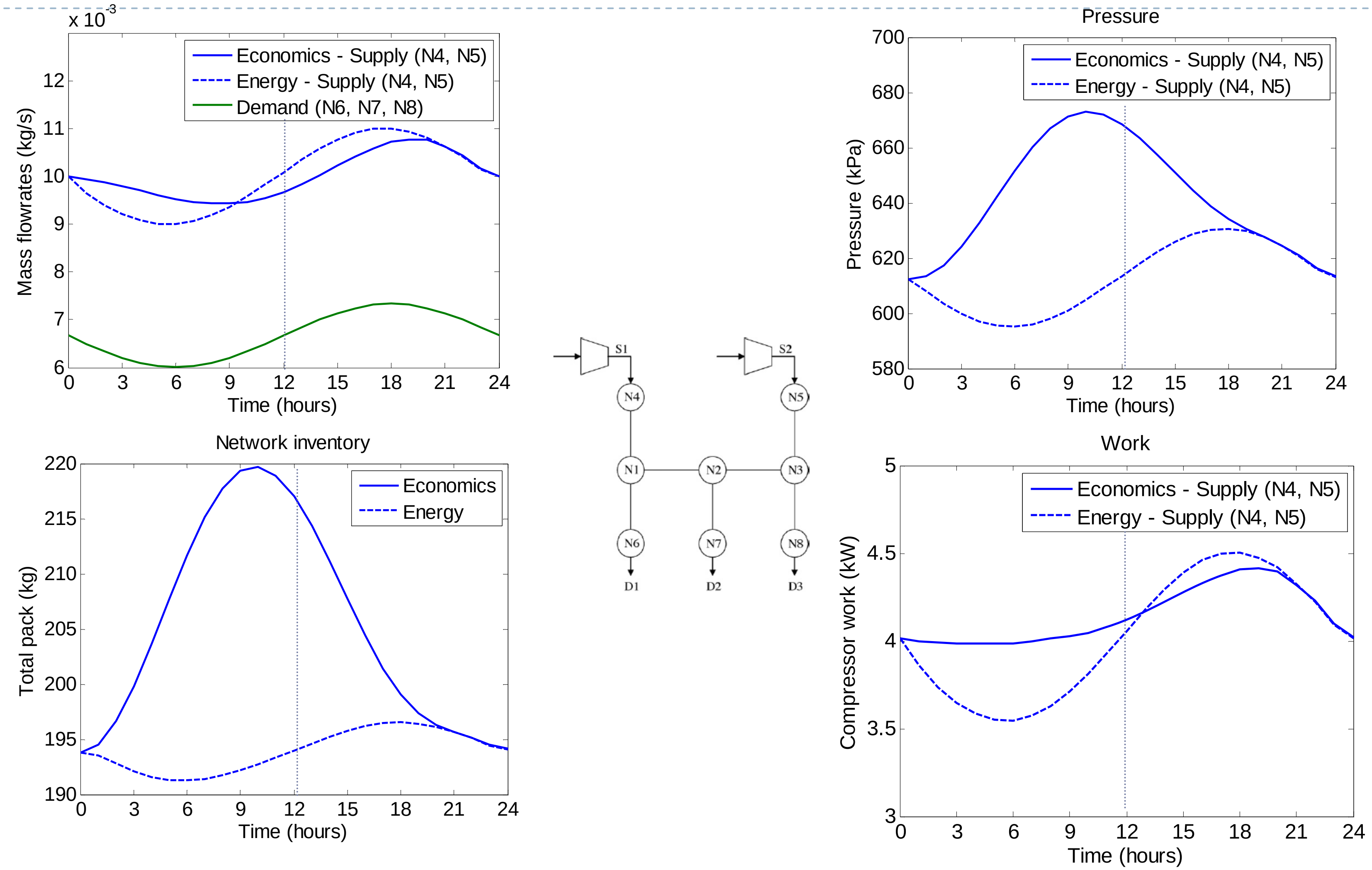
- Complex energy pricing schemes: Electricity prices vary through the day and consumers are encouraged to use more when power is cheaper.
  - *Time of day pricing*: Two 12-h time periods. Cheaper electricity during off-peak periods. Variations are seasonal.
  - *Day ahead pricing*: Hourly electricity prices decided at the beginning of the day.
  - *Real-time pricing*: Prices vary real-time (hourly) based on spot-market.
- Objective function modified to include economics:

$$J(t_0) = \underbrace{\sum_{s\in S} \int_{t=t_0}^{t=t_0+T_P} Power_{s,t}\mathbf{Cost_{s,t}}dt}_{\text{Energy charge}} + \underbrace{\rho \sum_{s\in S} \sum_{t\in T\setminus\{t_0\}} (Power_{s,t} - Power_{s,t-1})^2}_{\text{Smoothing term}} + \underbrace{c_d \max_t (\sum_{s\in S} Power_{s,t})}_{\text{Demand charge}}$$



- Time of day pricing data (SRP Utility):
  - Off-peak: $Cost_t$ = \$0.0662 per kWh
  - On-peak: $Cost_t$ = \$0.2130 per kWh
  - Demand charge coefficient (c_d) = 0.1435

Case study 3 – Economics



- Cost savings of 1.80% over a month due to flattening of work profiles & reduction of demand charge though 2.91% more work is done.

Conclusions

Pipeline optimization & Control:

- Moving horizon control scheme developed for optimal operation of pipelines – handles varying demand forecasts, inventory depletion, satisfies all constraints.
- State-of-art NLP solvers : ~10 secs of CPU time per moving horizon iteration.

Control theory:

- Stability of Economically-oriented NMPC with Periodic Constraints [Huang & Biegler, 2010]
  - With long enough prediction horizon and locally unique cyclic SS, stability can be proved for an equality terminal constraint.
  - Lyapunov function derived by subtracting the cyclic SS from the states.

Future:

- Test on a larger network.
- State estimation and feedback for plant-model mismatch.

Thank you

Questions & Comments?