Reliable Production Planning of Integrated Sites Under Uncertainty

Bruno A. Calfa, Ignacio E. Grossmann

Department of Chemical Engineering

Carnegie Mellon University

Pittsburgh, PA 15213

Anshul Agarwal, John M. Wassick, Scott J. Bury

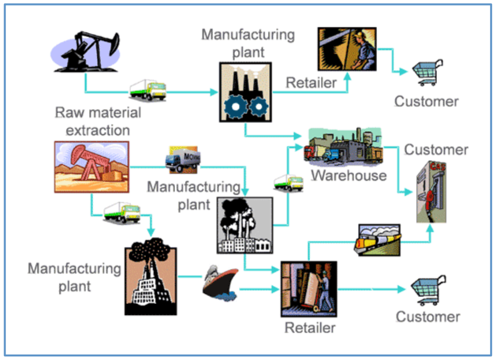
The Dow Chemical Company

Midland, MI and Freeport, TX

March 13h, 2013

Motivation

- **Enterprise-Wide Optimization (EWO)**
  - Optimize the operations of supply, manufacturing and distribution activities of a company

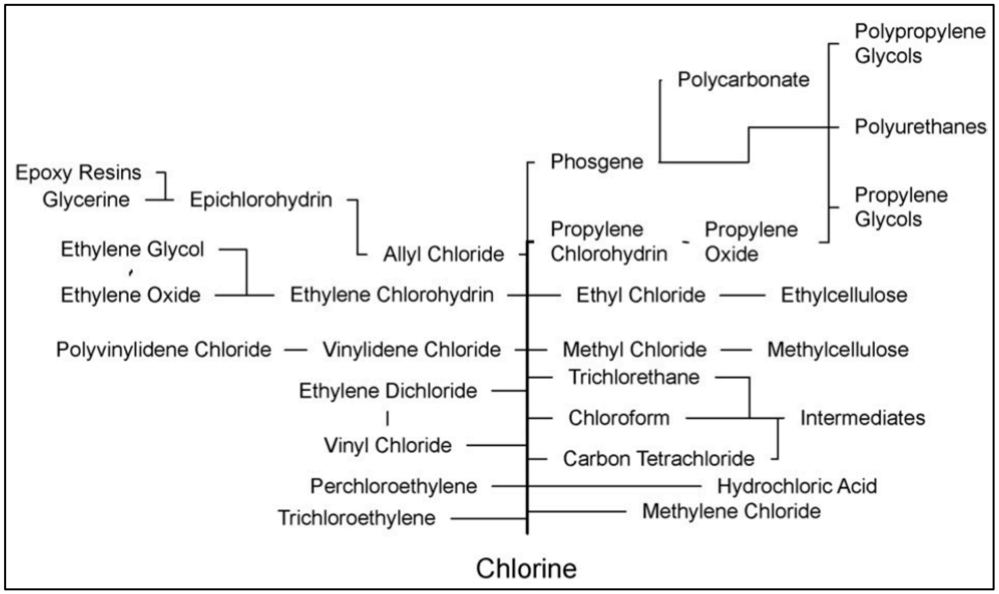


http://egon.cheme.cmu.edu/ewocp/

- Optimization models contain parameters that may be uncertain
- Solution to the deterministic model for average values of uncertain parameters may yield infeasible production plans when implemented in practice

Industrial Problem

Dow's Chlorine Envelope



Wassick (2009)

- **Envelope**
  - Network of integrated sites that produce basic chemical feedstocks (chlorine) and its derivatives
  - Majority of the chemicals above are produced in separate, dedicated production plants
  - Many plants are situated in the same geographical location
  - Network is supplemented by material imported from either external suppliers or other integrated sites

Problem Statement

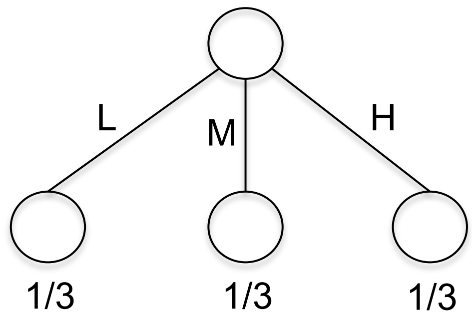
- **Given**
  - Future monthly demand by regions
  - Production capacity of each plant and its production costs
  - Availability of key raw material
  - Inventory holding costs, inventory capacity, and initial inventory

- **Objectives**
  - Set the production quota for all plants in the envelope
  - Satisfy customer demand across all markets
  - Achieve mass balance across the envelope
  - Plan the purchase of key intermediates

- **Enhancements Sought**
  - Account for unanticipated or uncertain events
    - First test case studied: raw material availability

Representation of Uncertainty

- A natural way of accounting for uncertainty is through a scenario tree (discrete probability distributions)
  - **Nodes:** state of the problem at a particular instant when decisions are made
  - **Branches:** different realizations of the random variables



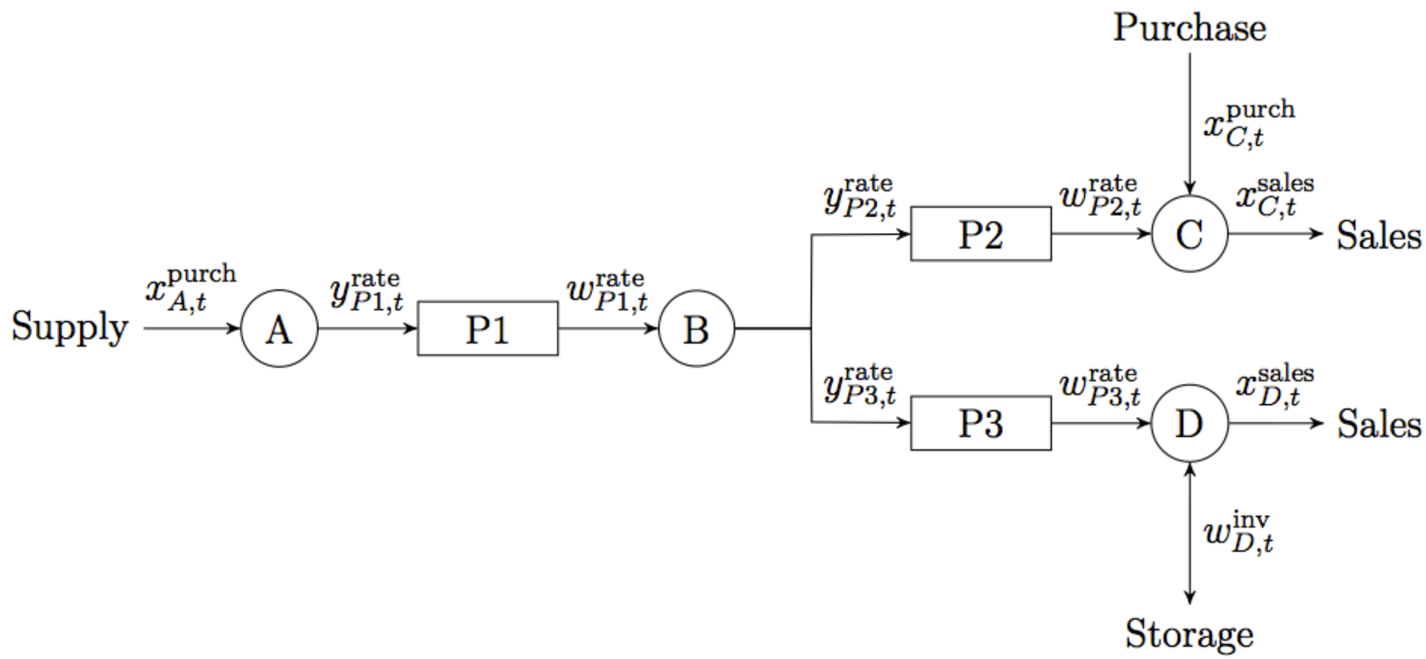
- General case: multi-stage stochastic programming problem

Stage 2 | Stage H

$$
\begin{aligned}
\min z = c^1 x^1 + E_{\xi^2}\big[\min c^2(\omega) x^2(\omega^2) + \cdots + E_{\xi^H}[\min c^H(\omega) x^H(\omega^H)] \cdots\big] \\
\text{s. t. } W^1 x^1 = h^1, \\
T^1(\omega) x^1 + W^2 x^2(\omega^2) = h^2(\omega), \\
\vdots \\
T^{H-1}(\omega) x^{H-1}(\omega^{H-1}) + W^H x^H(\omega^H) = h^H(\omega), \\
x^1 \geq 0;\ x^t(\omega^t) \geq 0,\ t = 2, \ldots, H;
\end{aligned}
$$

Motivating Example

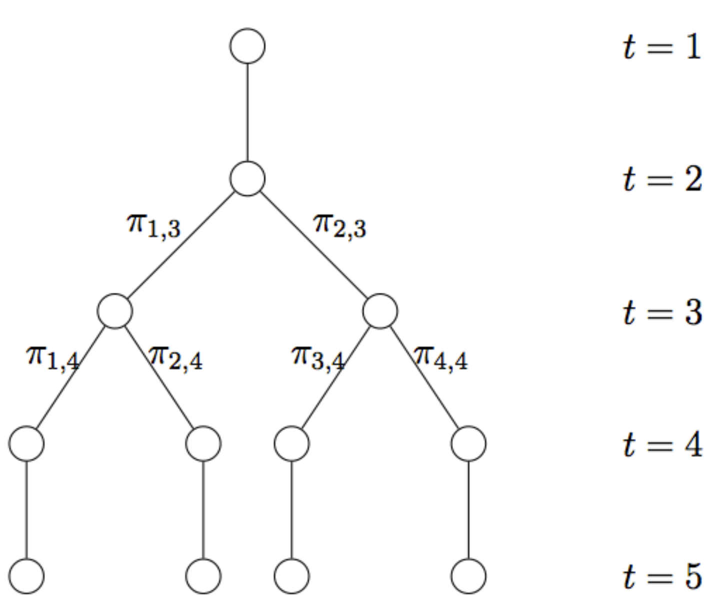
- Small-scale representation of the actual problem (1 site only)

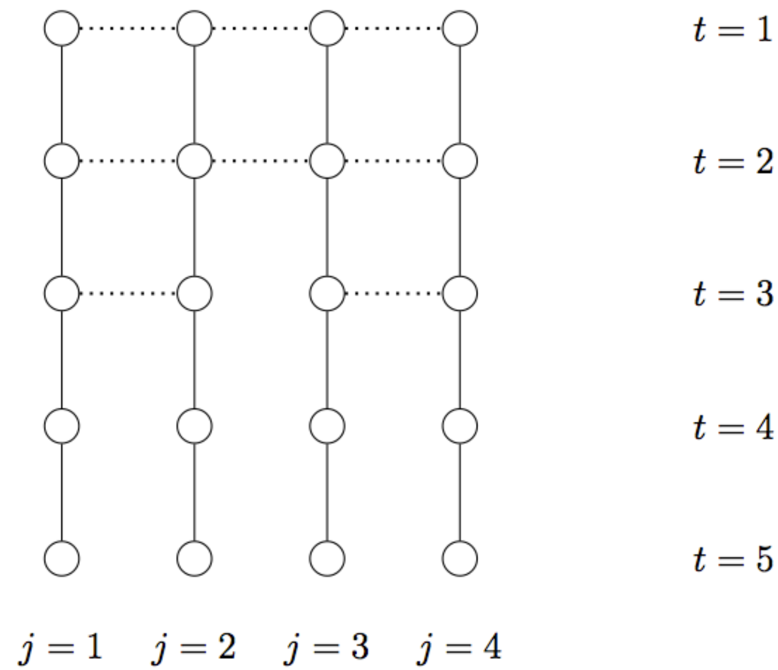


- 1 raw material (A), 1 intermediate product (B), two finished products (C and D), 1 site
- Only D can be stored and C can be purchased from elsewhere (may simulate inter-site transfers)

Motivating Example: Test Case

- Uncertain demand





- Modeled in AIMMS 3.12 and solved with Gurobi 5.0
- Hardware: Dell Optiplex 990 machine with 4 Intel® Core™ i7-2600 3.40 GHz CPUs (total 8 threads) and 8 GB of RAM
- Operating System: Windows 7

Motivating Example: Results

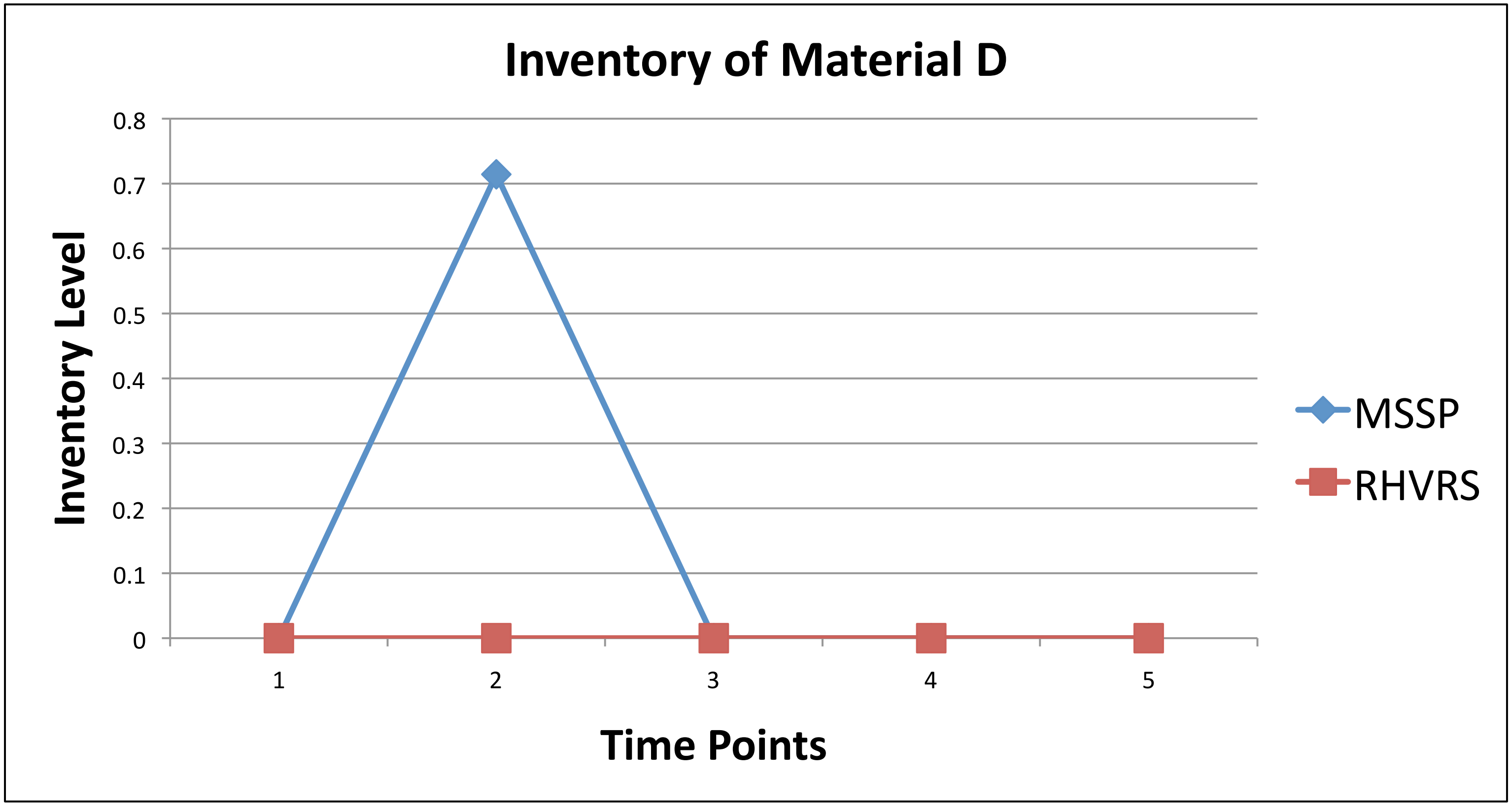
- Profits [$]

Deterministic	MS Stochastic	RHVRS	Stochastic Gain
92.92	88.07	64.87	22.92

- Do not directly compare the profits of deterministic and stochastic optimal objective function values
  - Stochastic model accounts for different scenarios when computing the values of the decision variables. Deterministic solution applied with uncertainties is infeasible.
- Actual profit of deterministic approach under an uncertain environment: RHVRS = $64.87, therefore RHVSS = $22.92
- RHVSS stands for Rolling Horizon Value of Stochastic Solution, which indicates the expected gain from solving a stochastic model rather than its deterministic counterpart, in which random parameters are replaced by their expected values

Maggioni, Allevi & Bertocchi (2012)

Motivating Example: Results

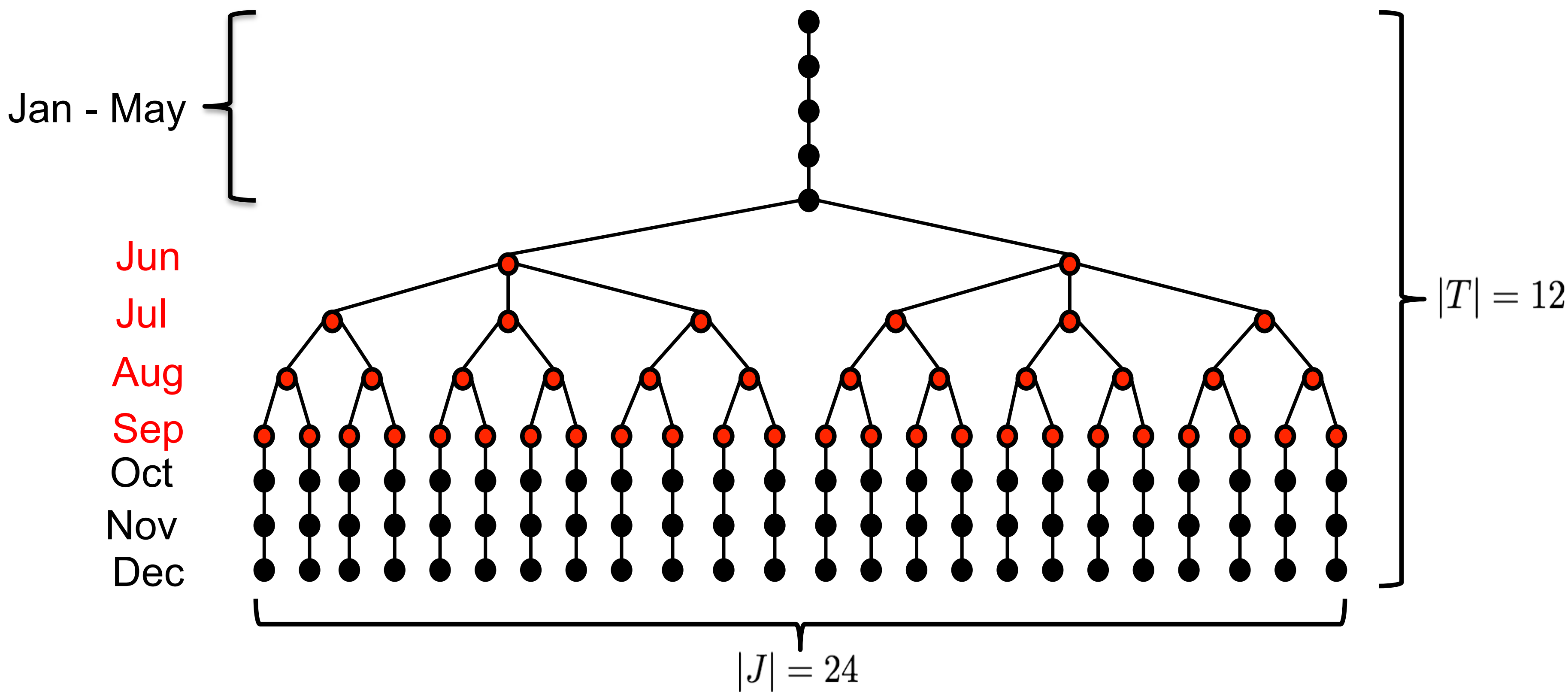


- RHVRS solution uses no inventory of D in the first stage as well as for the remaining time periods, whereas the MSSP solution indicates that some inventory of D has to be kept in order to cope with possible high demands in an uncertain environment.

Case Studies of Actual Problem

- Investigate the impact of uncertain availability of a key raw material
- Uncertain data given in a form of scenarios with shortage or no shortage of the key raw material
- All cases: 4 sites, 3 regions, 31 facilities, 33 materials, 15 products, and 9 raw materials
- **Case 1 (NS)**
  - Deterministic model
  - No shortage of the key raw material for any time period
- **Case 2 (AS)**
  - Deterministic model
  - Availability of the key raw material is averaged over all scenarios for each time period
- **Case 3 (ST)**
  - Stochastic model
  - Framework to directly account for uncertainty in the availability

Scenario Tree



- 12 time periods (months)
- 24 scenarios (discrete probabilities were given)

Total NPV [MM $]

Deterministic		MS Stochastic	RHVRS*	Stochastic Gain
Average	No Shortage			
851.06	851.88	850.64	798.49	52.15

* Only fixed run rates of a major plant in a major site

- Comments
  - As expected, higher NPV for No Shortage case (absence of uncertainty)
  - Actual NPV of deterministic approach under an uncertain environment = \$798.49 MM, therefore RHVSS = \$52.15 MM
  - First-stage decisions and state variables, run rates and inventory for example, had different levels between Stochastic and RHVRS (average case as reference), which also emphasizes the different production plans obtained

Conclusions and Future Work

- Stochastic Programming allows incorporating uncertainty explicitly into the optimization model
- The expected gain from solving the MSSP problem indicates that a deterministic model performs worse in an uncertain environment
- Work in progress:
  - Investigating scenario tree generation methods that take into account statistical properties of historical data

References

- Birge, J. R., & F. Louveaux. (1997). *Introduction to Stochastic Programming*. Springer-Verlag, New York.
- Maggioni, F., Allevi, E., & Bertocchi, M. (2012). *Measures of Information in Multi-Stage Stochastic Programming*. **Stochastic Programming E-Print Series***.* http://edoc.hu-berlin.de/docviews/abstract.php?id=39236.
- Ruszczyński, A. (1997). *Decomposition methods in stochastic programming*. **Mathematical Programming**, *79*(1-3), 333–353.
- Wassick, J. M. (2009). *Enterprise-wide optimization in an integrated chemical complex*. **Computers & Chemical Engineering**, 33(12), 1950–1963.