Inventory optimization in process networks

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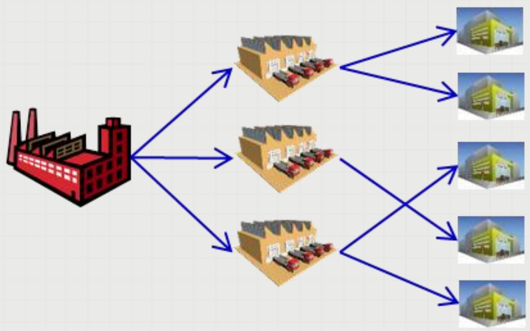
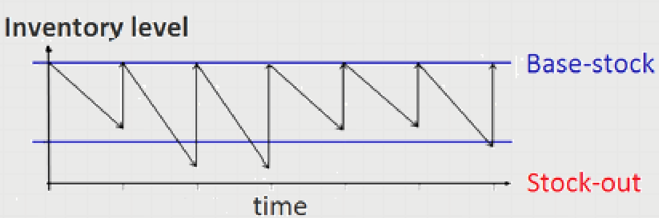
Motivation

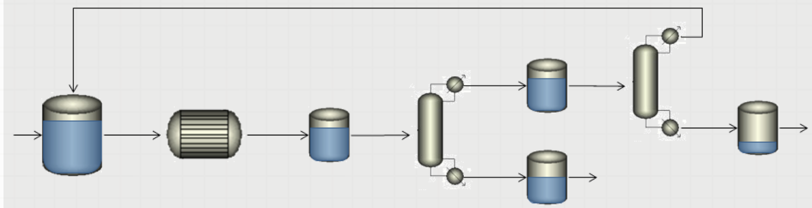
Supply Chain:



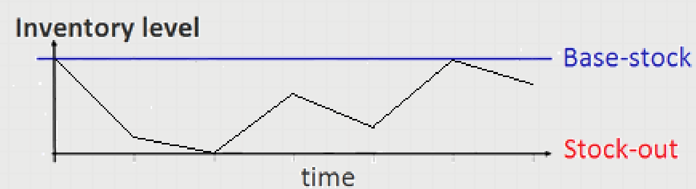
- Network of distant locations
- Transportation cost plays critical role
- Inventory availability is constraint by lead-times
- Inventory replenishment is instantaneous



Process Network:



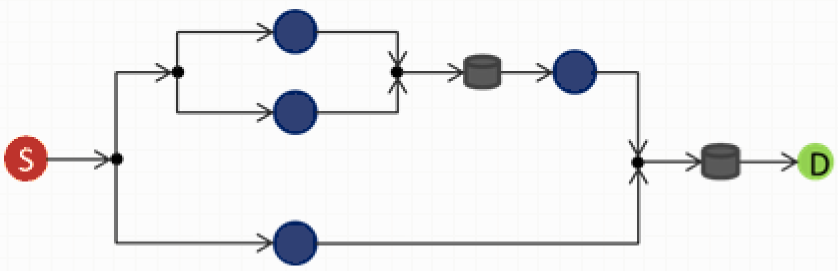
- Network of complex integrated operations
- Transportation is not critical
- Inventory availability is constrained by production capacity
- Continuous replenishment



Inventory Optimization in Process Networks

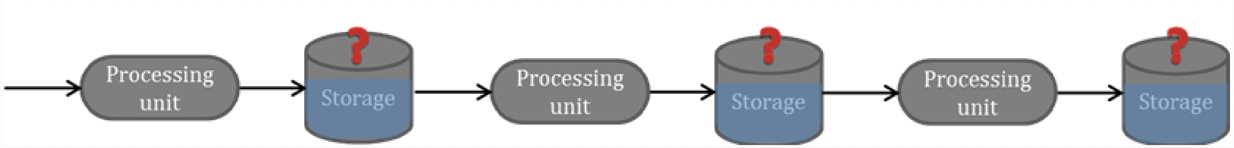
Given:

- A network with supply, processing, and demand nodes
- A discrete time horizon
- Probabilistic description of future supply, processing capacity, and demand
- Storage units



Minimize cost by:

- Establishing production rates at processing units
- Determining location and amount of inventories
- Balancing average inventory levels and stockouts



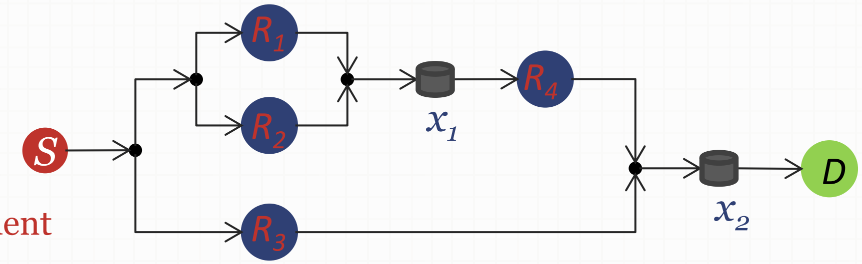
Stochastic Decision Processes

Conditions of the network are established by:

- Available supply (S_t)
- Production capacities ($R_{i,t}$)
- Demand (D_t)
- Stored inventories (x_i)

} Uncertain (supply, production capacities, demand)

} Decision and history dependent (stored inventories)

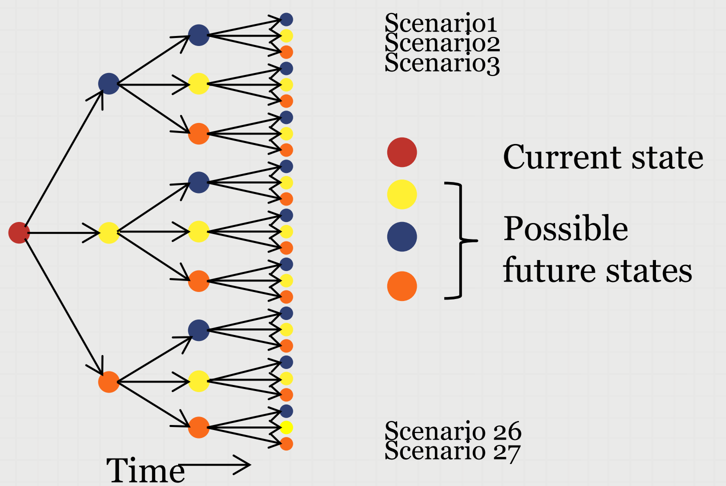


System dynamics:

Future state · Current state · Actions · Uncertainty

$$x_{t+1} = x_t + u_t + d_t$$

For fixed actions (u_t), the system evolves as a stochastic process:



Stochastic programming framework:

- Find optimal actions
- Optimize expected value

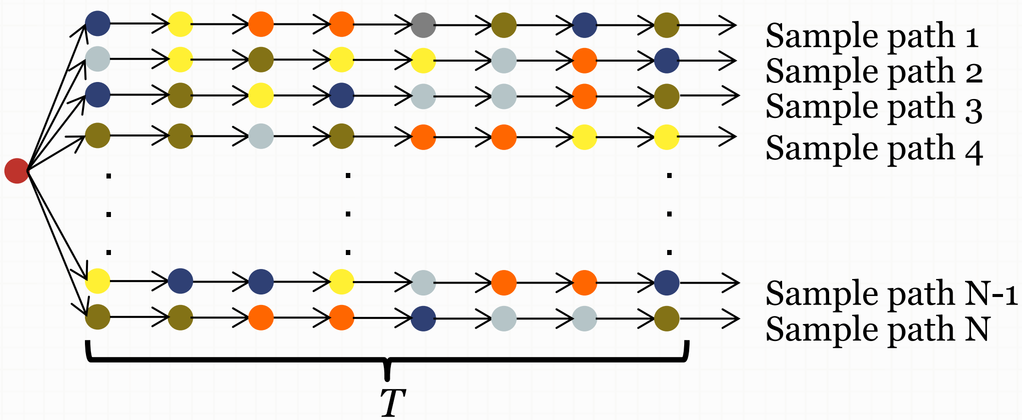
But…

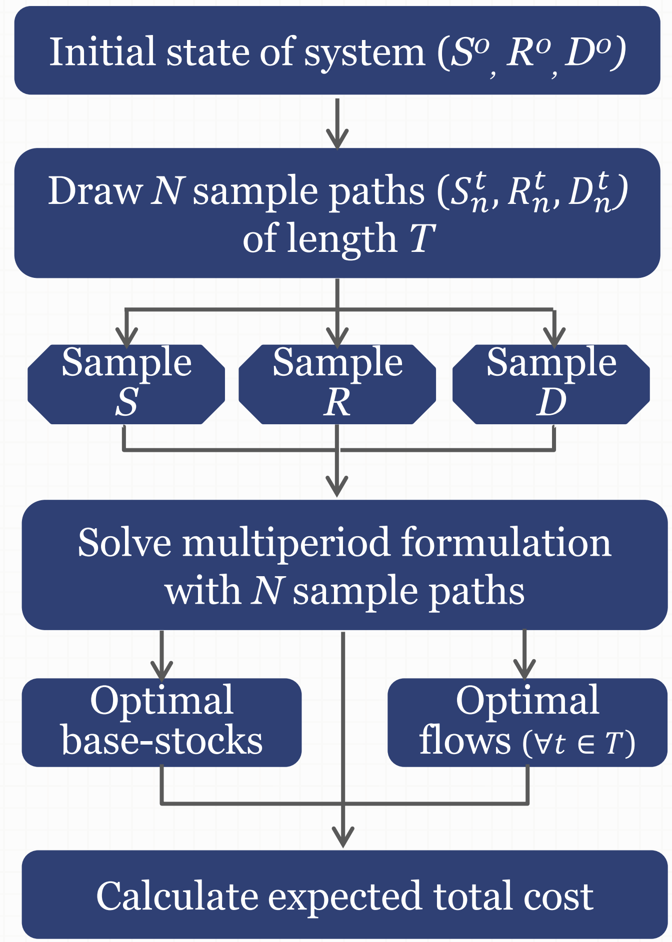
- Continuous state-space
- Arbitrary distribution of uncertain parameters (continuous, unbounded, autocorrelated, etc.)
- Probability distribution of states (inventory levels) is unknown and endogenous

Sample-path Optimization

Discrete-time samples of independent random parameters during planning horizon (0,T):

- Available supply: S_t
- Production capacities: $R_{i,t}$
- Demand rates: D_t





- Formulation approximates the optimal solution based on sample paths
- Each sample path is a deterministic trajectory of the uncertain parameters

Inventory Management Strategies

Two-stage Approach

$$\min_{u_{s,t}\in U} \mathbb{E}_{\omega\in\Omega}\left[\sum_{t=0}^{T} c_t x_{\omega,t}\right]$$

$$s.t. \quad x_{\omega,t+1} = x_{\omega,t} + u_{\omega,t} + d_{\omega,t} \quad \forall\, t \in T, \omega \in S$$

$$x_{\omega,t} \in X$$

- Optimize directly over recourse actions ($u_{s,t}$)

Decision-rule Approach

$$\min_{b_t\in B} \mathbb{E}_{\omega\in\Omega}\left[\sum_{t=0}^{T} c_t x_{\omega,t}\right]$$

$$s.t. \quad x_{\omega,t+1} = x_{\omega,t} + u_{\omega,t} + d_{\omega,t} \quad \forall\, t \in T, \omega \in S$$

$$u_{\omega,t} = \pi(b_{t,}, x_{\omega,t})$$

$$x_{\omega,t} \in X$$

- Optimize parameters (b_t) of operating policy ($\pi(b_t, x_{\omega,t})$)

Base-stock policy:

1. Use resources to satisfy demand
2. Use resources to bring inventory to base-stock level (b_t)
3. Stop replenishing inventory at base-stock level (b_t)

Auxiliary processes:

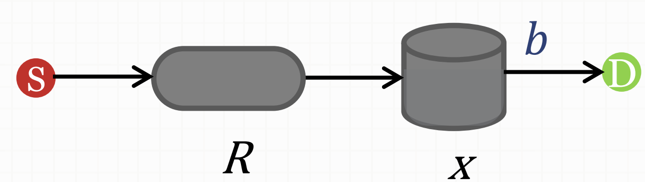
Throughput: $p_{\omega,t} = min(S_{\omega,t}, R_{\omega,t})$

Underutilization: $u_{\omega,t} = \begin{cases} 0 & if\ l < b \\ max(0, p_{\omega,t} - D_{\omega,t}) & if\ l = b \end{cases}$

Stock-outs: $so_{\omega,t} = \begin{cases} 0 & if\ l > 0 \\ max(0, D_{\omega,t} - p_{\omega,t}) & if\ l = 0 \end{cases}$

Mass balance:

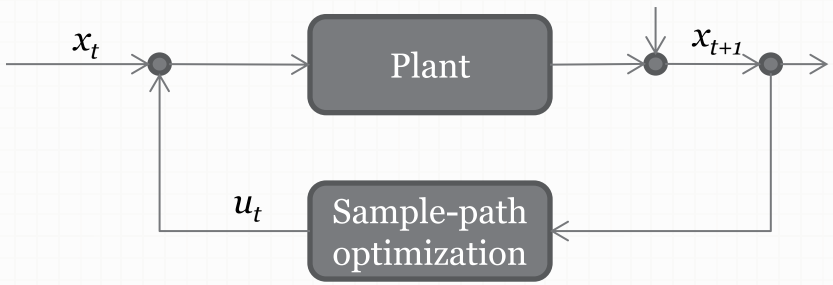
$$x_{\omega,t+1} - x_{\omega,t} = [p_{\omega,t} - u_{\omega,t}] - [D_{\omega,t} - so_{\omega,t}]$$

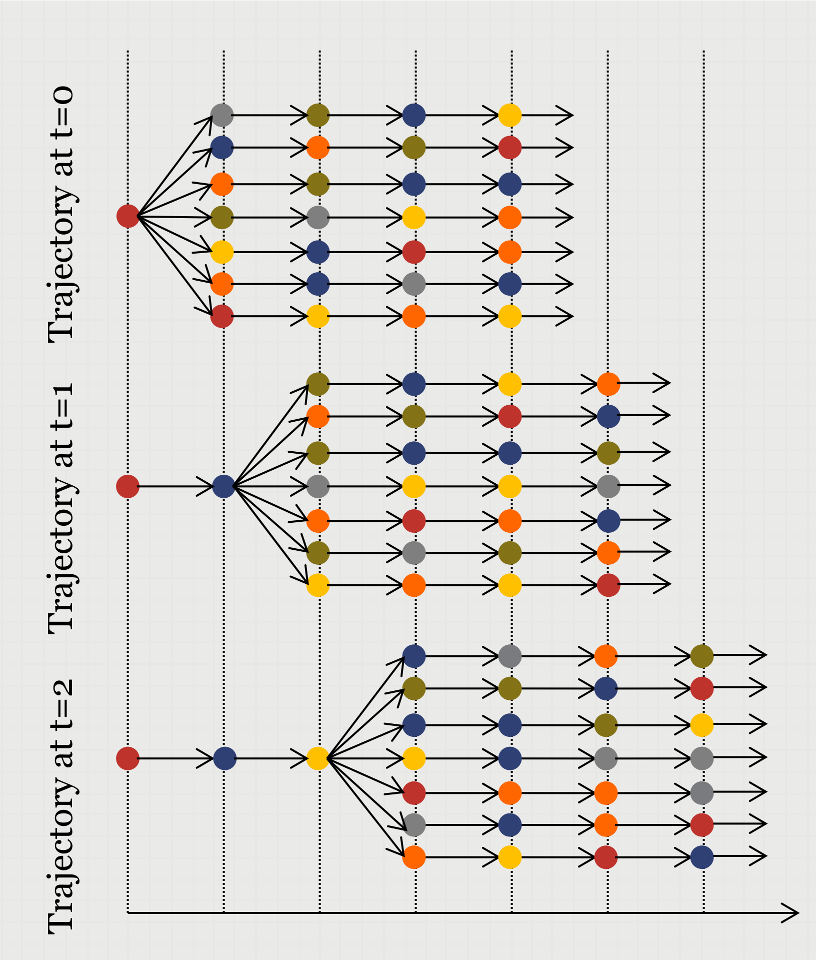


Evaluating Inventory Planning Strategies

Receding horizon:

Simulate the sequential implementation of optimal first-stage decisions





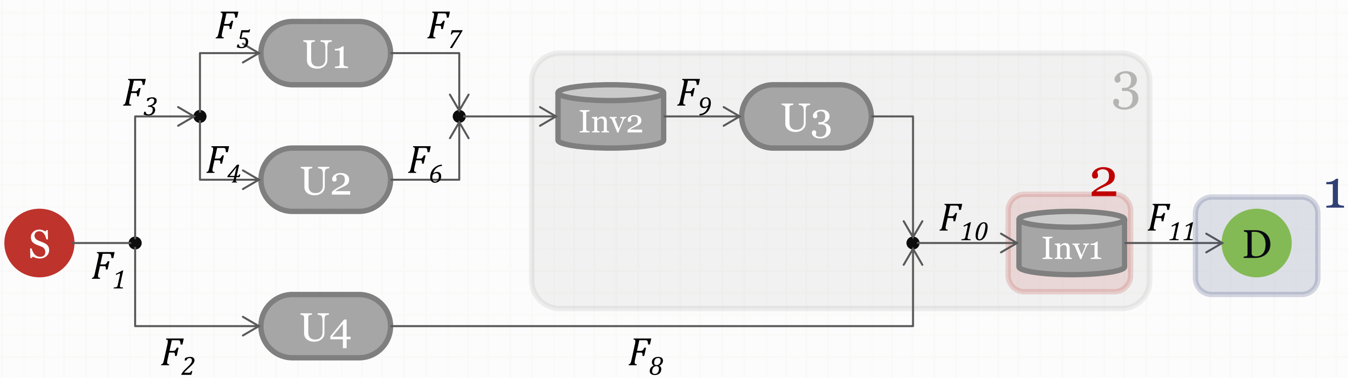
Algorithm:

1. Initial state (S^o, R^o, D^o, $cost^o=0$)
2. Draw N sample paths of length T
3. Solve sample-path optimization problem
4. Accumulate first-stage cost
5. End if the end of evaluation period is reached
   Else, update initial state and return to 2

Repeat implementation of the algorithm to estimate mean and variance of results

Example

Process network with random supply, random demand, and unit failures described as a Markov process

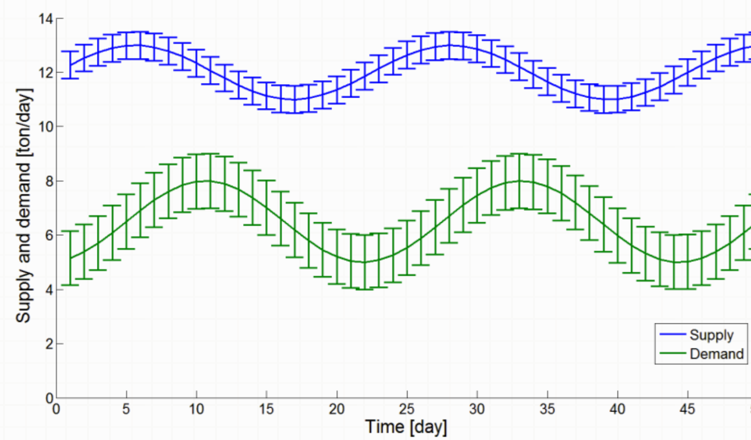


Resource priorities in multiechelon base-stock policy:

1. Demand
2. Inventory 1
3. Inventory 2

Data	U1	U2	U3	U4
Failure rate [1/day]:	0.011	0.021	0.017	0.030
Repair rate [1/day]:	0.200	0.200	0.200	0.200
Probability of operation:	$\pi_1 = 0.95$	$\pi_2 = 0.95$	$\pi_3 = 0.92$	$\pi_4 = 0.87$
Mass balance coefficients:	$\alpha_1 = 0.92$	$\alpha_2 = 0.90$	$\alpha_3 = 0.85$	$\alpha_4 = 0.75$
Processing capacity:	$R_1 = 5$	$R_2 = 5$	$R_3 = 7$	$R_4 = 9$

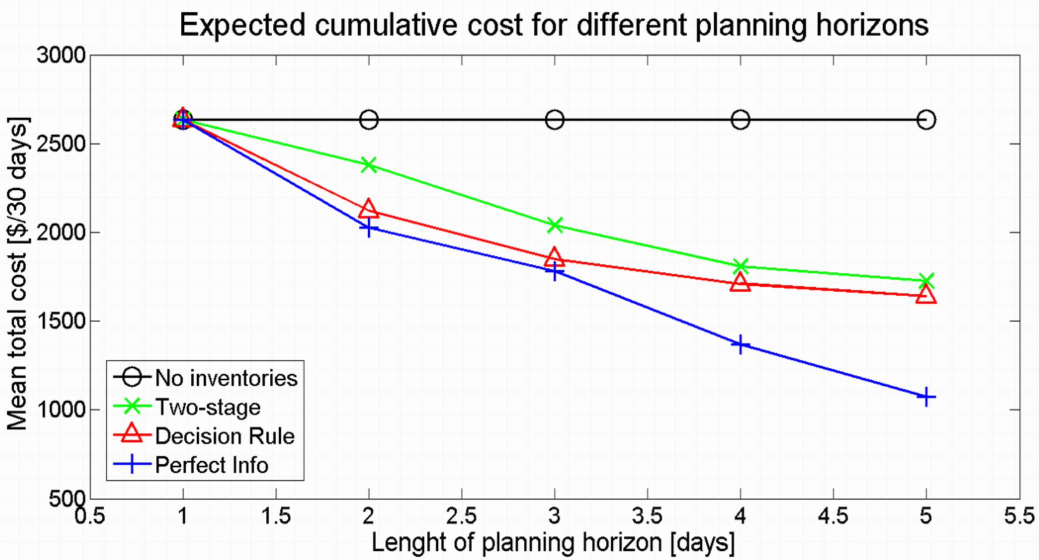
Cost coefficients:		
Inventory holding cost:	$h_1 = 5$ / (ton-day)	$h_2 = 10$ / (ton-day)
Backorder cost:	pen = \$100 / ton-day	



Example

Parameters of closed-loop simulations:

- 25 sample paths in each optimization problem
- Evaluate over horizon of 20 time periods
- 40 repetitions
- Planning horizons from 1 to 5 time periods



No inventories: no inventory planning

Two-stage: solves stochastic formulation with sample paths using two-stage stochastic programming

Decision rule: solves stochastic formulation with sample paths using multiechelon inventory policy

Perfect Info: deterministic problem for the realizations in the horizon

- Longer planning horizons yield better inventory management strategies
- Decision rule approach yields lower cost because it considers the distribution of uncertain parameters for decisions of all time periods

Analysis

Novelty:

- Approach for inventory planning in industrial networks
- Arbitrary distributions for the uncertain parameters
- Inventory policies are used as decision rules in sample-path optimization

Impact for industrial applications:

- Historical data and forecasts can be used for investment planning
- Cost reductions achieved through optimization
- Solutions are easy to implement in practice