Inventory Optimization for Industrial Network Flexibility

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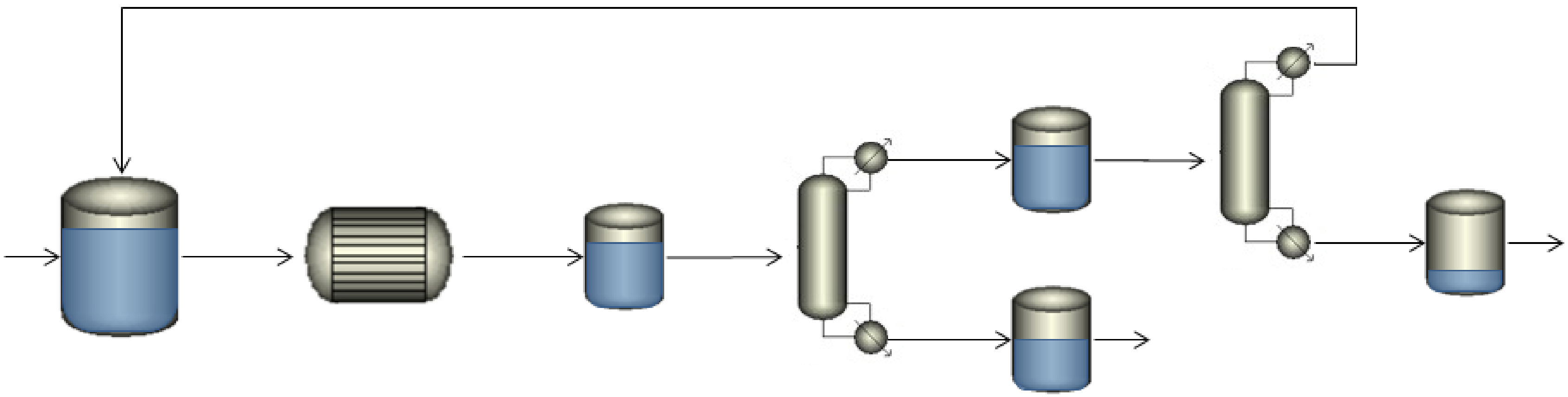
Motivation

Process networks describe the operation of chemical plants

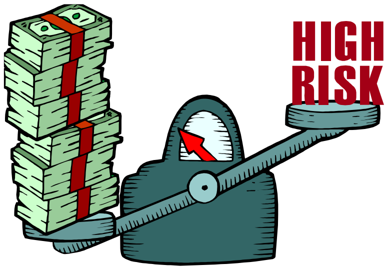


Inventories are necessary because of process uncertainty:

- Raw material storage tanks hedge against supply variability
- Intermediate storage tanks hedge against production rates variability
- Finished product inventories hedge against demand variability

Holding inventory is expensive!

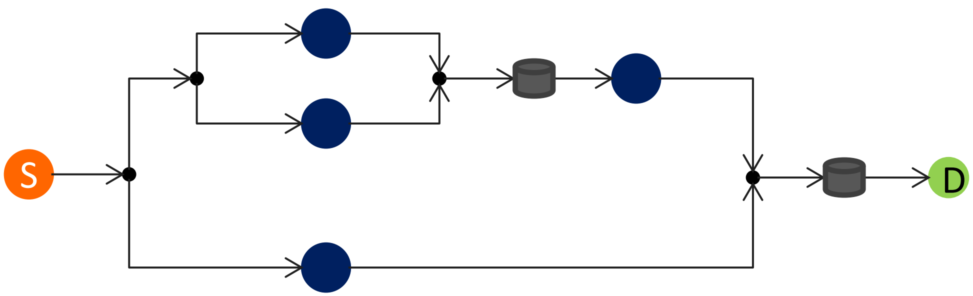
Need to trade-off between inventory and stock-out cost



Problem Statement

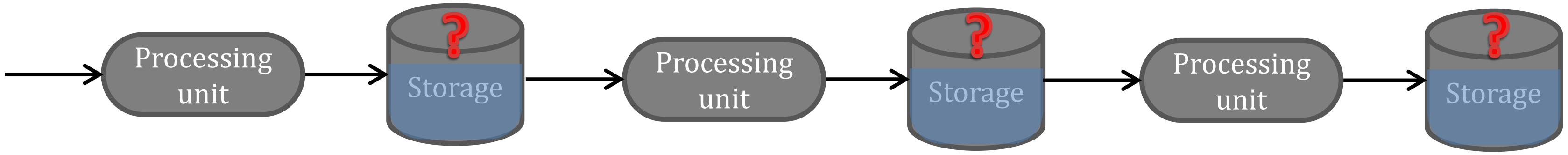
Given:

- A process network with continuous processing units
- Uncertain supply, processing rates, and demand with known probabilistic description
- Candidate locations for storage units

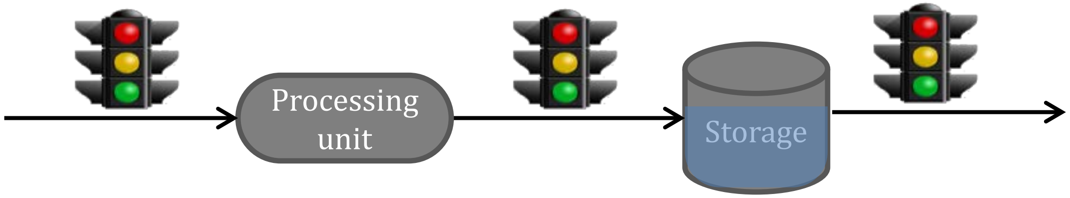


Find optimal trade-off between inventory cost and service level by:

- Determine at which stages of the process to hold inventory



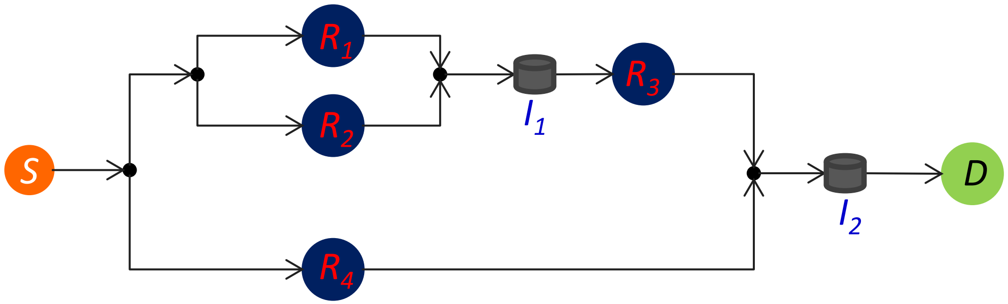
- Establish rules to deplete and replenish inventories



Modeling Inventories in Process Networks

Operating policies in discrete-event simulation:

Rules to establish when inventories are depleted, replenished, and their priorities



In every time-period during a time-horizon (0, T):

1. Satisfy demands (*D*) according to priorities using available supply (*S*) and production capacities (*R*)

   ➡ Update D, S, R_1, R_2, and R_3

2. Satisfy demands (*D*) according to priorities using inventories (*I*)

   ➡ Update D, S, R_1, R_2, R_3, I_1, and I_2

3. Replenish inventories according to priorities using left over supply (*S*) and production capacities (*R*)

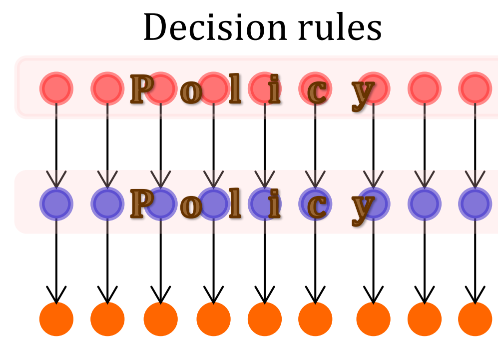
   ➡ Update S, R_1, R_2, R_3, I_1, and I_2

4. Stop inventory replenishments at base-stock levels (*B*)
5. Repeat for next time-period

Inventory decisions connect the state of consecutive **time-periods**

Sequential Decision-making

Modeling decisions without foreseeing the future



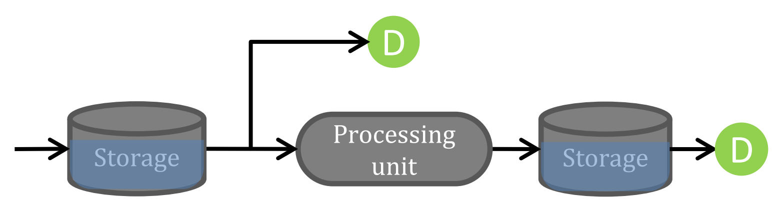
Policy: rules to determine decisions as functions of the state of the system

- Decisions rules do not change as the information reveals
- Optimal policy ➡ Optimal decisions ➡ Optimal objective value

Anticipativity issues in inventory management

Inventories might be accumulated anticipating adverse realizations of random parameters:

- Use raw material inventories to anticipate price increases that have not been revealed ✗
- Make-to-stock products anticipating unexpected increases in demand that will be realized ✗

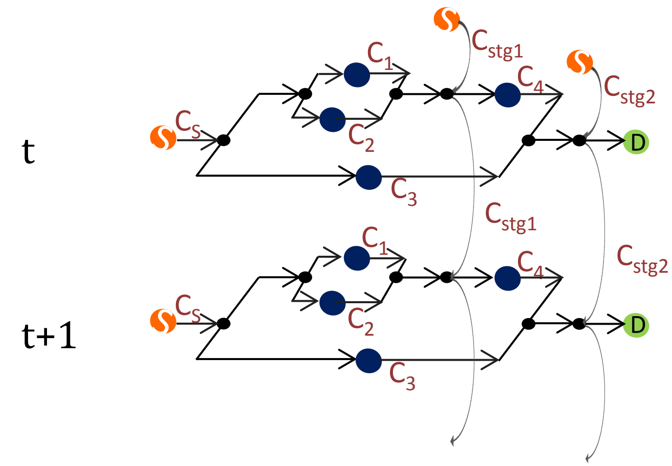


Multi-period Formulation (MILP)

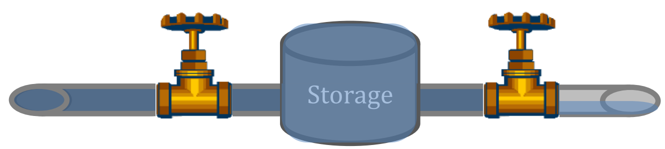
Minimize: expected inventory cost + expected lost sales cost

Subject to:

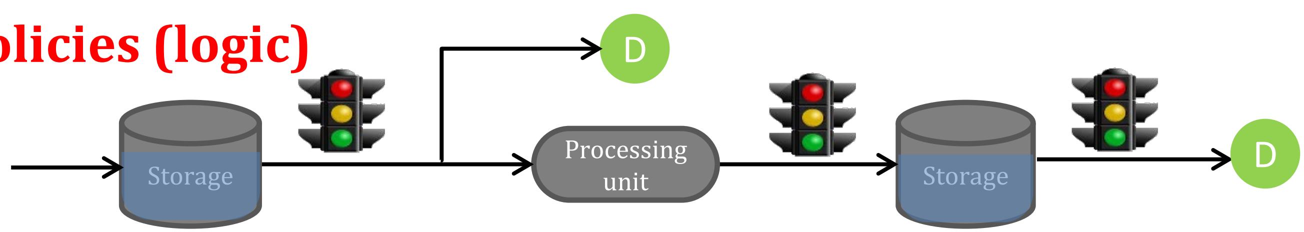
- Mass balances



- Capacity constraints



- **Operating policies (logic)**



- Bounds: Base-stock levels ($B_i \geq 0$)
  Flows ($F_j \geq 0$)

Solution Strategy

Challenges:

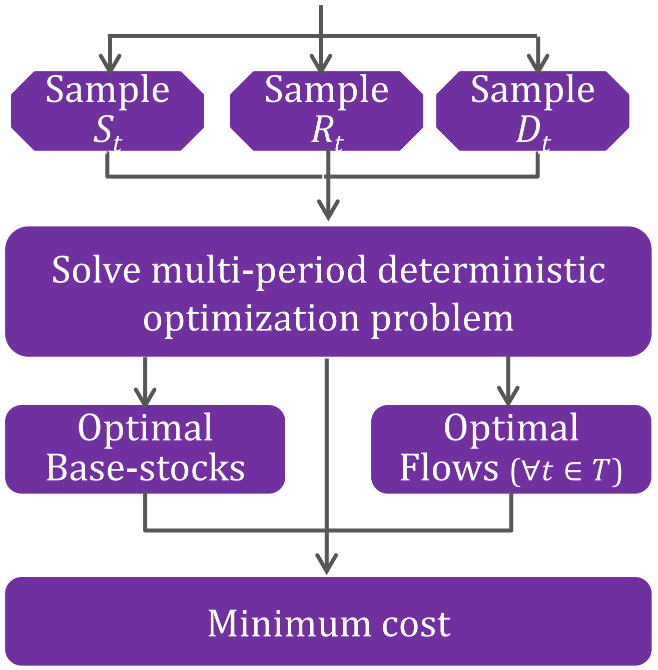
- Inventory levels depend on the history of random parameters
- Arbitrary distributions for uncertain parameters

Solution strategy: Sample-path optimization

Discrete-time samples of random parameters in a time-interval (0,T):

- Available supply: S_t
- Maximum processing rates: R_t
- Demand rates: D_t

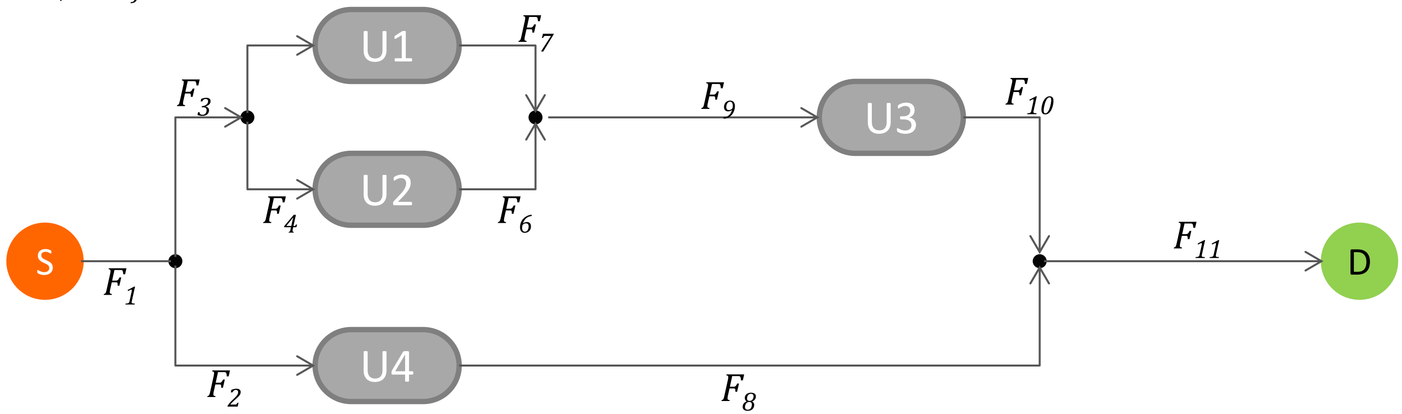
For **long time-horizons**, the solution is a **good estimate** of the optimal solution



Illustrative Example

Process network with failures (no storage):

(Straub & Grossmann, 1990)



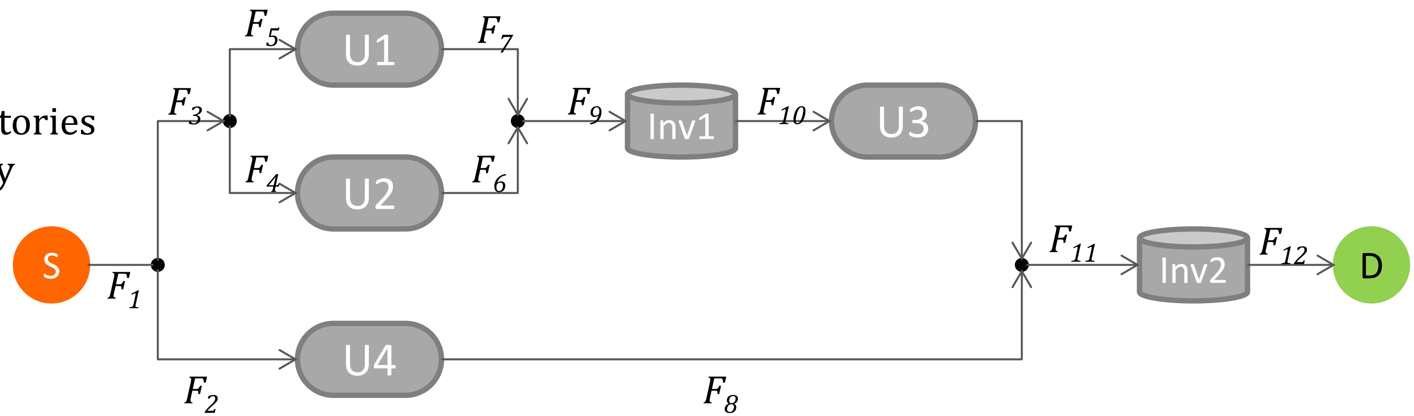
Data				
Supply: $S \sim N(12,1)$ ton/day		Demand: $D \sim N(7,1)$ ton/day		
Probability of operation:	$\pi_1 = 0.95$	$\pi_2 = 0.95$	$\pi_3 = 0.92$	$\pi_4 = 0.87$
Mass balance coefficients:	$\alpha_1 = 0.92$	$\alpha_2 = 0.90$	$\alpha_3 = 0.85$	$\alpha_4 = 0.75$
Processing capacity:	$R_1 = 5$	$R_2 = 5$	$R_3 = 7$	$R_4 = 9$

Probability of satisfying demand	
Expected stochastic flexibility:	**0.81**

Illustrative Example

Process network with failures

Inventory rule:
only replenish inventories when demand is fully satisfied



Cost coefficients:				
Production costs:	$\mu_1 = 0$	$\mu_2 = 0$	$\mu_3 = 0$	$\mu_4 = 0$
Inventory holding cost:	$h_1 = 5$ / (ton-day)		$h_2 = 10$ / (ton-day)	
Penalty for unmet demand:	$p = \$25$ / ton			

Optimal results on 25,000 time-periods

	Base-stock 1	Base-stock 2	Exp. holding cost	Exp. penalty cost	Exp. total cost	Exp. Stoch. flexibility
No storage	0	0	\$ 0 / day	\$ 7.25 / day	\$ 7.25 / day	**0.81**
Storage	7.04	7.60	\$ 2.81 / day	\$ 1.25 / day	\$ 4.06 / day	**0.97**

Expected flexibility **increases** from 0.81 to 0.97

Conclusions

Novelty:

- Specialized approach for inventory management in process networks
- Arbitrary distributions for the uncertain parameters
- Discrete-event simulation principles in an optimization framework
- Operating policies are modeled with logic constraints

Impact for industrial applications:

- Historical data can be used directly for inventory optimization
- Cost reductions can be achieved by implementing optimal strategies for inventory management in process networks
- Process resilience can be improved with the addition of intermediate storage units