Multi-Stage Scenario Tree Generation via Statistical Property Matching

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Motivation

- **Uncertainty in Optimization**
  - In reality, the decision-making process of an enterprise involves multiple sources of uncertainty
    - Product demand, selling price
    - Production yield
    - Unplanned plant shutdown
  - Stochastic Programming with Recourse is a powerful modeling framework to explicitly account for uncertainty and provide corrective actions
  - Practical issues
    - Unknown "true" distributions (scenarios usually assumed to be known)
    - Need meaningful scenarios and probabilities
    - Use of historical data and forecasts (data-driven approach)
    - Quality of scenario tree ➔ quality of the decisions

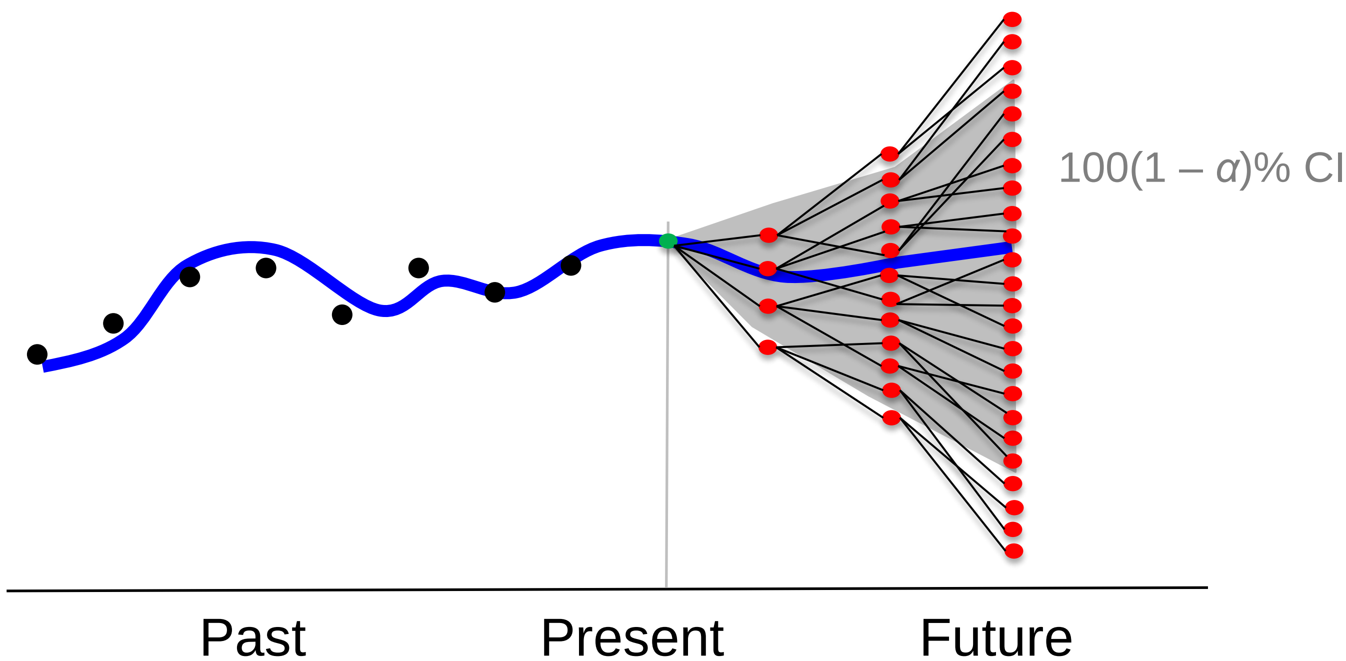
Problem Statement

- **Given**
  - Topology of the network of chemical plants
  - Deterministic multi-period production planning model with all its parameters (costs, capacities etc.)
  - Historical data and forecasts of product demand (uncertain parameter)

- **Objectives**
  - Generate multi-stage scenario tree from historical and forecast data
  - Formulate the multi-stage stochastic production planning model
  - Assess the quality of the stochastic solution with Monte Carlo simulation method

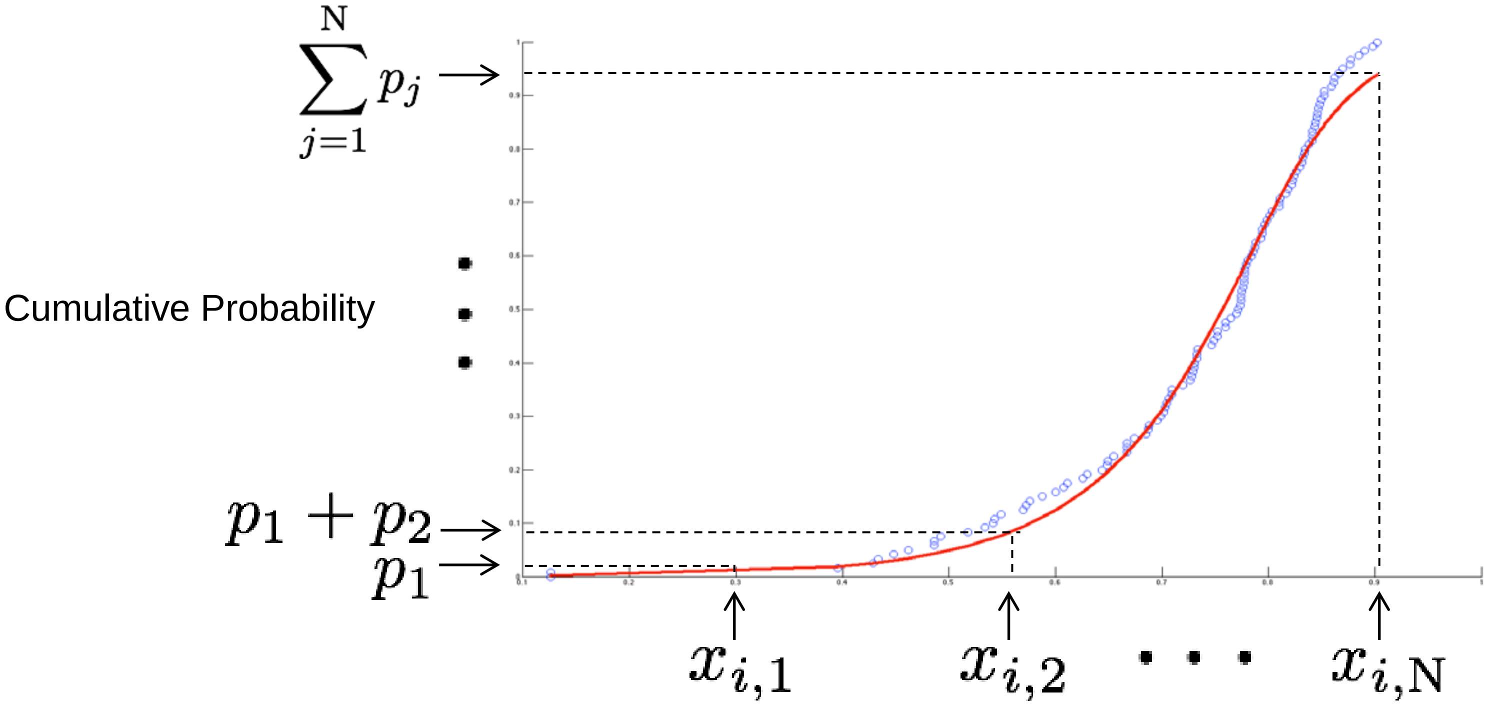
Multi-Stage Scenario Tree Generation

- **Stochastic Processes**
  - Random data that are autocorrelated in time
  - Scenario generation is aided by *time series forecasting*
  - Solve Distribution Matching Problem at each non-leaf node



New Contribution

- Only matching moments ➔ underdetermined problem
- Consequences:
  - Multiple combinations of demand and probability values
  - Probabilities may not capture the shape of the underlying distribution
- Additional information: marginal (Empirical) Cumulative Distribution



Distribution Matching Problem (DMP)

$$\min_{x,\, p} \quad z_{\text{DMP}}^{L^2} = \sum_{i \in I} \sum_{k \in K} \mathrm{w}_{i,k} (m_{i,k} - \mathrm{M}_{i,k})^2 + \sum_{\substack{(i,\, i') \in I \\ i < i'}} \mathrm{w}_{i,i'} (c_{i,i'} - \mathrm{C}_{i,i'})^2 + \sum_{i \in I} \sum_{j=1}^{\mathrm{N}} \omega_{i,j} \delta_{i,j}^2$$

Min weighted error between tree and data

s.t.

$$\sum_{j=1}^{\mathrm{N}} p_j = 1$$

Probabilities add up to 1

$$m_{i,1} = \sum_{j=1}^{\mathrm{N}} x_{i,j} p_j \qquad \forall\, i \in I$$

$$m_{i,k} = \sum_{j=1}^{\mathrm{N}} (x_{i,j} - m_{i,1})^k p_j \qquad \forall\, i \in I,\ k > 1$$

Moments calculated from the tree

$$c_{i,i'} = \sum_{j=1}^{\mathrm{N}} (x_{i,j} - m_{i,1})(x_{i',j} - m_{i',1}) p_j \qquad \forall\, (i,\, i') \in I,\ i < i'$$

Covariances calculated from the tree

$$x_{i,j} \in [\mathrm{x}_{i,j}^{\mathrm{LB}},\ \mathrm{x}_{i,j}^{\mathrm{UB}}] \qquad \forall\, i \in I,\ j = 1, \ldots, \mathrm{N}$$

$$p_j \in [0,\ 1] \qquad \forall\, j = 1, \ldots, \mathrm{N}$$

Bounds on variables

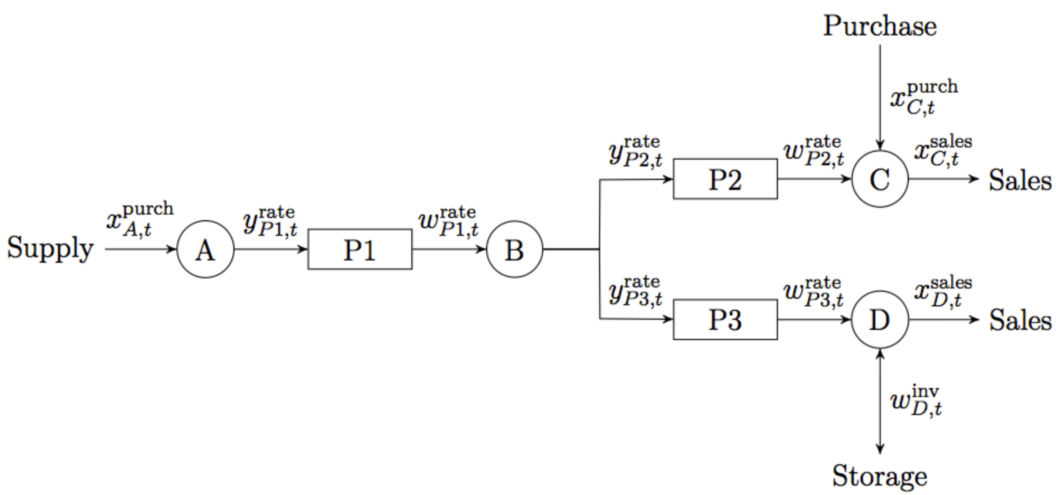
$$\widehat{ECDF}(x_{i,j}) - \sum_{j'=1}^{j} p_{j'} = \delta_{i,j} \qquad \forall\, i \in I,\ j = 1, \ldots, \mathrm{N}$$

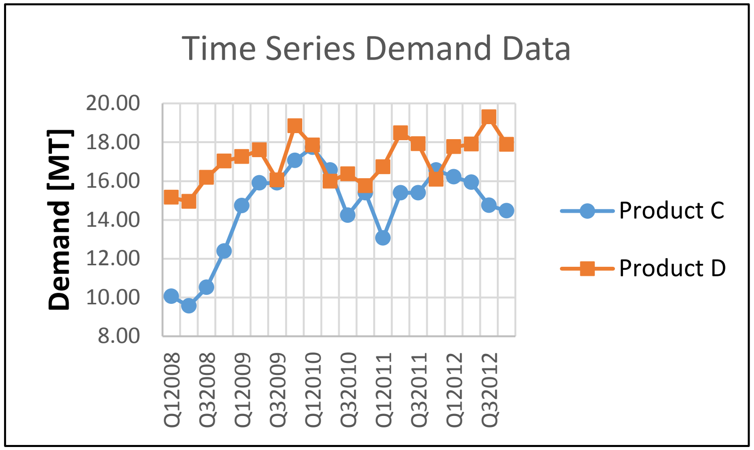
$$x_{i,j} \leq x_{i,j+1} \qquad \forall\, i \in I,\ j = 1, \ldots, \mathrm{N} - 1$$

ECDF information

Motivating Example

- Network of chemical plants





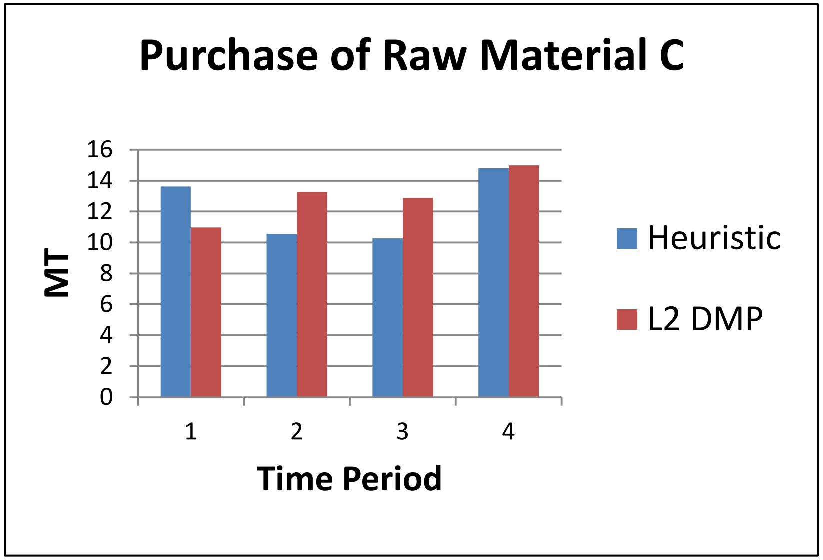
- 1 raw material (A), 1 intermediate product (B), two finished products (C and D), 1 site
- Only D can be stored and C can be purchased from elsewhere (may simulate inter-site transfers)

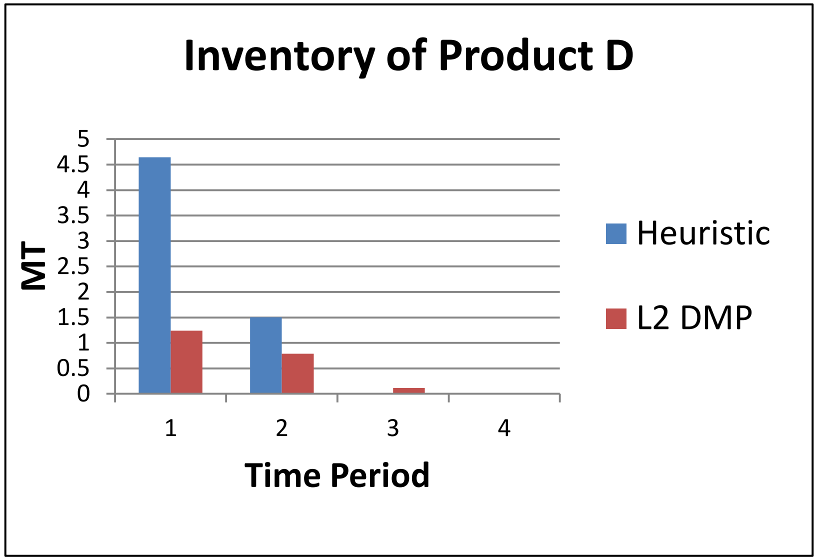
Results and Discussion

- Expected profits [$]

Heuristic	L^2 DMP
79.95	82.39

- Production plan variables





- The choice of the scenario tree impacts the decisions
- DMP accounts for correlations between the product demands

Literature and Practical Implications

- Current research on **scenario generation**
  - Mostly addressed by the OR&MS community
  - Predominantly financial applications (portfolio optimization)
  - Key idea: data-driven approach, *i.e.* no or very few assumptions about distributions (nonparametric rather than parametric models)
- This work
  - Additional information to be matched: (E)CDF
  - Improved numerics (degeneracy) when solving DMP
- Implementation in an industrial setting
  - Take advantage of forecasting and data mining software suites (e.g., SAS)
    1. Collect plant and market data (sources: Global Insight, Chemical Markets Associates Inc. (CMAI), government sources etc.)
    2. Obtain *good* forecasts (crucial part, see Rey *et al.*, 2012)
    3. Solve DMP (using information from step 2.) and generate tree
    4. Solve stochastic optimization model
  - Takeaway: poor quality data (scenario trees) yield poor quality decisions

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