Extended Resource Task Network Formulation for Reactive Scheduling of a Mixed Batch/Continuous Process

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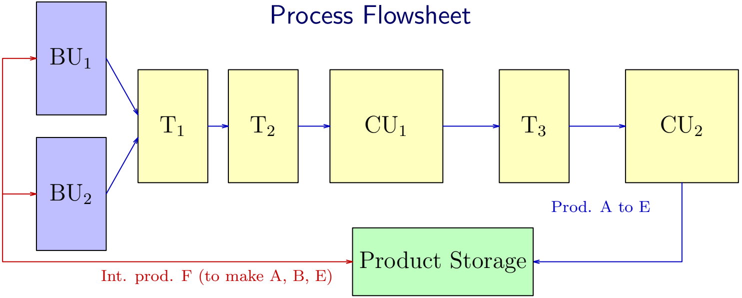
Introduction

Project motivation

- Industrial application of optimization-based scheduling approaches



BU batch unit; T tank; CU continuous unit

- Problem characteristics
  - Mixed equipment types: batch and continuous
  - Multi-purpose plant: network structure and multiple products
  - Operating rules:
    - Equipment cleaning for product transitions
    - Replenishment task of CU_2
  - Re-optimization of disrupted schedules: rescheduling capability

Introduction

Problem statement

- Objective
  - Meet product orders and maintain product inventory levels
- Decision-to-make
  - Batch units: number of batches and their timing (fixed size)
  - Tanks: inlet and outlet flow rates
  - Continuous units
    - CU_1: processing rate and transition
    - CU_2: processing rate, transition, and maintenance
- Desired performance
  - Generic modeling method
  - Fast solution
  - Ease of re-optimization

Modeling Framework

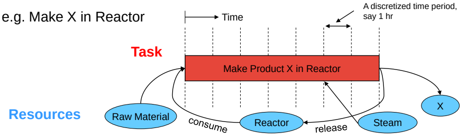
Resource Task Network (RTN) formulation

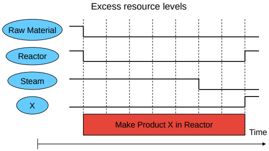
- A process can be represented as interactions between resources and tasks

  Resources Process units, materials, utilities, etc.

  Tasks Process operations, such as reaction, separation, and storage

e.g. Make X in Reactor





- Core (discrete-time) RTN equations

  - Excess resource balances:

$$R_{r,t} = R_{r,t-1} + \sum_i \sum_{\theta=0}^{\tau_i} \mu_{i,r,\theta} N_{i,t-\theta} + \sum_i \sum_{\theta=0}^{\tau_i} \nu_{i,r,\theta} \xi_{i,t-\theta} + \Pi_{r,t}$$

  - Excess resource limits:

$$R_{r,t}^{min} \leq R_{r,t} \leq R_{r,t}^{max}$$

Indices

i	task
r	resource
t	time slot
θ	time shift

Variables

N	number of tasks
R	excess resource
τ	task duration
μ	interaction parameter
ν	interaction parameter
ξ	batch extent
Π	external resource transfer

Modeling Framework

RTN and its extensions

- Advantages of discrete-time RTN
  - + Concise and consistent model structure
  - + Tight mixed integer linear program (MILP)
- RTN extensions*
  - Multi-extent resource balance

$$R_{r,t} = R_{r,t-1} + \sum_{i} \sum_{\theta=0}^{\tau_i} \mu_{i,r,\theta} N_{i,t-\theta} + \sum_{i} \sum_{m} \sum_{\theta=0}^{\tau_i} \nu_{i,m,r,\theta} \xi_{i,m,t-\theta} + \Pi_{r,t}$$

Resource interaction routes of synchronized on/off:

→ Inlet/outlet flow rates of a buffer tank

  - Resource limit balance:

$$R_{r,t}^{max} = R_{r,t-1}^{max} + \sum_{i} \sum_{\theta=0}^{\tau_i} \alpha_{i,r,\theta} N_{i,t-\theta} + \sum_{i} \sum_{m} \sum_{\theta=0}^{\tau_i} \beta_{i,m,r,\theta} \xi_{i,t-\theta}$$

Time-varying resource limits

→ Emptying a tank with variable inventory level

*Wassick and Ferrio. Extending the resource task network for industrial applications. *Computers and Chemical Engineering*, 2011

Modeling Framework

State space RTN model

- Rescheduling is essential in practice
  - Model Predictive Control (MPC) based approach for scheduling?
  - Iterative optimization coupled with receding time horizon
- Convert RTN model to state space form
- Lifting: additional state variables to record decision history*

$\bar{N}_{i,t,\theta} \triangleq N_{i,t-\theta}$, Task i at time t has been running for θ periods

State evolution: $\bar{N}_{i,t,\theta} = \bar{N}_{i,t-1,\theta-1}, \quad \bar{N}_{i,t,0} = N_{i,t}$

- Rewrite RTN equations, e.g., resource balance

$$R_{r,t} = R_{r,t-1} + \sum_i \sum_{\theta=0}^{\tau_i} \mu_{i,r,\theta} N_{i,t-\theta} + \sum_i \sum_m \sum_{\theta=0}^{\tau_i} \nu_{i,m,r,\theta} \xi_{i,m,t-\theta} + \Pi_{r,t}$$

Influence from past decisions

*Subramanian et al., A state-space model for chemical production scheduling. *Computers and Chemical Engineering*, 2012

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First order dynamics

*Subramanian et al., A state-space model for chemical production scheduling. *Computers and Chemical Engineering*, 2012

Scheduling Formulation

Scheduling optimization with state space RTN model

- Objective function evaluates process economic performance in
  1. Order delivery
  2. Product inventory maintenance
  3. Operation smoothness (avoid excessive ons/offs for continuous units)

$$\min \phi = \sum_{o} \sum_{r} \sum_{t \geq E_o}^{t \leq D_o} (-c_{o,r,t}^{order})(-\Pi_{r,t}) + \sum_{r} c_r^{slack} S_{r,H}^{min} + \sum_{r} \sum_{t} c_{r,t}^{invt} E_{r,t}$$

- Model constraints
  1. State space RTN: resource/limit balance, task history evolution
  2. Auxiliary constraints: safety stock, order fulfillment
- Solution approach
  - Mixed-integer program: GAMS/Gurobi
  - Rolling horizon scheme

Case Study

Nominal schedule design

- Three day schedule design with 1-hour grid

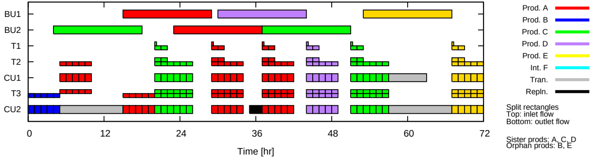
Product demand

Prod.	A	B	C	D	E
Day 1	70				
Day 2	70		70		70
Day 3	70	70	70	70	
Day 4	70			70	

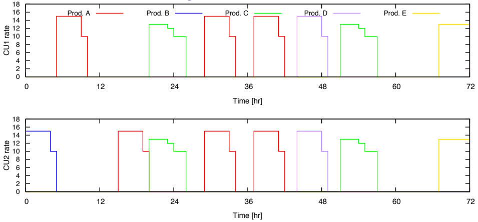
Model solution and statistics

Var(binary) #	Cons. #	Opt. gap	CPU [s]
134, 148(4, 380)	140, 382	2.6%	600

Optimal plant schedule of Day 1-3



Processing rates of continuous units

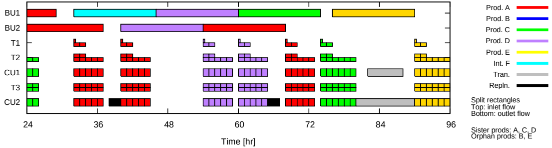


Case Study

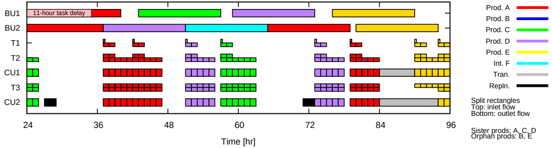
Reactive schedule design

- Reschedule at $t = 24$ hr with three scenarios:

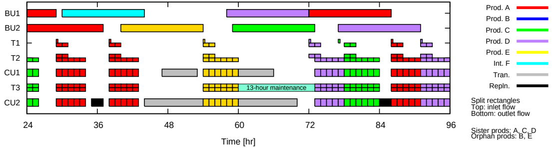
Optimal schedule without process disruptions



Optimal schedule with task delay in BU_1



Optimal schedule with T_3 maintenance



Conclusions

- Novelty of work
  - Generic framework of wide applicability
    - Model customization minimized for different application complications (storage policy, disturbance source, etc.)
    - Hybrid plant: Nott and Lee 1999, Castro et al. 2004
    - Reactive scheduling: Mendez and Cerda 2004, Li and Ierapetritou 2008
  - State space scheduling model of the extended RTN
    - Extended RTN model: Wassick and Ferrio 2011
    - State space STN model: Subramanian et al 2012
- Benefits and scope of industrial application
  - Enable responsive schedule design for economic optimization
  - Satisfactory solution performance with thousands of discrete variables
    - Weekly schedule design of medium size processes
    - 7 units and 6 products in the case study example

Thank you

I am glad to take your questions