Integrated Production Planning and Scheduling for Batch Operations

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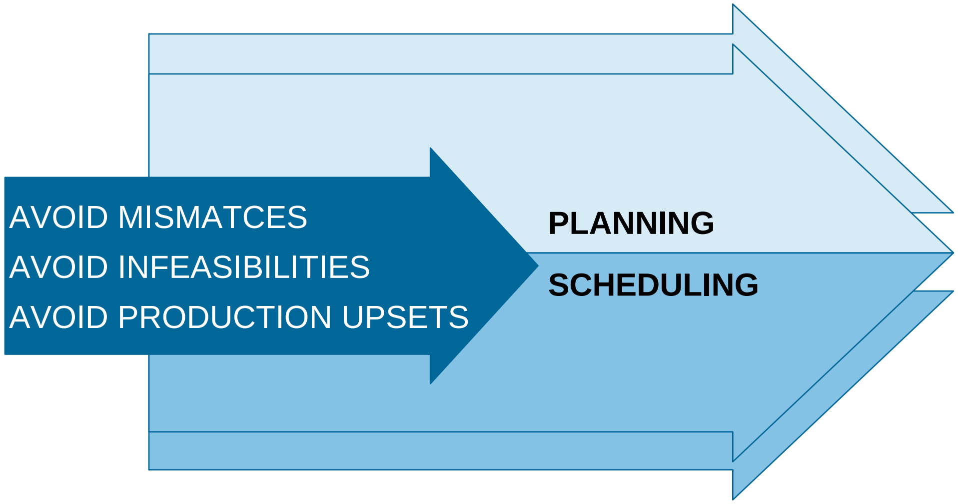
AIM OF THE PROJECT

- Develop **Accurate Planning Models** for Batch Operated Plants using mixed integer optimization techniques by integrating planning and scheduling for the Dow Chemical Company

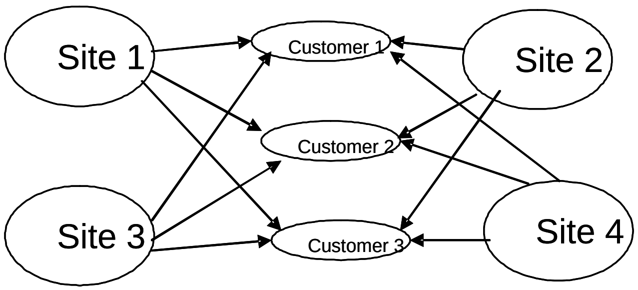
INTEGRATION OF PLANNING AND SCHEDULING



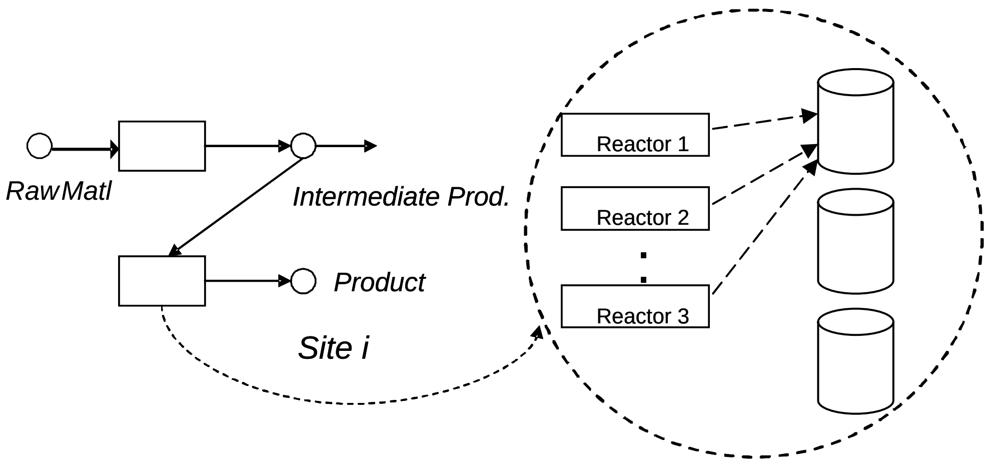
ULTIMATE GOAL:

- Propose a novel decomposition algorithm to integrate planning and scheduling for Multisite Batch Reactors.
- Ensure optimality and consistency between the two levels.

Multi-Site Problem of the Dow Chemical Company



- •Multiple sites
- •Multiple markets
- •Multiple products
- •Price of each product
- •Transportation costs to each customer
- •Monthly demand forecasts for each product



- •Batch reactors in parallel
- •Raw material availabilities and costs
- •Reactor batch times and batch sizes
- •Sequence dependent change-over times
- •Intermediate and final storage units

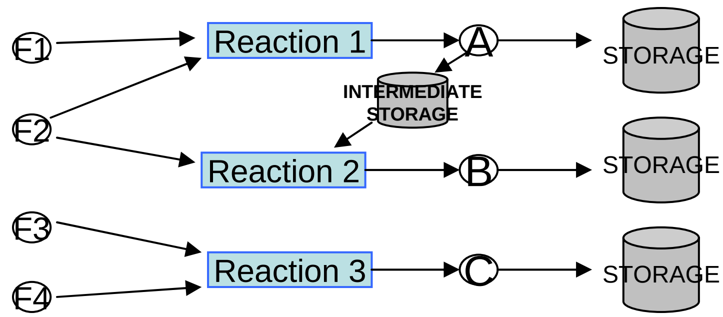
Maximize profit

- •What products are shipped to each market from each site
- •Total sales of each product in each site
- •Monthly production quantities for each reactor
- •Assignment of end products to storage tanks

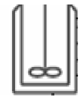
PROBLEM STATEMENT

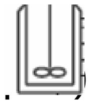
Focus: Planning and Scheduling of a Single Site

Production Site:



- Raw material availability and Raw material costs
- Storage tanks with associated capacity
- Transportation costs to each customer
- Reactors:
  - Materials it can produce
  - batch sizes (lbs) for each material it can produce
  - operating costs ($/hr) for each material
  - ***Sequence dependent clean out times*** (hrs per transition for each material





  - Time the reactor is available during a given month (hrs)

Customers:

- Monthly forecasted demands for desired products
- Price paid for each product

Materials:

- Raw materials, Intermediates, Finished products
- Unit ratios (lbs of needed material per lb of material produced)

PROBLEM STATEMENT

DETERMINE:

- **PLANNING PERSPECTIVE:**
  - Monthly production quantities
  - Amount of raw materials to be purchased
  - Monthly Inventory levels

- **SCHEDULING PERSPECTIVE:**
  - Assignments of products to available processing equipment
  - Detailed timing and sequence of production in each processing equipment
  - Daily Production and Inventory levels

OBJECTIVE:

To Maximize Profit.

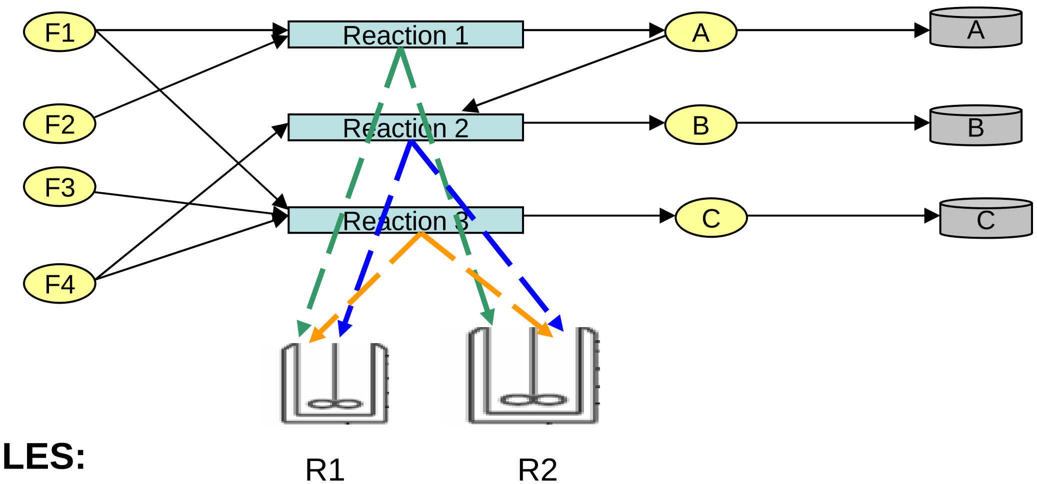
Profit = Sales –Raw Material Costs-Operating Costs – Inventory Costs – Transition Cost-Transportation Costs

BASIC IDEAS OF THE MULTIPERIOD PLANNING MODEL

- RELAXATION OF STATE-TASK NETWORK

Detailed timing constraints are relaxed

- States: Raw materials, intermediates, end products, (J)
- Tasks: Physical and Chemical transformations between adjacent states (i.e. reactions), (I)
- Equipment: Physical devices that will execute a given task (i.e. reactors), (L)



KEY VARIABLES:

$$Y_{i.l.t} = \begin{cases} 1, & process\ i\ is\ assigned\ to\ equipment\ l\ at\ period\ t \\ 0, & otherwise \end{cases}$$

$W_{i,l,t}$: batch size of process i, in unit l at time period t (lb)

$N_{i,l,t}$: number of batches of process i in unit l at time period t

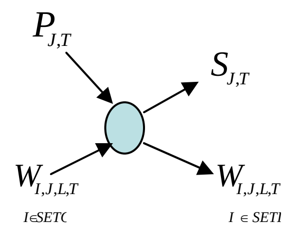
Integrality requirements enforced on the number of batches

AGGRAGATE MILP PLANNING MODEL

MASS BALANCE FOR EACH CHEMICAL AT EACH TIME PERIOD:

$$P_{j,t} + \sum_{i\in SETO} RHO_{i,j} \sum_{i\in N(I,L)} W_{i,l,t} = S_{j,t} + \sum_{i\in SETI} RHO_{i,j} \sum_{i\in N(I,L)} W_{i,l,t} + INV_{j,t} - INV_{j,t-1} \qquad \forall j,t$$

purchases | production | sales | consumption | change in inventory



$$W_{i,l,t} \le BOUND_{i,l} * Y_{i,l,t} \qquad \forall\, i,l,t$$

TIME BALANCE:

$$\sum_{i\in N(I,L)} N_{i,l,t} * TAU_{i} + \sum_{i} TRA_{i} * Y_{i,l,t} - U_{l,t} \le H_{t} \qquad \forall\, l,t$$

Number of batches | Batch time | Lower bounds for transitions | Total available time

$$N_{i,l,t} = \frac{W_{i,l,t}}{Q_l} \qquad \forall i,l,t$$

Integer number of Batches

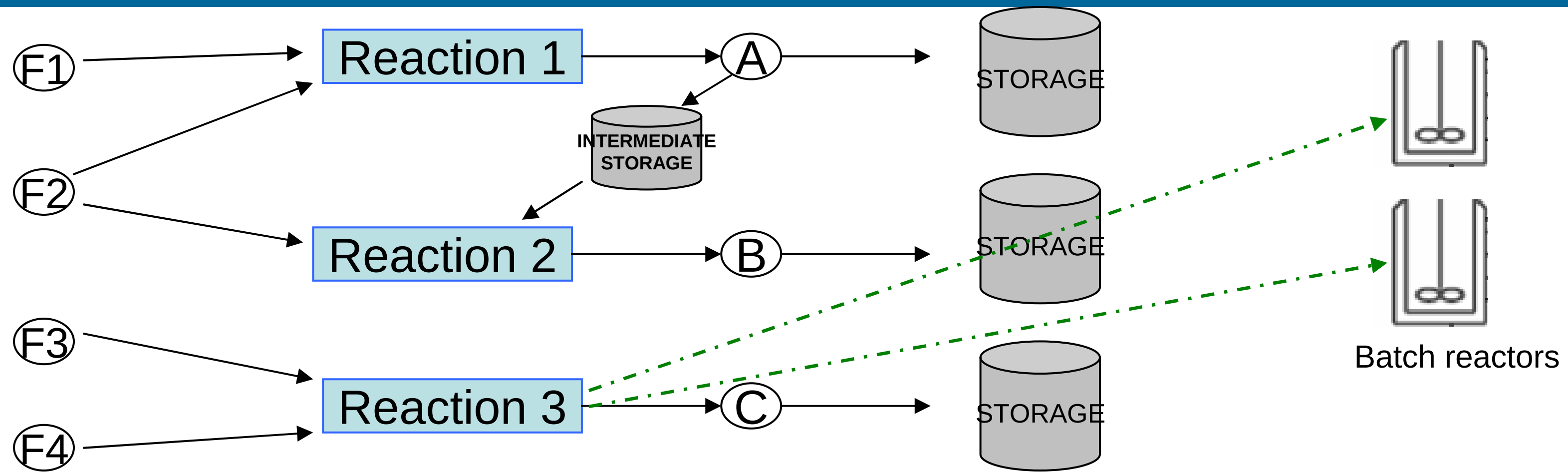
OBJECTIVE FUNCTION:

Maximize:

$$NPV = \sum_{t}\sum_{j} CP_{j,t} * S_{j,t} - \sum_{t}\sum_{j} CR_{j,t} * P_{j,t} - \sum_{t}\sum_{i}\sum_{l\in N(I,L)}\sum_{j\in M(I,J)} COP_{i,t} * W_{i,l,t} - \sum_{t}\sum_{i}\sum_{l\in N(I,L)} COPF_{i,t} * Y_{i,l,t}$$

sales | purchases | variable operating costs | fixed costs

DETAILED SCHEDULING MODEL



DETAILS:

- Several batch reactors operating in parallel
- Each reactor is connected to any final product storage tank.
- Raw materials and end products– **end product** of one process might be used as a **raw material** of another process.
- Once an end product is sent to the dedicated storage tanks, it can’t be fed back into the plant –requires modeling of **intermediate storage**

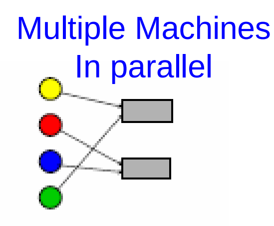
Difficulties with STN/RTN Models or Sequential Models

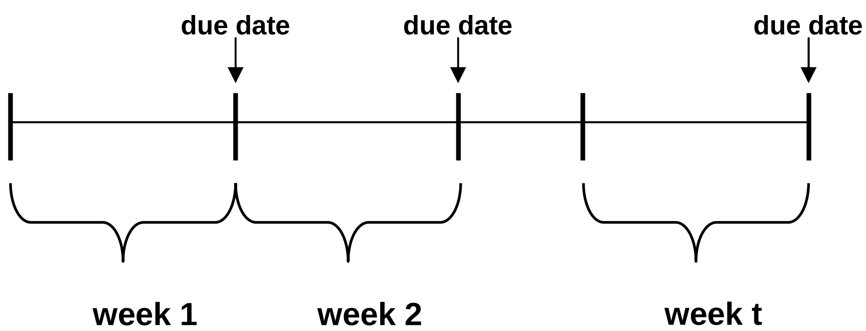
•Sequence dependent transition times are handled via the slot based representation.
•To handle intermediate storage, process that produces the intermediate is duplicated.

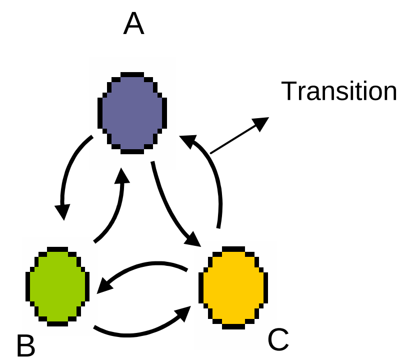
Problem Statement for the Single Site Scheduling

Given

- **Multiproducts** to be processed on a multipurpose batch plant with equipment in parallel.
- Time horizon subdivided into **weeks** at the end of which demands are specified.
- Transition times are **sequence dependent**.
- **Continuous** time representation is used.
- **Time slot** representation is used.







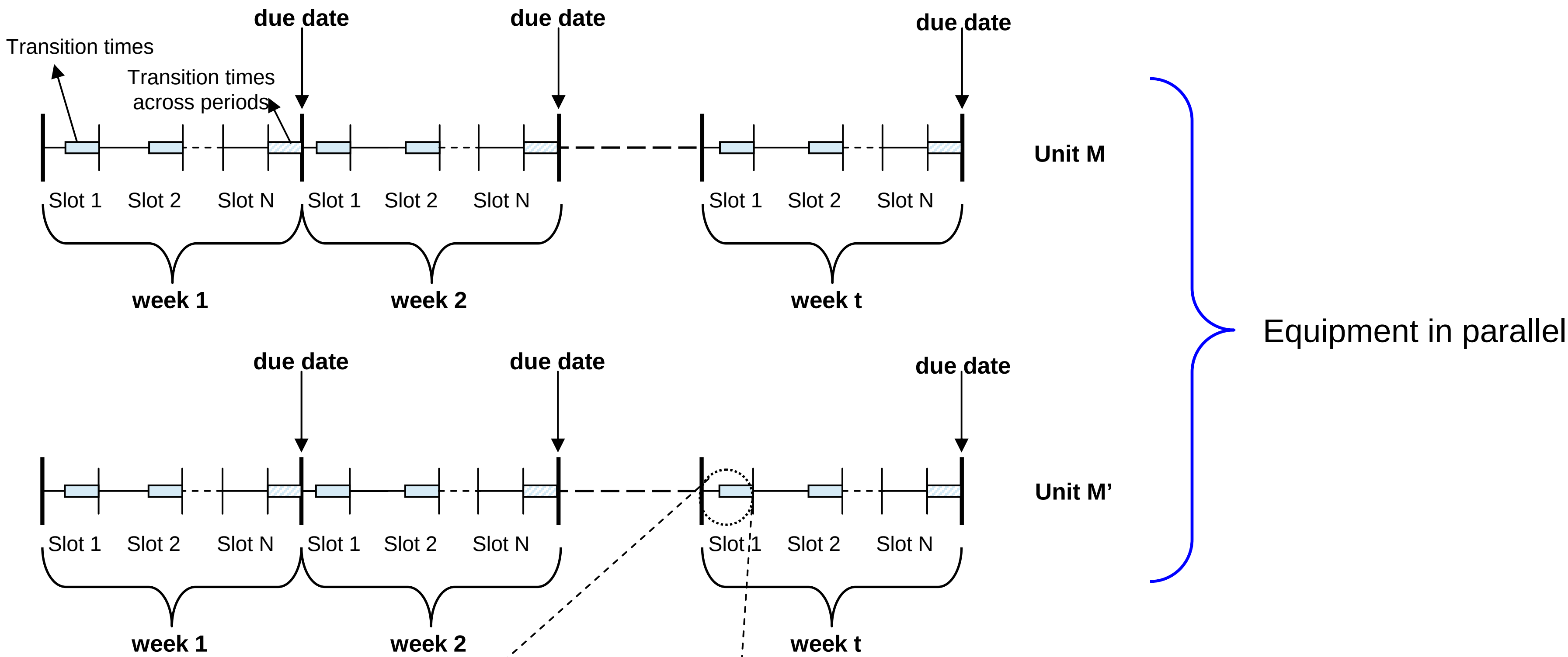
Decisions

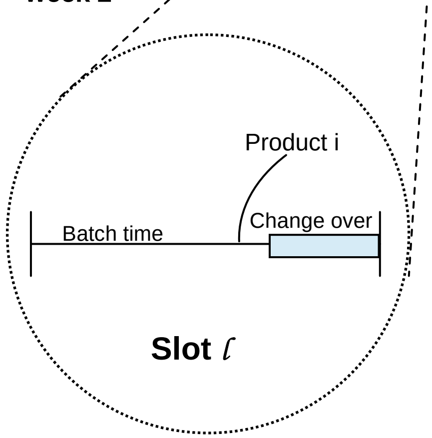
- Number of batches of each product
- Amounts to be produced
- Product inventories
- Sequencing of products
- Assignments of batches to equipments

Objective

- Max Profit = Sales –Operating Costs – Inventory Costs – Transition Costs

Problem Formulation





Key variables for assignments:

$W_{im\ell t}$=1: Product *i* is produced in slot ℓ of of unit *m* of time period *t*

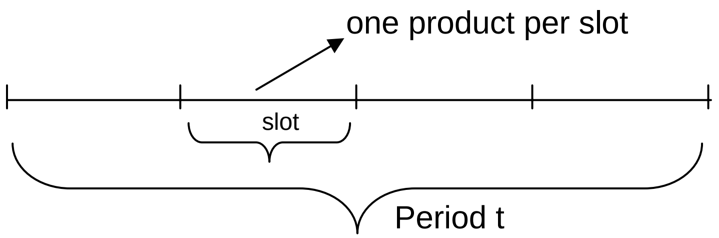
binary variable

$Y_{\ell mt}$=1: slot ℓ of of unit *m* of time period *t* is occupied.

continuous variable

Assumptions

- Batch times and batch sizes are fixed parameters.
- Each slot represents one batch, at most one batch is produced in each slot.

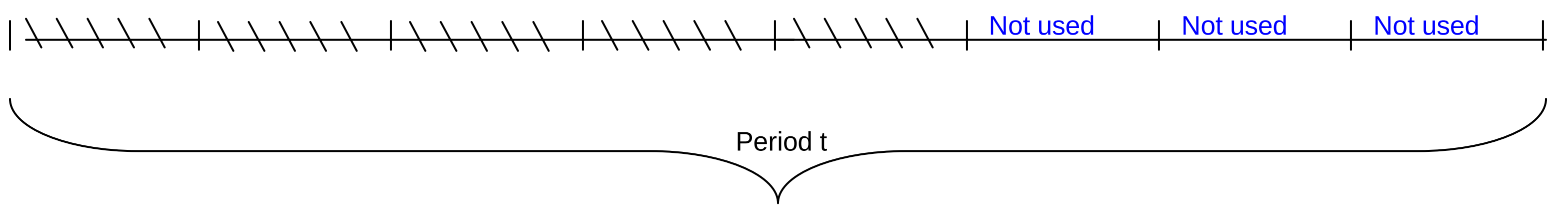


- Same product can be produced in more than on slot.

- Consecutive utilization of the slots is ensured by the model.

- Number of batches is a variable to be determined by the model, hence exact number of slots to be postulated is unknown

- To avoid infeasible or suboptimal solutions, more than necessary number of slots are postulated at each time period. Hence some slots might be left unutilized.



Detailed Scheduling Model (MILP)

$$Z=\sum_i\sum_t CP_{i,t}\cdot S_{i,t}-\sum_i\sum_m\sum_l\sum_t COP_{i,t}\cdot XB_{i,m,l,t}-\sum_i\sum_t CINV_{i,t}\cdot INV_{i,t}-\sum_i\sum_k\sum_m\sum_l\sum_t CTRANS_{i,k}\cdot\left(Z_{i,k,m,l,t}+ZDEL_{i,k,m,l,t}+ZBEL_{i,k,m,l,t}\right)$$

Assignment and production constraints

$$\sum_i W_{i,m,l,t}\le 1 \qquad \forall\, m,l,t$$

$$\sum_i W_{i,m,l,t}\ge\sum_i W_{i,m,l+1,t} \qquad \forall\, m,l,t$$

$$PT_{i,m,l,t}=TA_{i,m}\cdot W_{i,m,l,t} \qquad \forall i,m,l,t$$

$$XB_{i,m,l,t}=R_{i,m}\cdot PT_{i,m,l,t} \qquad \forall i,m,l,t$$

$$Y_{m,l,t}=\sum_i W_{i,m,l,t} \qquad \forall m,l,t$$

Sequence dependent transitions

$$Z_{i,k,m,l,t}\ge W_{i,m,l,t}+W_{k,m,l+1,t}-1 \qquad \forall\, i,k(k\ne l),m,l\ne N,t$$

$$ZDEL_{i,k,m,l,t}\ge W_{i,m,l,t}+W_{k,m,1,t+1}-1 \qquad \forall\, i,k(k\ne l),m,l,t\ne HT$$

$$ZBEL_{i,k,m,l,t}\ge W_{i,m,l,t}+W_{k,m,1,t+1}-1 \qquad \forall\, i,k(k\ne l),m,l=N,t<HT$$

$$Te_{m,l,t}=Ts_{m,l,t}+\sum_i PT_{i,m,l,t}+\sum_i\sum_k TAU_{li,k}\cdot Z_{i,k,m,l,t}+\left(\sum_i\sum_k TAU_{i,k}\cdot ZDEL_{i,k,m,l,t}\right)\cdot\left(1-Y_{m,l+1,t}\right)+\left(\sum_i\sum_k TAU_{Ii,k}\cdot ZBEL_{i,k,m,l,t}\right)$$

Slot location and slot Availability constraints

$$TE_{mm,ll,t}\le TS_{m,l,t}+BIGW\cdot(1-YY_{ll,l,mm,m,t}) \qquad \forall ll,l,mm,m,t\,(mm\ne m)$$

$$TS_{m,l,t}\le TE_{mm,ll,t}+BIGW\cdot(YY_{ll,l,mm,m,t}) \qquad \forall ll,l,mm,m,t\,(mm\ne m)$$

$$\begin{bmatrix} YY_{ll,l,mm,m,t} \\ AA_{A',mm,ll,t}=XB_{A',mm,ll,t}\end{bmatrix}\vee\begin{bmatrix}\neg YY_{ll,l,mm,m,t} \\ AA_{A',mm,ll,t}=0\end{bmatrix}$$

$$\alpha\cdot XB_{B,m,l,t}\le INV_{A',t-1}+\sum_{ll}^{ll\le l-1} XB_{A',m,ll,t}+\sum_{mm\ne m}\sum_{ll} AA_{A',mm,ll,t}$$

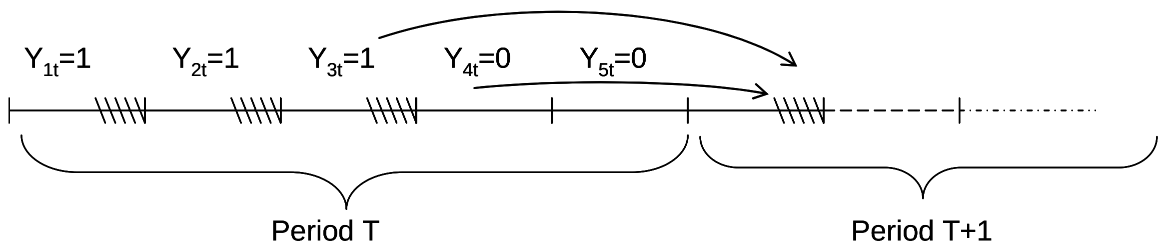
Inventory balances

$$INV_{i,t-1}+\sum_m\sum_l XB_{i,m,l,t}=S_{i,t}+INV_{i,t} \qquad \forall t,i\ne A'$$

$$INV_{i,t}+\sum_m\sum_l XB_{i,m,l,t}=\alpha\cdot\sum_m\sum_l XB_{B,m,l,t}+INV_{i,t} \qquad \forall t,i=A'$$

Linearization of the Timing Constraint

$$Te_{m,l,t} = Ts_{m,l,t} + \underbrace{\sum_i PT_{i,m,l,t}}_{\text{Batch time}} + \underbrace{\sum_i \sum_k TAU_{Ii,k} \cdot Z_{i,k,m,l,t}}_{\text{Transitions within the time period}} + \underbrace{(\sum_i \sum_k TAU_{i,k} \cdot ZDEL_{i,k,m,l,t}) \cdot (1 - Y_{m,l+1,t})}_{\text{Transitions across time periods}} + \underbrace{(\sum_i \sum_k TAU_{Ii,k} \cdot ZBEL_{i,k,m,l,t})}_{\text{Transitions across time periods if the last slot is Being utilized}}$$



$$Te_{m,l,t} = Ts_{m,l,t} + \sum_i PT_{i,m,l,t} + \sum_i \sum_k TAU_{Ii,k} \cdot Z_{i,k,m,l,t} + \underbrace{(\sum_i \sum_k TAU_{i,k} \cdot ZDEL_{i,k,m,l,t}) \cdot (1 - Y_{m,l+1,t})}_{\text{Non-linear}} + (\sum_i \sum_k TAU_{Ii,k} \cdot ZBEL_{i,k,m,l,t})$$

$$TRT_{m,l,t} = \sum_i \sum_k TAU_{i,k} \cdot ZDEL_{i,k,m,l,t}$$

$$Te_{m,l,t} = Ts_{m,l,t} + \sum_i PT_{i,m,l,t} + \sum_i \sum_k TAU_{i,k} \cdot Z_{i,k,m,l,t} + TRT_{m,l,t} - \underbrace{Y_{m,l+1,t} \cdot TRT_{m,l,t}}_{TX_{m,l,t}}$$

$$Te_{m,l,t} = Ts_{m,l,t} + \sum_i PT_{i,m,l,t} + \sum_i \sum_k TAU_{i,k} \cdot Z_{i,k,m,l,t} + TRT_{m,l,t} - TX_{m,l,t}$$

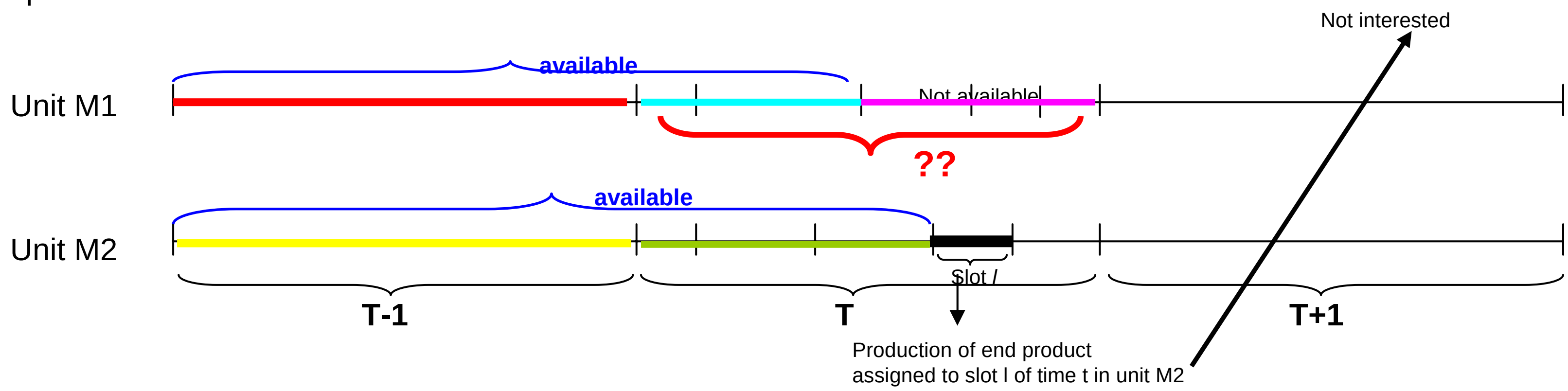
$$\begin{bmatrix} Y_{m,l+1,t} \\ TX_{m,l,t} = TRT_{m,l,t} \end{bmatrix} \vee \begin{bmatrix} \neg Y_{m,l+1,t} \\ TX_{m,l,t} = 0 \end{bmatrix}$$

Using Convex Hull Representation →

$$TRT_{m,l,t} = TRT1_{m,l,t} + TRT2_{m,l,t}$$
$$TX_{m,l,t} = TRT1_{m,l,t}$$
$$TRT1_{m,l,t} \le BIGM \cdot Y_{m,l+1,t}$$
$$TRT2_{m,l,t} \le BIGM \cdot (1 - Y_{m,l+1,t})$$

Details of the Slot Location and Availability Constraints

•Production of the intermediate to be used as a raw material for the end products should be completed before the production of that particular end starts!



Key variable:

$$YY_{ll,l,m1,m2,t} = \begin{cases} 1, & slot\ ll\ of\ unit\ m1\ is\ completed\ before\ slot\ l\ of\ unit\ m2\ starts \\ 0, & otherwise \end{cases}$$

Determining the location of the slots relative to slot l of unit M2 at time T:

$$TE_{mm,ll,t} \le TS_{m,l,t} + BIGW \cdot (1 - YY_{ll,l,mm,m,t}) \qquad \forall ll, l, mm, m, t\ (mm \ne m)$$

$$TS_{m,l,t} \le TE_{mm,ll,t} + BIGW \cdot (YY_{ll,l,mm,m,t}) \qquad \forall ll, l, mm, m, t\ (mm \ne m)$$

Defining the availabilities of the slots:

$$\begin{bmatrix} YY_{ll,l,mm,m,t} \\ AA_{A',mm,ll,t} = XB_{A',mm,ll,t} \end{bmatrix} \vee \begin{bmatrix} \neg YY_{ll,l,mm,m,t} \\ AA_{A',mm,ll,t} = 0 \end{bmatrix}$$

Using Convex Hull Representation: →

$$XB_{A',mm,ll,t} = XB1_{A',mm,ll,t} + XB2_{A',mm,ll,t}$$

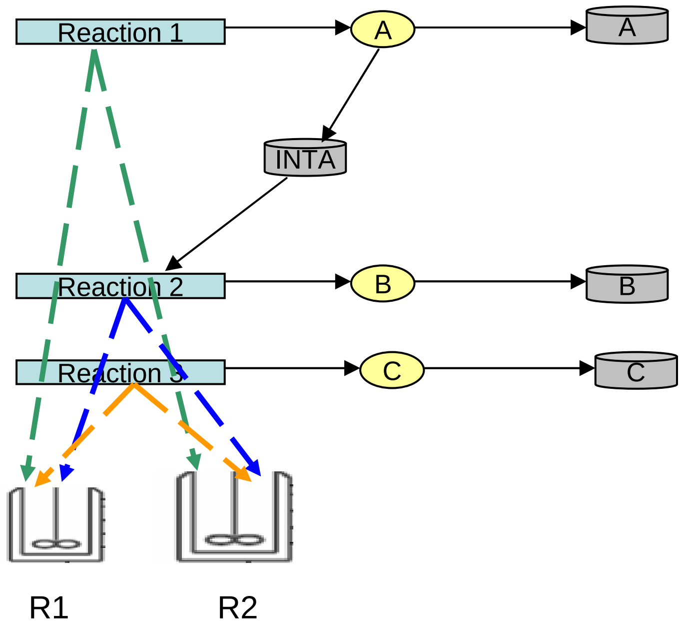
$$AA_{A',mm,ll,t} = XB1_{A',mm,ll,t}$$

$$XB1_{A',mm,ll,t} \le BIGU \cdot YY_{ll,l,mm,m,t}$$

$$XB2_{A',mm,ll,t} \le BIGU \cdot (1 - YY_{ll,l,mm,m,t})$$

Example and Preliminary Results

- Determine **plan and schedule** for **3** product, 2 reactors plant for a planning horizon of **3 weeks** so as to maximize **profit.**



- •3 Products, A,B,C
- •To produce 1 lb of "B", 0.2lb of "A" is required.
- •2 Reactors, R1,R2
- •End time of each week is defined as due dates
- •Demands are upper bounds

Problem Data:

	Demand values (lb)		
	T1	T2	T3
A	80,000.00	160,000.00	160,000.00
B	96,000.00	192,000.00	480,000.00
C	120,000.00	240,000.00	360,000.00

	Batch times (hrs)	Batch sizes (lb)
A	16.00	80,000.00
B	10.00	96,000.00
C	25.00	120,000.00

Transition times (hrs)			
	A	B	C
A	0	4	6
B	8	4	8
C	4	4	0

Results

	#of discr. Vrbs	# of cont. vrbs	# of eqns	Time (CPUs)	Solution ($)
Planning Model	36	90	102	0	1,070,830.96
Scheduling Model	1,440	4,534	6,002	2,464	1,008,119.46

% 5.86 difference in the predicted profit!

($)	Planning	Scheduling	Difference	%Difference
Profit	1,070,830.96	1,008,119.47	62,711.49	5.86
Sales	1,788,320.00	1,693,280.00	95,040.00	5.31
InvCost	134.04	100.53	33.51	25.00
TransCost	1,035.00	1,380.00	-345.00	-33.33
OperCost	716,320.00	683,680.00	32,640.00	4.56

Difference in prediction of Planning and Scheduling Models

Difference in the amounts produced in each time period:

	Planning- Production Values (lb)		
	T1	T2	T3
A	160,000.00	160,000.00	240,000.00
B	96,000.00	192,000.00	480,000.00
C	120,000.00	240,000.00	360,000.00
	Scheduling- Production Values (lb)		
	T1	T2	T3
A	80,000.00	240,000.00	240,000.00
B	0.00	192,000.00	480,000.00
C	120,000.00	240,000.00	360,000.00

Difference in the sales values in each time period:

	Planning- Sales Values (lb)		
	T1	T2	T3
A	80,000.00	160,000.00	160,000.00
B	96,000.00	192,000.00	480,000.00
C	120,000.00	240,000.00	360,000.00
	Scheduling- Sales Values (lb)		
	T1	T2	T3
A	80,000.00	160,000.00	160,000.00
B	0.00	192,000.00	480,000.00
C	120,000.00	240,000.00	360,000.00

Difference in the total amounts produced :

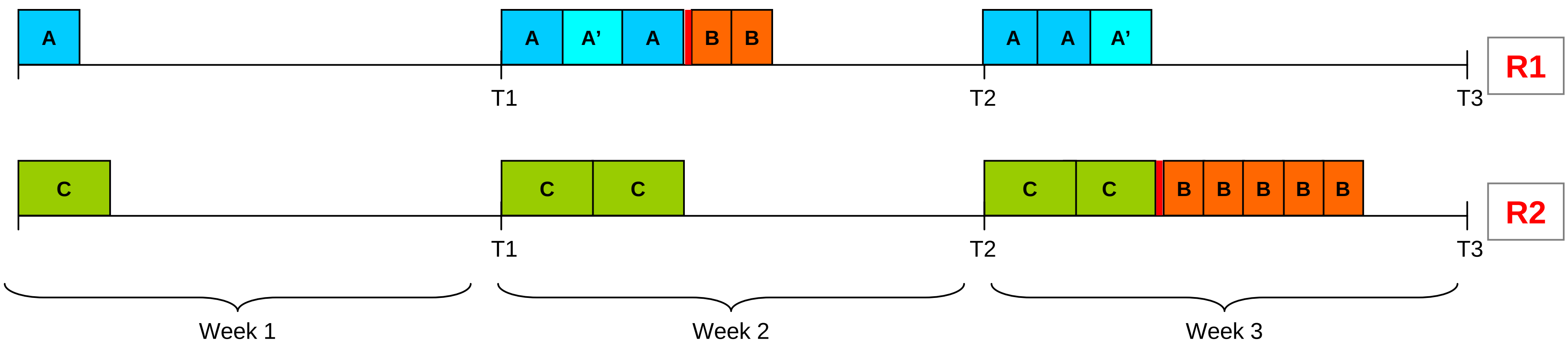
	Planning	Scheduling
Total A Production (lb)	560,000	560,000
Total B Production (lb)	768,000	672,000
Total C Production (lb)	720,000	720,000

Difference in the total sales values :

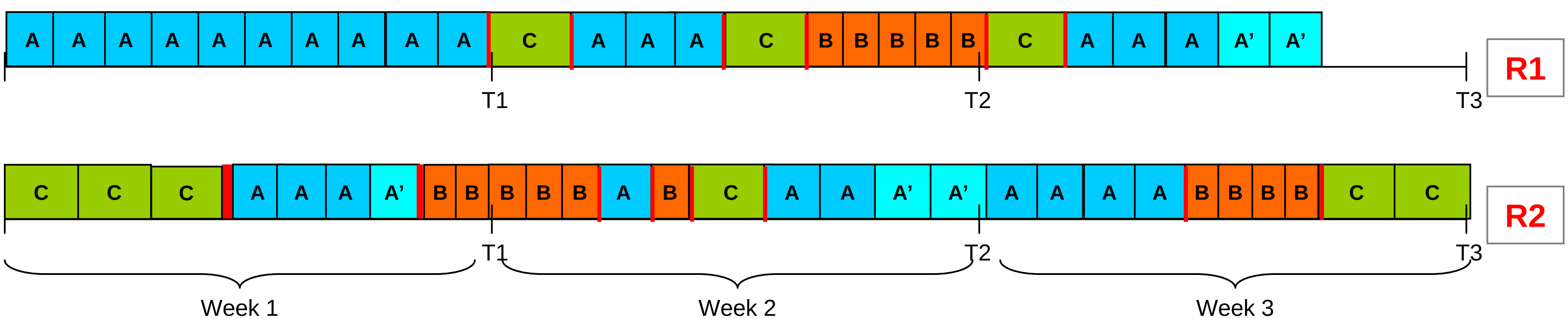
	Planning	Scheduling
Total A Sales (lb)	400,000	400,000
Total B Sales (lb)	768,000	672,000
Total C Sales (lb)	720,000	720,000

Results

Gantt Chart for the scheduling model for the demands as upper bounds case:



Gantt Chart for the scheduling model for the demands as lower bounds case:



Future Work

- Further investigate improvements for scheduling model.
- Develop bi-level decomposition algorithm for the single site problem to integrate planning and scheduling.
- Apply the proposed bi-level decomposition scheme to the entire system of sites and customers.
- Further decompose the problem via temporal Lagrangean decomposition so as to handle multiple sites.