An efficient algorithm for designing reliable large-scale process networks

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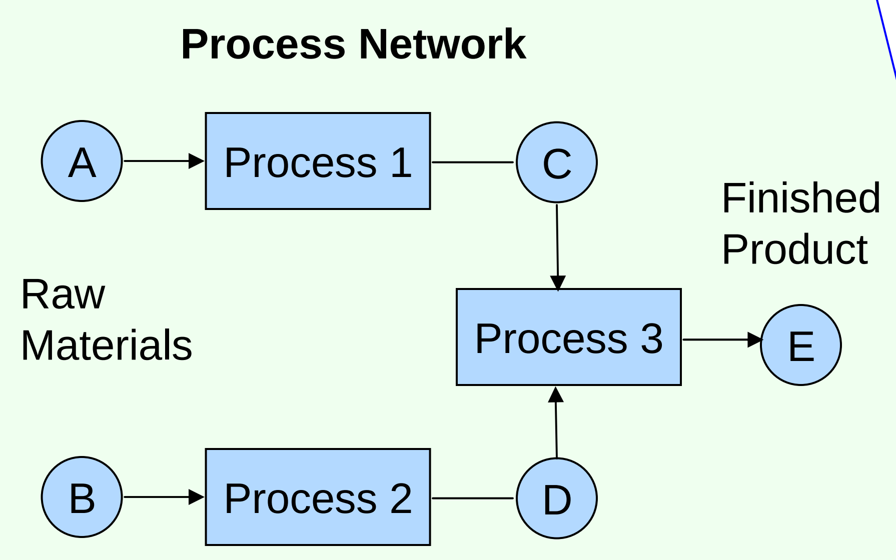
Design of reliable process networks takes random process failures into consideration

- Chemical plants are networks of processes
- All processes in real plants are subject to stochastic (random) failures
- The network has to be designed so that failures do not propagate (think about bullwhip effect in a supply chain).

Photograph of Ras Tanura Refinery from www.arabianoilandgas.com/pictures/gallery

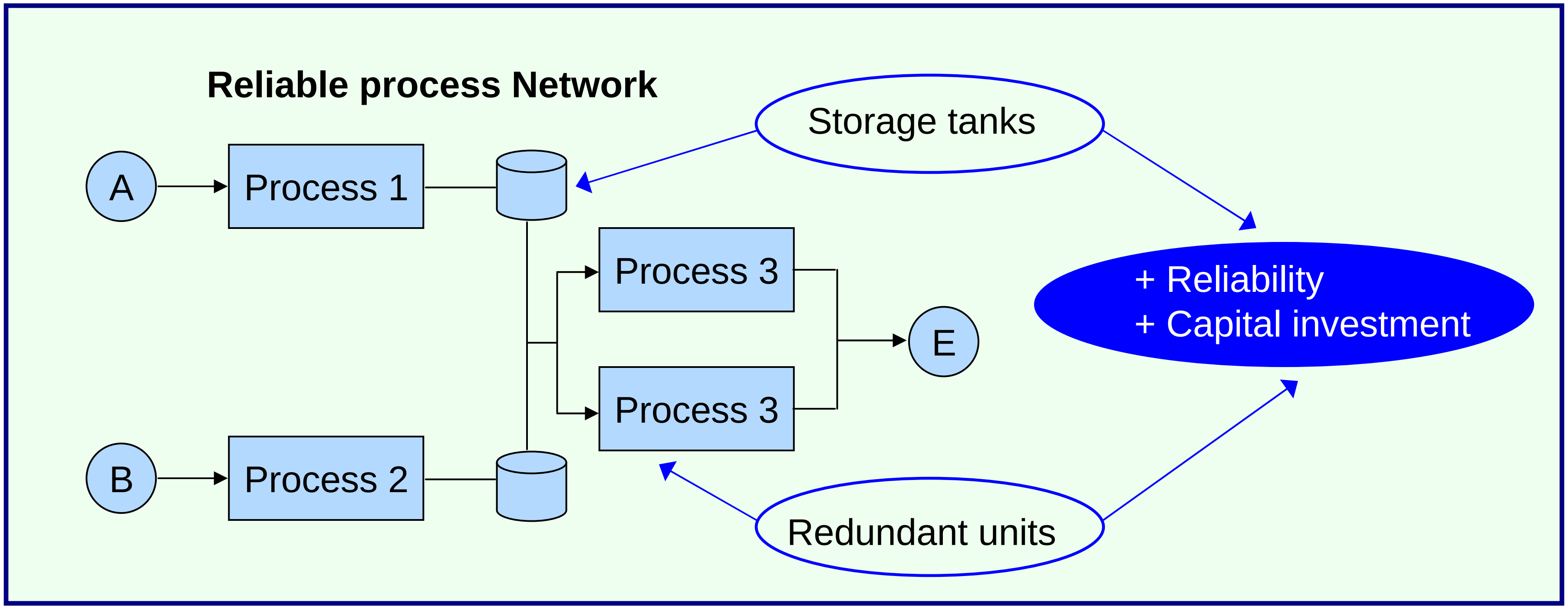
Design of reliable process networks takes random process failures into consideration



- Each individual process has an availability of 97% (percent of time the process is functional)
- Random failure in any of the three plants forces all plants to stop

Without **intermediate storage** or **redundant parallel** units the network is NOT functional ~ 10% of the time

Objective: determine number and size of redundant units and storage tanks needed for reliable operation



Challenges involved in designing reliable process networks:

- How much investment is required to ensure certain reliability ?
- How to invest a given amount of capital in the best possible way ?
  - Redundancies vs. storage tanks
  - There are 27 combinations of redundancies and storage tank configurations in the simple network above

Mathematical optimization provides a systematic method for achieving the objective of reliable design

Given:
- The superstructure of an integrated site
- Process specifications (maximum allowable capacities, supply of raw material, etc.)
- Number of failure modes and their corresponding probability

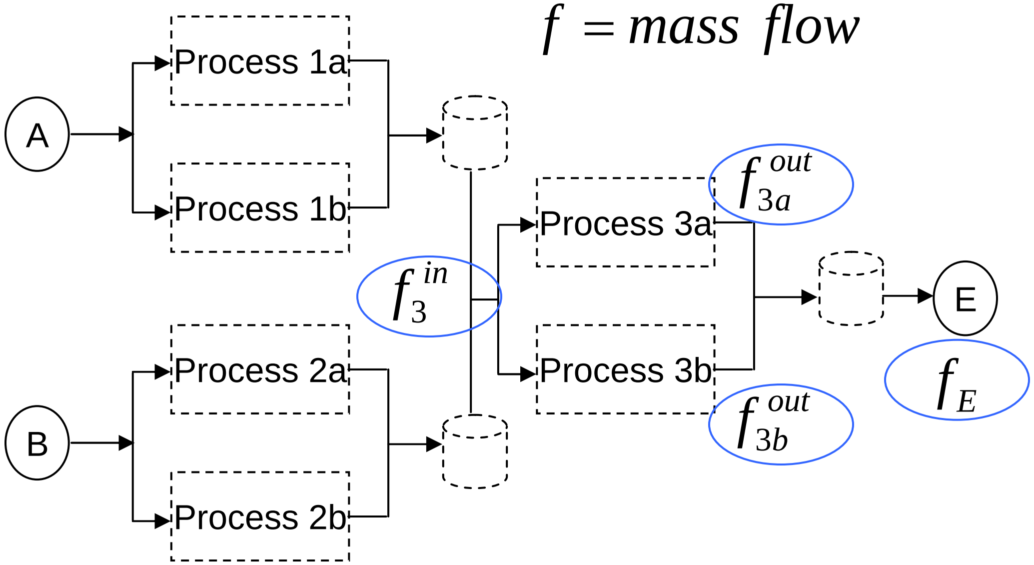
Determine:
- The number of parallel production units for each process.
- Sizes of intermediate storage between processes.

With the objective of:
- Maximizing the average production rate (function of reliability)
- Minimizing the capital investment.

Mathematical optimization provides a systematic method for achieving the objective of reliable design

Postulate a *superstructure*

$$f = mass\ flow$$



Assign **decision variables** to redundancies and tanks

y_{3a} 1 if process 3a is installed; 0 otherwise

y_{3b} 1 if process 3b is installed; 0 otherwise

v_3 Size of storage tank (0 if tank is not included)

Superstructure constraints

$$f_{3a}^{out} \le y_{3a} Capacity_{3a}$$

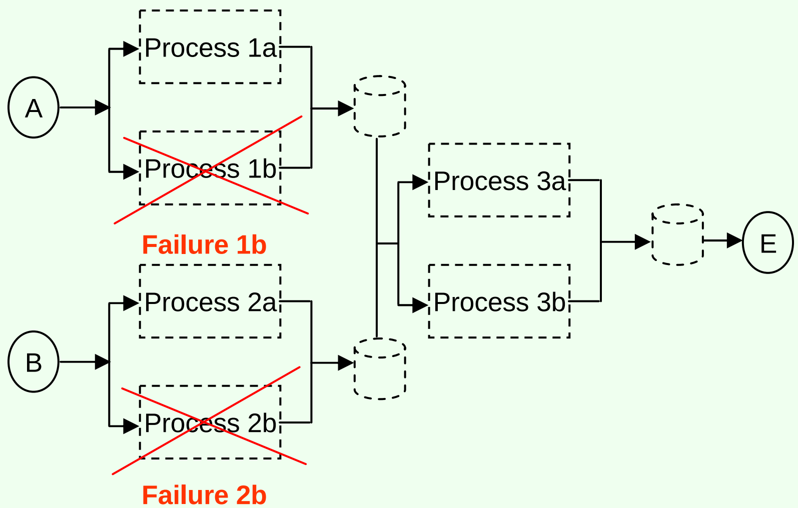
$$f_{3b}^{out} \le y_{3b} Capacity_{3b}$$

$$(f_{3a}^{out} + f_{3b}^{out}) - f_E \le \frac{v_3}{\text{repair time}}$$

The uncertainty in process failures leads to a stochastic programming (SP) formulation

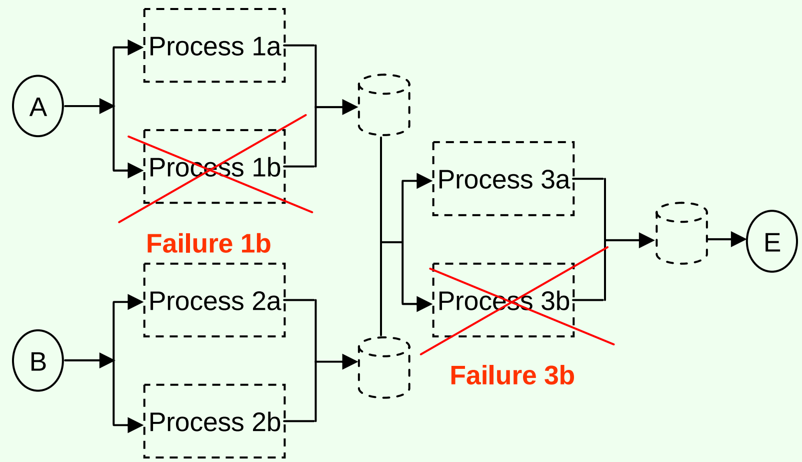
The network is subject to failures; in each failure scenario the model of the network (mass balance, superstructure constraints, etc.) has a different solution, i.e.:

Scenario 1 (s = 1)



Flows in the network, $f^{s=1}$ are different in scenario 1…

Scenario 2 (s = 2)



… than in scenario 2, $f^{s=2}$.

The mass balance equations in the model are repeated for each scenario:

$$m^s(f^s, y, v, \theta^s) \leq 0$$

⇨ *Compact representation of the equations in the model for each scenario*

The uncertainty in process failures leads to a stochastic programming (SP) formulation

Two-stage stochastic programming formulation

$$\max \sum_{s} p_s f_E^s$$

- p_s: probability of each scenario
- f_E: flow of finished product

$$s.t.$$

$$m^s(f^s, y, v, \theta^s) \leq 0$$

$$investment(y, v) \leq \varepsilon$$

- Flow variables f^s are assigned to each scenario
- Design variables y,v, are not a function of the scenario

$$s = 1,2,\ldots,S$$

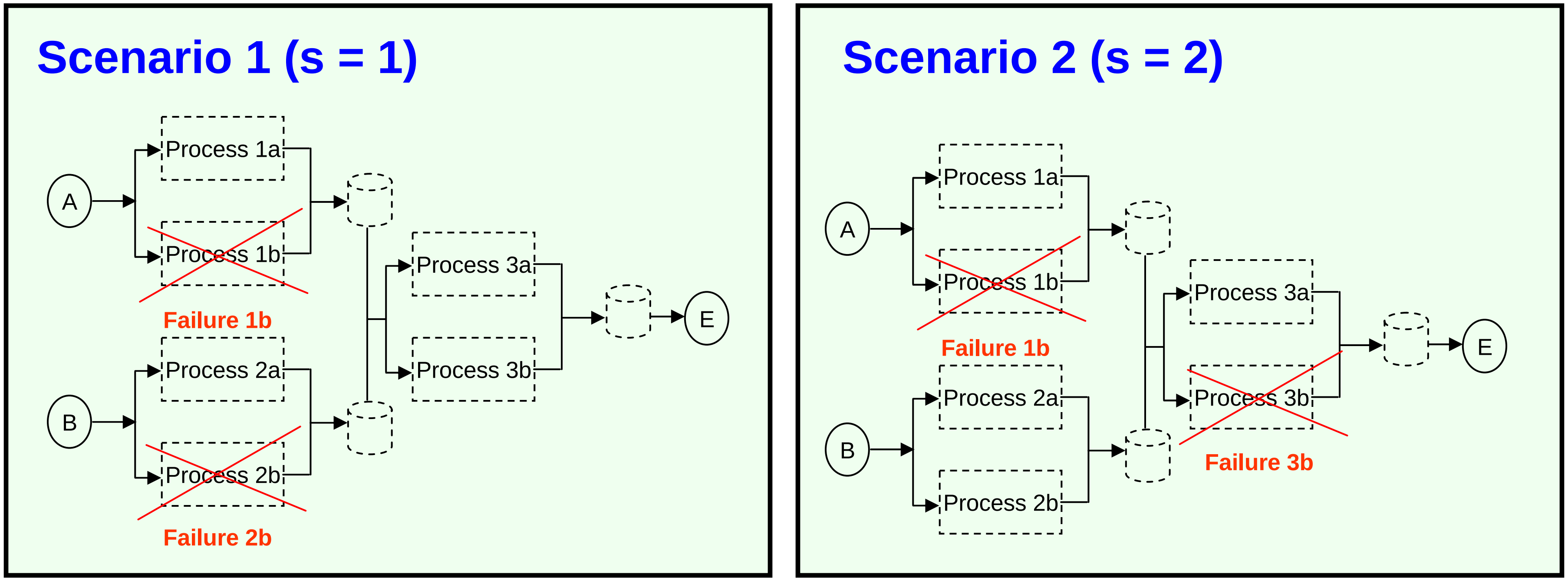
- One scenario for each combination of failure modes ***in the superstructure***

- Objective function represents the average productivity
- We want to determine the design that maximizes the average productivity

Modeling endogenous uncertainties in the SP formulation requires non-anticipativity (NA) constraints

We still need more constraints…

Scenarios 1 and 2 are indistinguishable IF Process 1b, 2b, and 3b are not chosen from the superstructure



$$\left.\begin{array}{l} \textbf{IF} \quad y_{1b}, y_{2b}, y_{3b} = 0 \\ f^{s=1} = f^{s=2} \end{array}\right\}$$

- Equation for each possible pair of scenario
- Equation is activated or deactivated as a function y
- These are Non-anticipativity constraints

Modeling endogenous uncertainties in the SP formulation requires non-anticipativity (NA) constraints

$$\max \sum_{s} p_s f_E^s$$

$$s.t.$$

$$m^s(f^s, y, v, \theta^s) \le 0$$

$$investment(y, v) \le \varepsilon$$

$$f^s \le f^{s'} + \alpha(y)$$

$$f^s \ge f^{s'} - \alpha(y)$$

$$s = 1,2,\ldots,S$$

$$s' = 1,2,\ldots,S$$

Remarks about formulation

- **Optimizes the average performance** of network
- The probability of each scenario is affected by the design decisions: **endogenous uncertainty**.
- To model endogenous uncertainty we need **non-anticipativity (NA) constraints**
- Solve for different values of ε to obtain **Pareto-optimal solutions**

The number of NA constraints turns most industrial applications into large-scale optimization problems

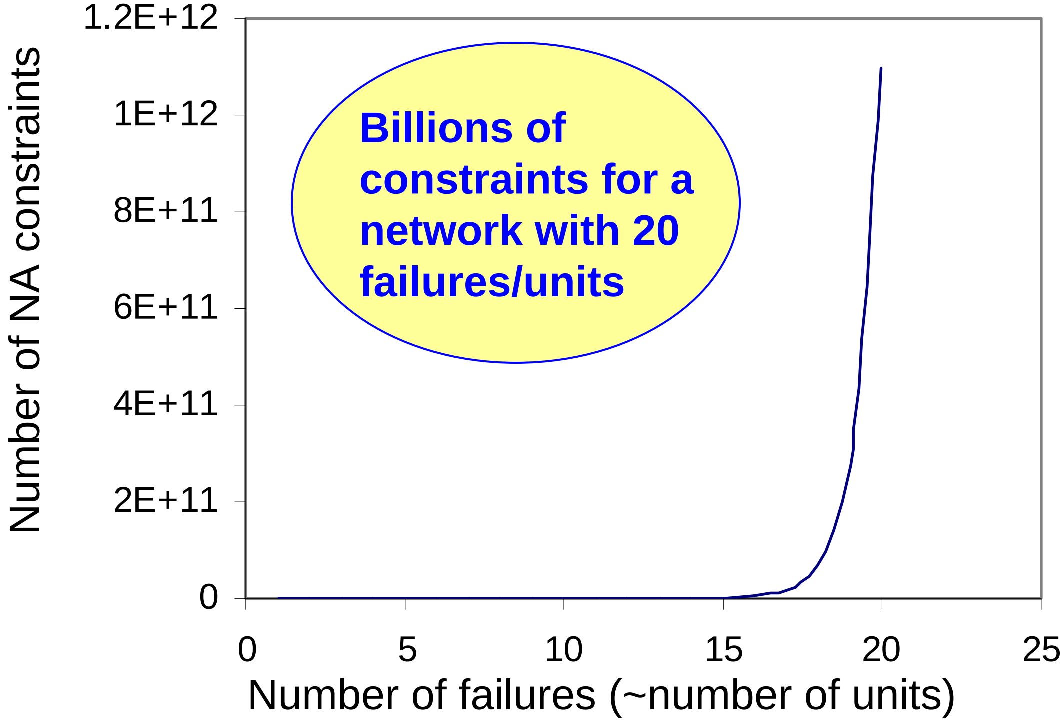
- Number of scenarios as a function of possible failures in the network:

$$scenarios = 2^n$$

$$n = \text{Number of possible failures}$$

- Number of non-anticipativity constraints required

$$\# NA = s^2 = 2^{2n}$$

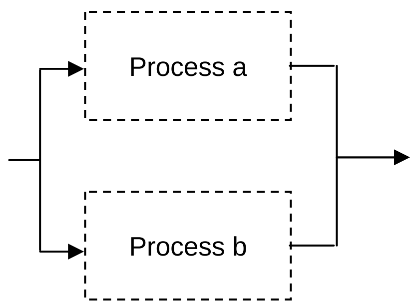


Largest Mixed-integer (continuous and discrete variables) problems can involve a **few million constraints**

We propose an algorithm based on Benders decomposition to reduce the number of NA constraints

Basic idea: If design variables are fixed there is NO need for NA constraints

For a *superstructure* with two parallel redundant units we have 4 scenarios:



Scenario	Process A	Process B
1	Functional	Functional
2	Failure	Functional
3	Functional	Failure
4	Failure	Failure

and two NA constraints*

$$f^{s=1} \leq f^{s=2} + \alpha(y_1, y_2)$$
$$f^{s=1} \geq f^{s=2} - \alpha(y_1, y_2)$$

We propose an algorithm based on Benders decomposition to reduce the number of NA constraints

Basic idea: If design variables are fixed there is NO need for NA constraints

But if the designed is fixed we need only construct the scenarios relevant for the process installed

Scenario	Process A
1	Functional
2	Failure

and we don’t need NA constraints

Fewer scenarios, no NA constraints:
Problem smaller by orders of magnitude

We propose an algorithm based on Benders decomposition to reduce the number of NA constraints

Proposed algorithm:

1. Fix design variables y,v
2. Construct only the scenarios relevant for the processes included in the design
3. Optimize the network for the given design
4. Use a different trial value for y and v

Add dual cut between steps 3. and 4.

The resulting algorithm is equivalent to Benders Decomposition

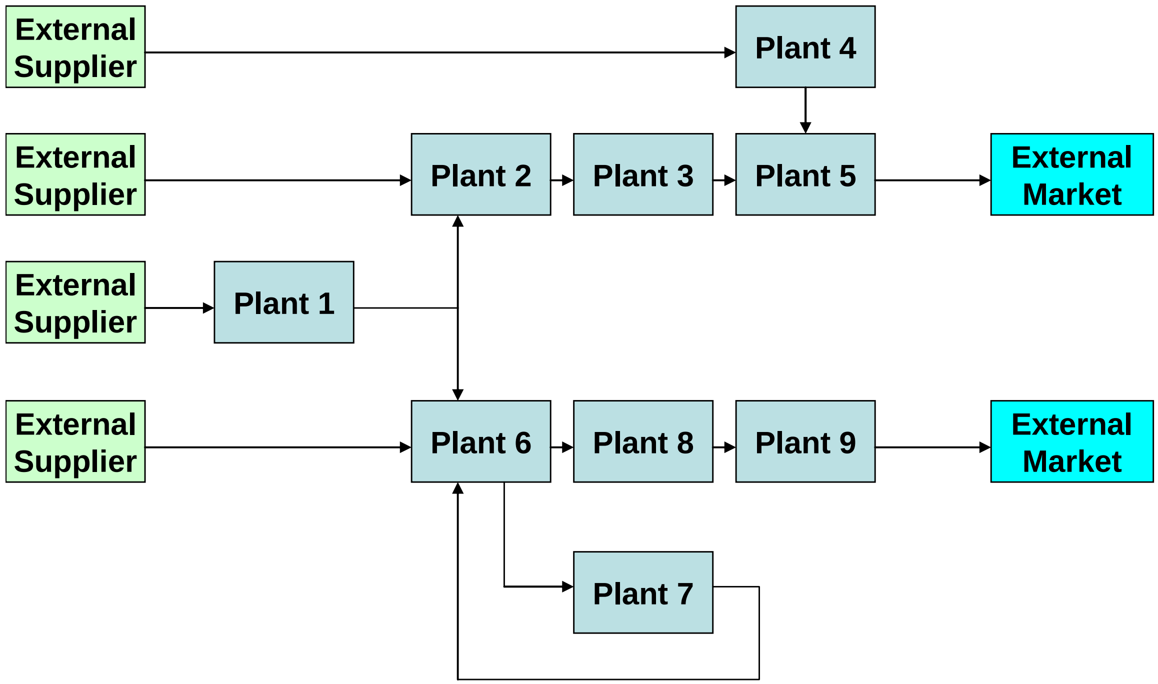
Our contribution is to allow the solution of the Benders sub-problem in a reduced space

	Subproblem	Dual Cut	Master Problem	Construct scenarios
Description	$\max \sum_s p_s f_E^s$ $s.t.$ $m^s(f^s, v, y, \theta^s) \le 0$ • Reduced scenarios • No NA constraints	The dual cut is derived in $S_{Subproblem}$ but must be valid in S^2_{Master}	$\max \sum_s p_s f_E^s$ $s.t.$ $m^s(f^s, y, v, \theta^s) \le 0$ $investment(y,v) \le \varepsilon$ $f^s \le f^{s'} + \alpha(y)$ $f^s \ge f^{s'} - \alpha(y)$ $Dualcut^k(y,v) \le 0$	The scenarios for the subproblem: $S_{Subproblem}$ can be different in each iteration
Relevant space	$s \in S_{Subproblem}$		$(s,s') \in S^2_{Master}$	

Lower Bound ⇨ **Converge to optimum** ⇦ **Upper Bound**

An industrial case study illustrates the impact of using the proposed algorithm (1/2)

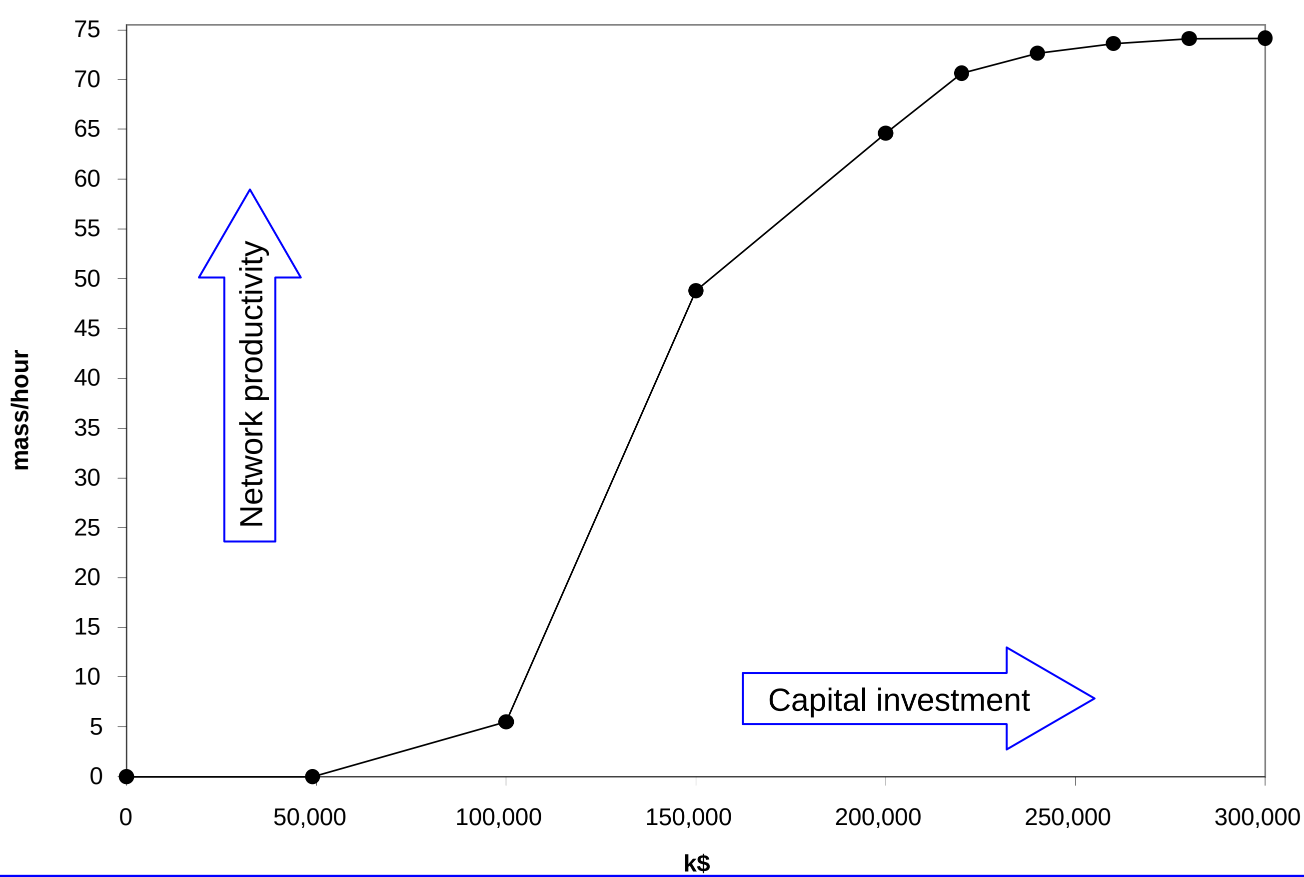
Case study provided by Dow



- Real integrated process called a product "envelope"
- 1000 failure scenarios were included in the problem formulation
- Work station (2G RAM) is not able to generate the model in GAMS

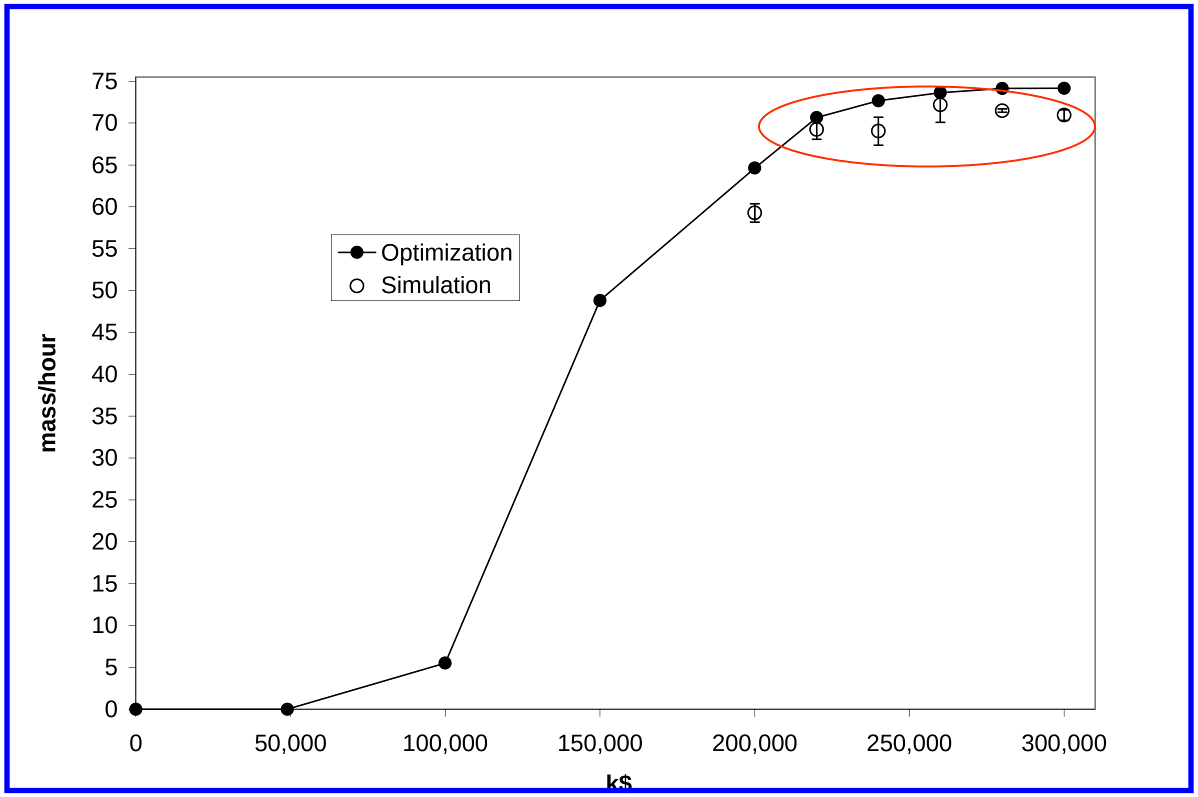
An industrial case study illustrates the impact of using the proposed algorithm (2/2)

- **Problem cannot be solved in workstation (2G RAM) due to insufficient memory**
- **A complete Pareto front for 10 values of available capital for investing (problem solved 10 times) can be generated in less than 6 hrs**



We validated the design obtained using the proposed decomposition technique

- Dow provided us with a simulation model that has been validated internally as an adequate representation of the real system
- We use this simulation to test the optimal designs found by the two-stage stochastic simulation



- **Results of simulation validate the accuracy of the optimization model**

Summary and next steps

- We formulated the design of reliable process networks under uncertainty as a two-stage stochastic program with endogenous uncertainties

- This a powerful representation that can be used to model a variety of problems that arise in industrial practice

- We proposed an algorithm to partly overcome the computational burden associated with this type of formulation

- We successfully applied this algorithm to the bi-criterion optimization of an industrial case study

- Next steps involve extending the modeling framework and solution methodology to batch and discrete manufacturing processes in multiproduct process networks