Inventory Optimization for Process Network Reliability

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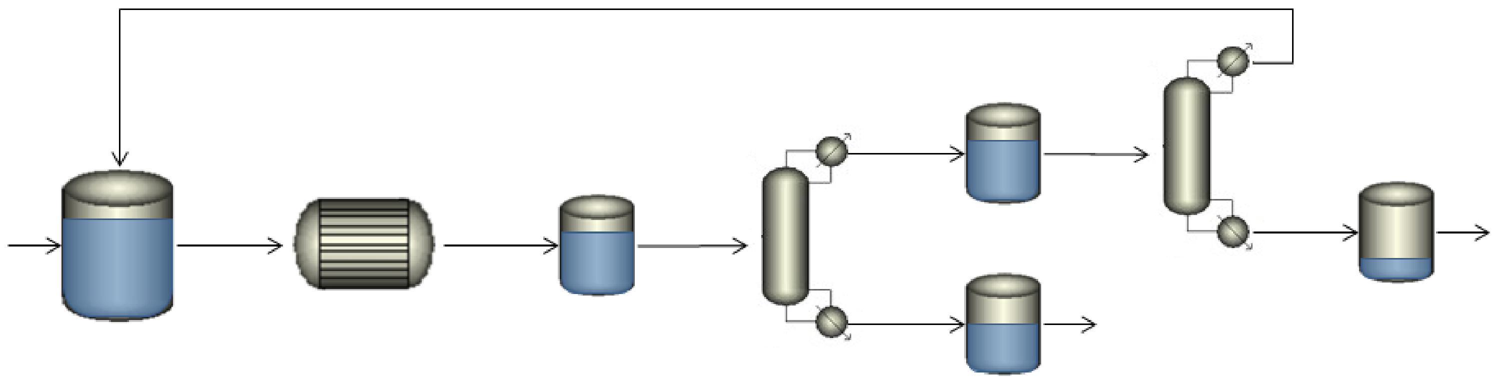
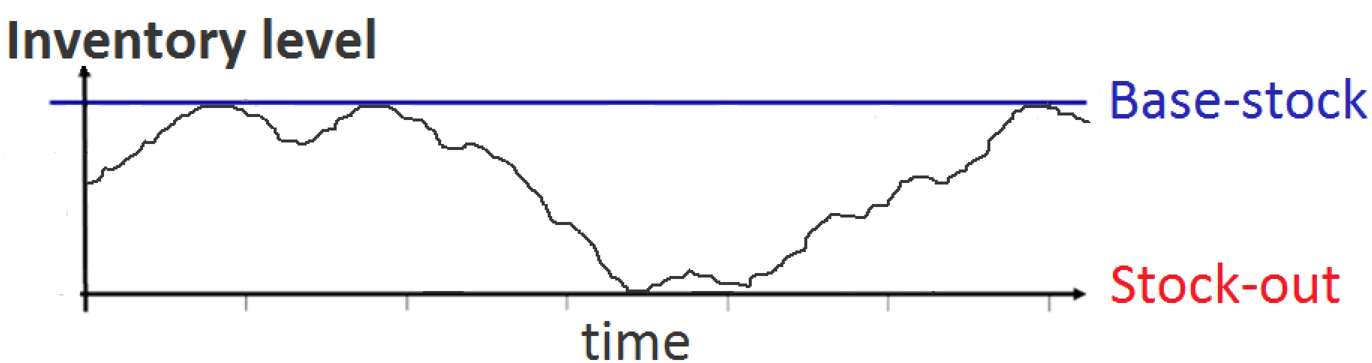
Introduction



Process networks describe the operation of chemical plants

- Integration of complex operations
- Continuous flowrates
- Inventory availability is constrained by production capacity
- Inventory changes continuously (continuous replenishment)

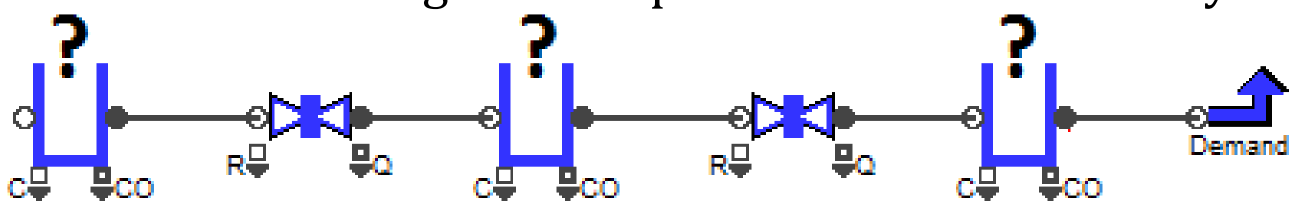


Problem Statement

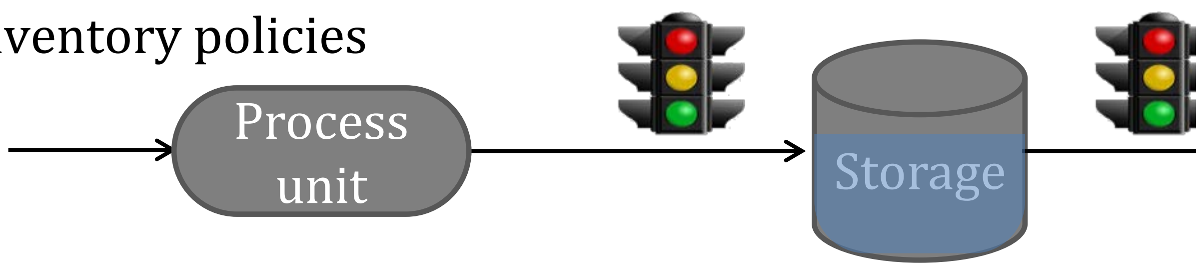
Stochastic Inventory Optimization

Minimize inventory levels that guarantee high demand satisfaction under uncertain production rates

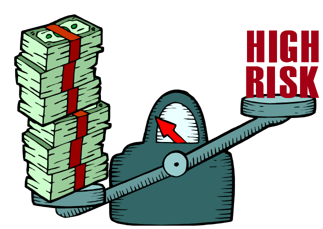
- Determine at which stages of the process to hold inventory



- Establish inventory policies



- Balance holding cost and service



Inventory Policy

Inventory depletion:

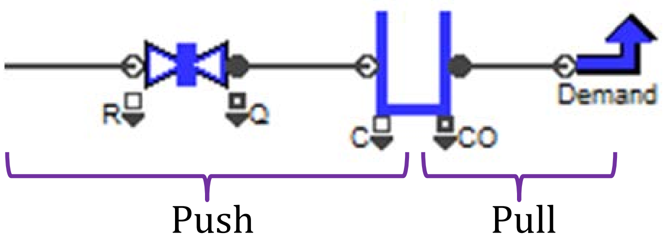
1. Satisfy demand rate with available inventory
2. If inventory is stocked-out, constrain demand satisfaction rate to inventory replenishment rate

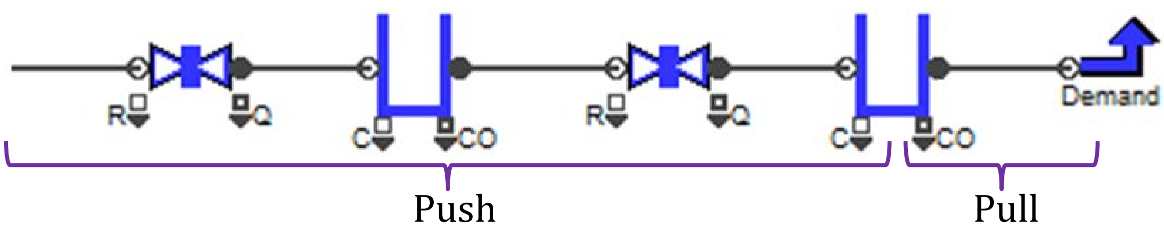
Inventory replenishment:

1. Replenish inventory at the upstream production rate
2. If inventory target is reached, decrease replenishment rate to match demand rate

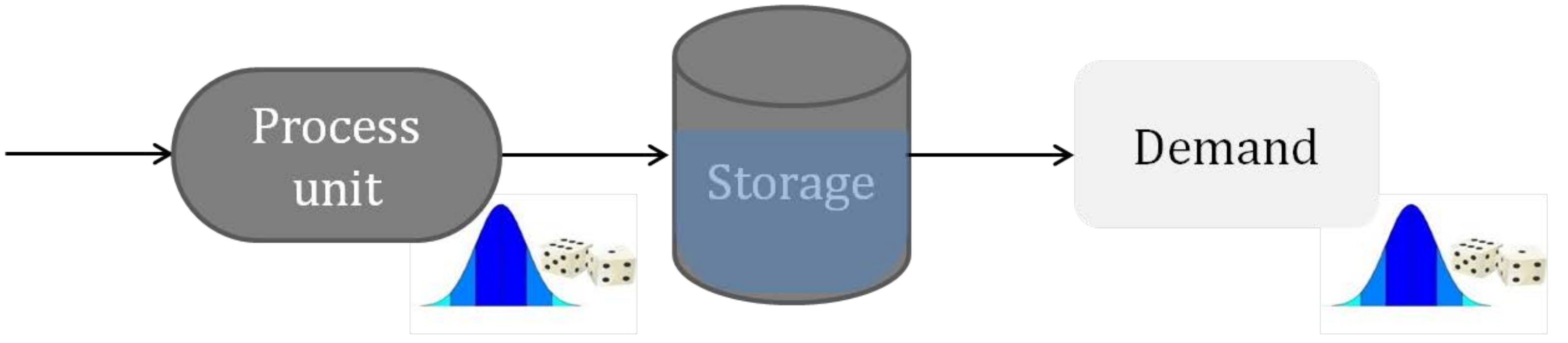
Inventory policy is characterized by an inventory target

Policy is a combination of **push and pull system** with inventories as buffer





Single Inventory System



Maximum production rate (*R*) and demand rate (*D*) are random:

- Potential input rate to the tank: *R*
- Potential output rate: *D*

Actual input and output rates depend on the inventory level and inventory target

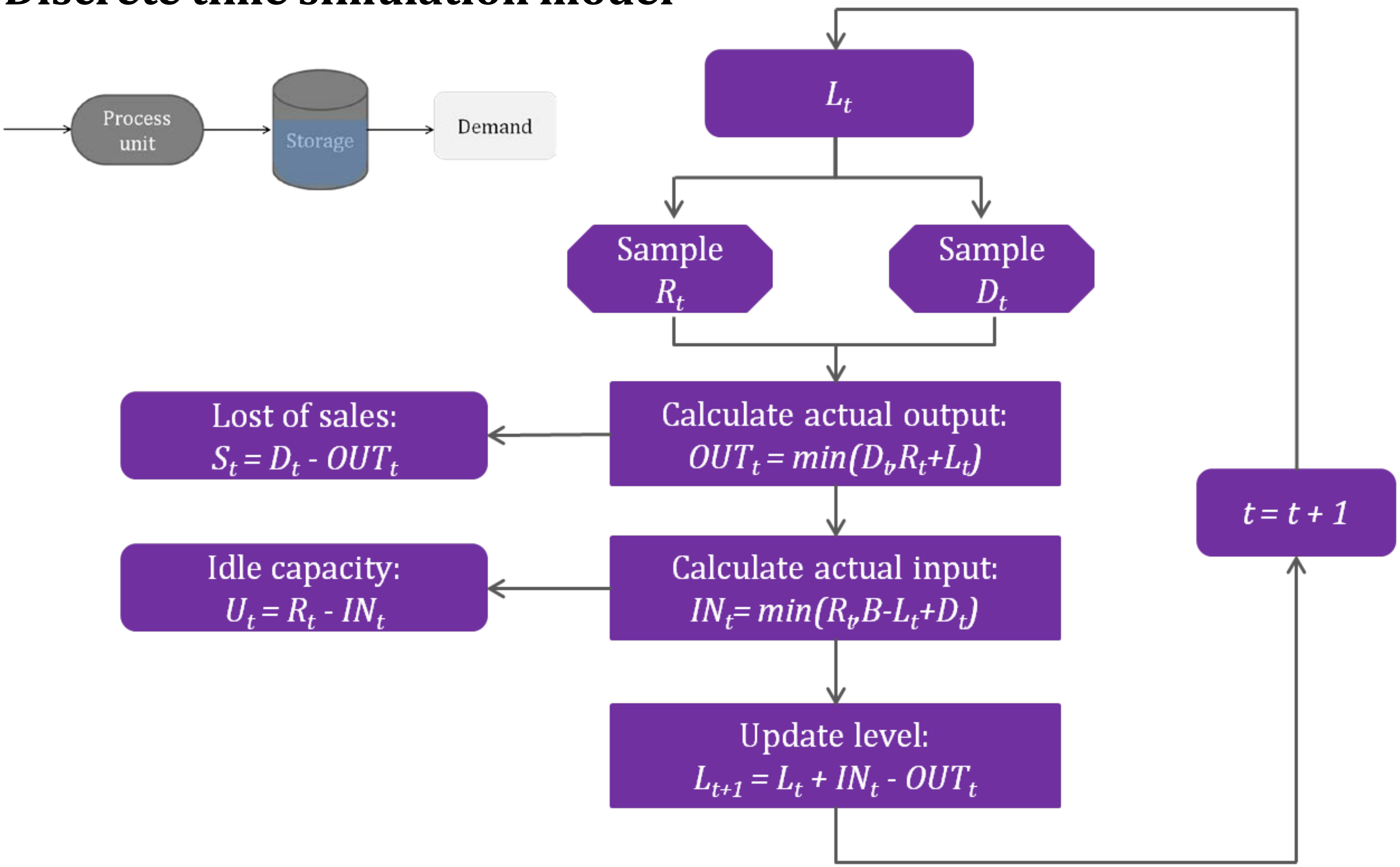
Inventory buffers input and output mismatches

Inventory level is a random variable: depends on the history of *R* and *D*

Measure of performance: expected fraction of demand that is satisfied (β-service level)

Numerical Approach

Discrete time simulation model



Numerical Approach

Discrete time optimization model:

Find minimum inventory target (B) that provides service level (SL^{β}) for given sample paths (n) of random variables ($R_{n,t}$, $D_{n,t}$)

$$\min \quad B$$

$$s.t. \quad L_{n,t} = L_{n,t-1} + \left(R_{n,t} - U_{n,t}\right) - \left(D_{n,t} - S_{n,t}\right) \qquad \forall t \in T,\ \forall n \in N$$

$$\begin{bmatrix} L_{n,t-1} + R_{n,t} - D_{n,t} < 0 \\ S_{n,t} = D_{n,t} - R_{n,t} - L_{n,t-1} \\ U_{n,t} = 0 \end{bmatrix} \vee \begin{bmatrix} L_{n,t-1} + R_{n,t} - D_{n,t} > B \\ U_{n,t} = L_{n,t-1} + R_{n,t} - D_{n,t} - B \\ S_{n,t} = 0 \end{bmatrix} \vee \begin{bmatrix} U_{n,t} = 0 \\ S_{n,t} = 0 \end{bmatrix} \qquad \forall t \in T,\ \forall n \in N$$

$$SL^{\beta} \le 1 - \frac{1}{N} \sum_{n=1}^{N} \left(\sum_{t=1}^{T} S_{n,t} \Big/ \sum_{t=1}^{T} D_{n,t} \right)$$

$$L_{n,t} \le B \qquad \forall t \in T,\ \forall n \in N$$

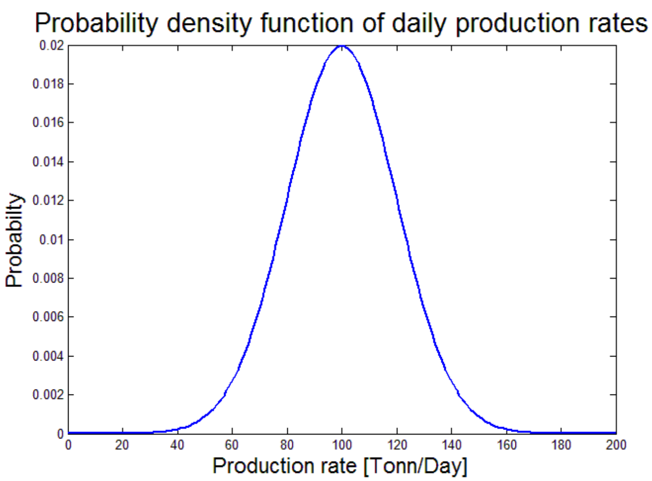
$$L_{n,t} \ge 0; \quad U_{n,t} \ge 0; \quad S_{n,t} \ge 0; \qquad \forall t \in T,\ \forall n \in N$$

Case Study

Normally distributed production rate

Problem data:

- Production rate: $R \sim N(100,20)$ Tonn/day
- Demand rate: $D = 100$ Tonn/day
- Service level: $SL \geq 0.95$



Optimization parameters:

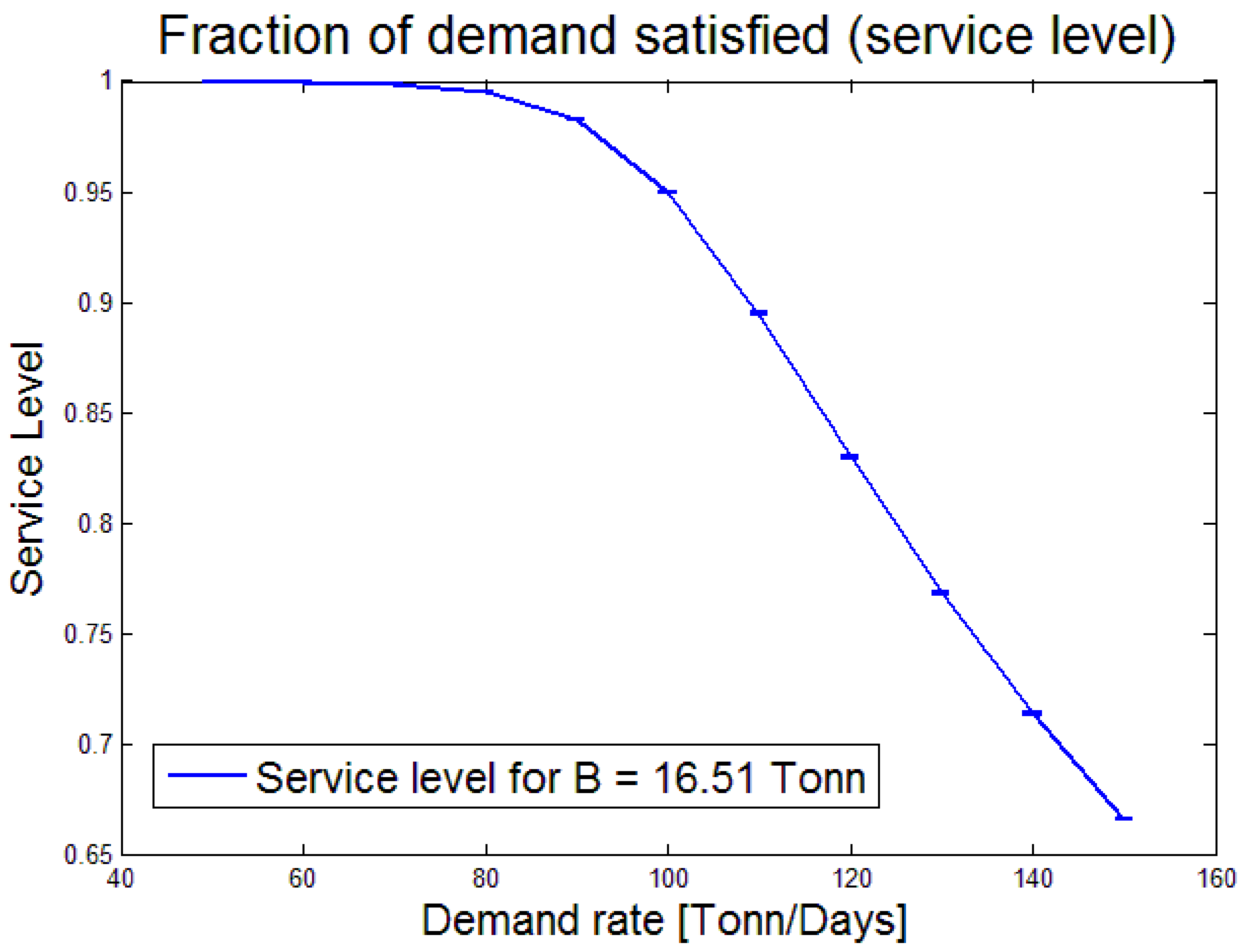
- Simulated time horizon: 5,000 days
- Number of runs: 50

Results:

- Optimal inventory target: $E[B] = 16.55$
  $Var(B) = 1.22^2$

Case Study

Normally distributed production rate



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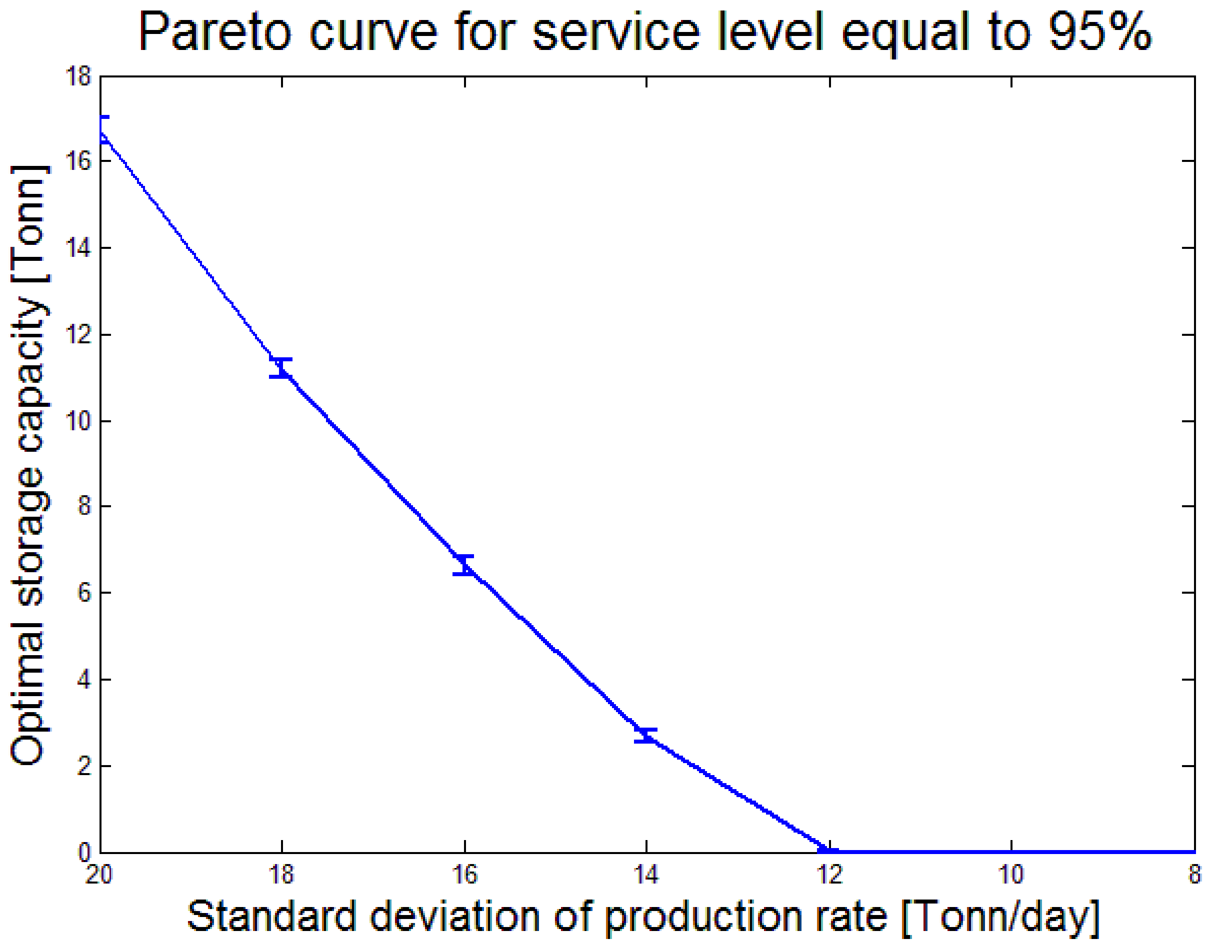
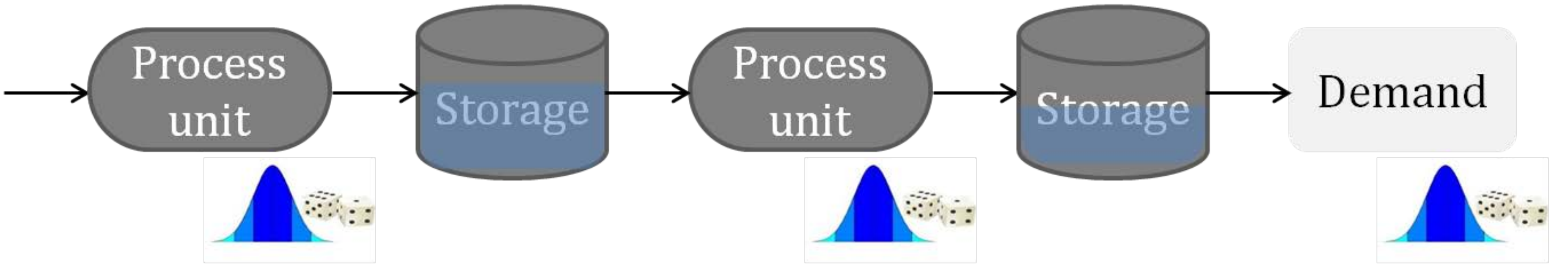


Processes in Tandem



Maximum production rates (*R_1* and *R_1*) and demand rate (*D*) are random:

- Potential input rate to tank 1: R_1
- Potential output rate from tank 1: R_2
- Potential input rate to the tank 2: R_2
- Potential output rate from tank 2: D

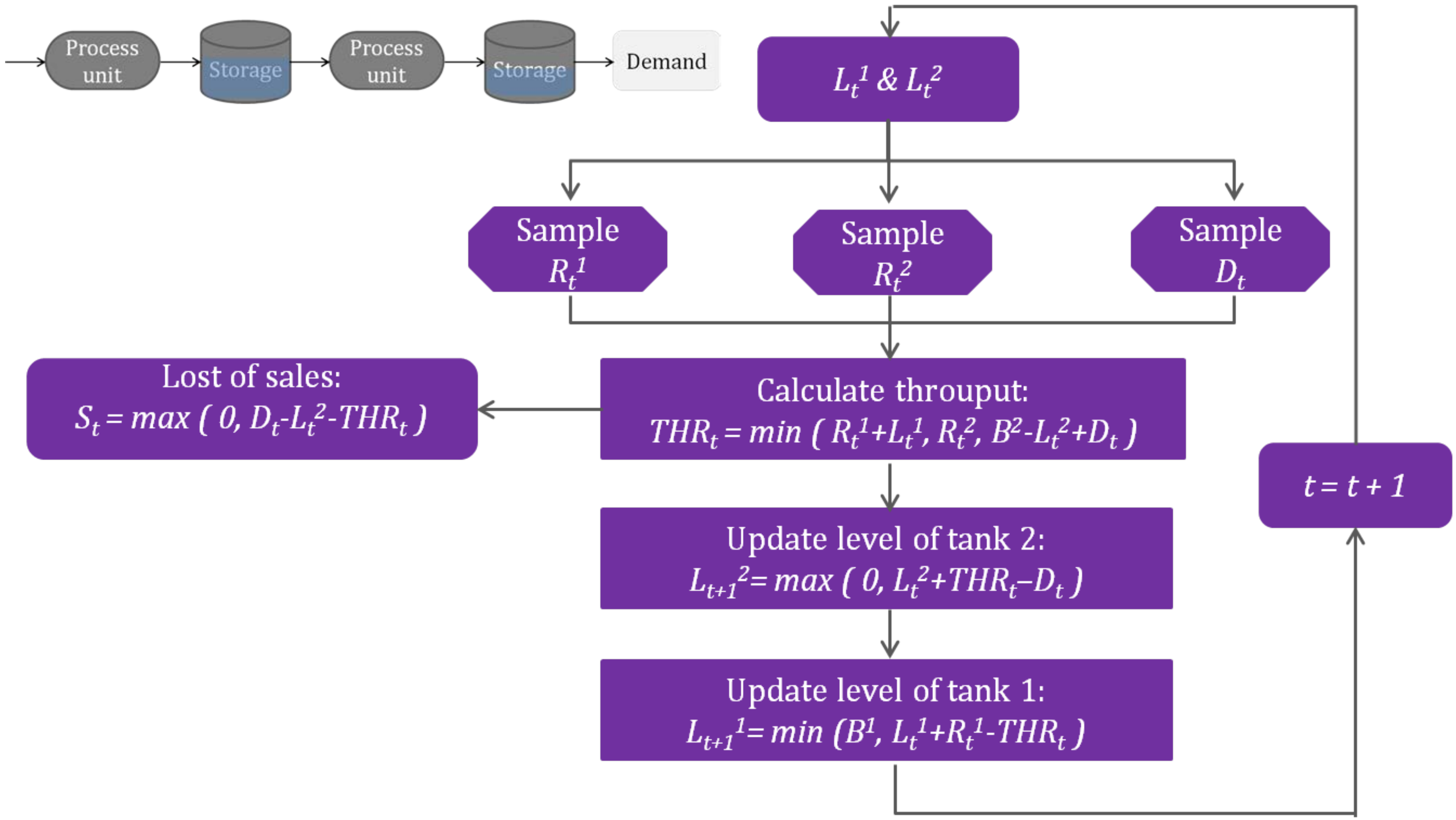
Actual input and output rates depend on inventory levels and targets

Inventory levels are random variables

Measure of performance: expected fraction of demand that is satisfied

Numerical Approach

Discrete time simulation model of two processes in tandem



Numerical Approach

Discrete time optimization model:

Find minimum inventory targets (B^1 *and* B^2) that provides service level (SL^β) for given sample paths (n) of random variables (R_t^1, R_t^2, D_t)

$$\min \quad B^1 + B^2$$

$$\text{s.t.} \quad L^1_{n,t} = L^1_{n,t-1} + \left(R^1_{n,t} - U^1_{n,t}\right) - THR_{n,t} \qquad \forall t \in T,\ \forall n \in N$$

$$L^2_{n,t} = L^2_{n,t-1} + THR_{n,t} - \left(D_{n,t} - S^2_{n,t}\right) \qquad \forall t \in T,\ \forall n \in N$$

$$THR_{n,t} \le B^2 - L^2_{n,t} + D_{n,t} \qquad \forall t \in T,\ \forall n \in N$$

$$THR_{n,t} \le L^1_{n,t} + R^1_{n,t} \qquad \forall t \in T,\ \forall n \in N$$

$$THR_{n,t} \le R^2_{n,t} \qquad \forall t \in T,\ \forall n \in N$$

$$\forall t \in T,\ \forall n \in N$$

$$\begin{bmatrix} L^1_{n,t-1} + R^1_{n,t} - THR_{n,t} > B^1 \\ U^1_{n,t} = L^1_{n,t-1} + R^1_{n,t} - THR_{n,t} - B^1 \end{bmatrix} \vee \left[U^1_{n,t} = 0\right] \qquad \forall t \in T,\ \forall n \in N$$

$$\begin{bmatrix} L^2_{n,t-1} + THR_{n,t} - D_{n,t} < 0 \\ S^2_{n,t} = D_{n,t} - L^2_{n,t-1} + R^1_{n,t} - THR_{n,t} \end{bmatrix} \vee \left[S^2_{n,t} = 0\right] \qquad \forall t \in T,\ \forall n \in N$$

$$SL^\beta \le 1 - \frac{1}{N}\sum_{n=1}^{N}\left(\sum_{t=1}^{T} S^2_{n,t} \Big/ \sum_{t=1}^{T} D_{n,t}\right) \qquad \forall t \in T,\ \forall n \in N$$

$$L^1_{n,t} \ge 0 \quad L^2_{n,t} \ge 0 \quad U^1_{n,t} \ge 0; \quad S^2_{n,t} \ge 0; \qquad \forall t \in T,\ \forall n \in N$$

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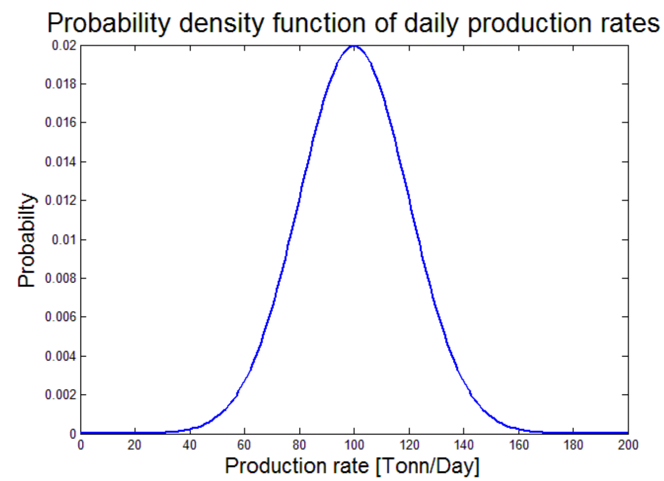


Optimization parameters:

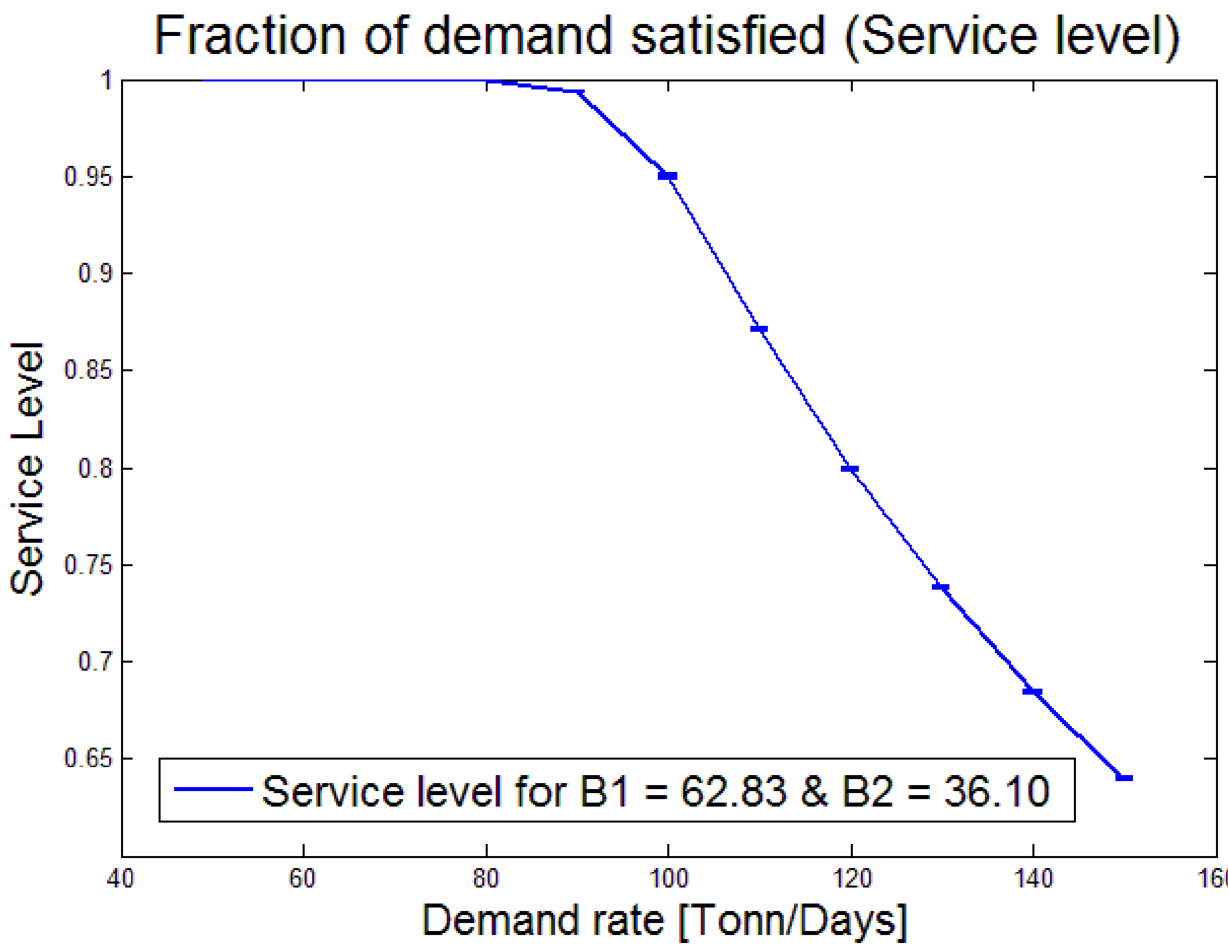
- Simulated time horizon: 5,000 days
- Number of runs: 20

Results:

- Optimal inventory target: $E[B^1] = 62.83$; $Var(B^1) = 3.20^2$
  $E[B^2] = 36.10$; $Var(B^2) = 1.64^2$

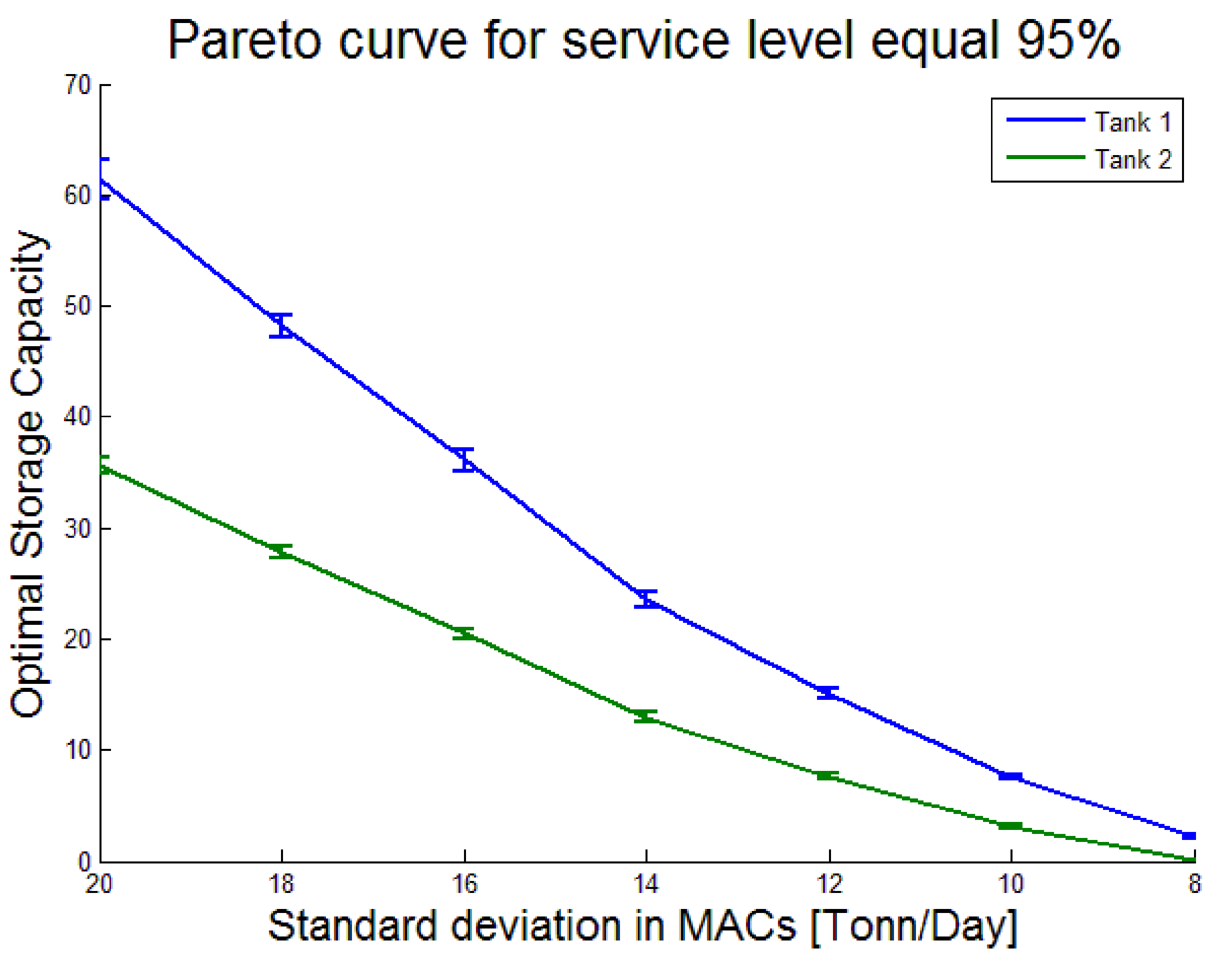
Case Study

Normally distributed production rates



Case Study

Normally distributed production rates



Conclusions

- **Simulation-optimization** approaches are suitable for inventory optimization in **uncertain** production systems
- **Inventory policy** can be modeled by using **disjunctions**
- Significant **increases in inventory** are needed when variability in **tandem production** units is considered
- **Inventory** is more effective if stored **up-stream**: avoids downstream starving