Optimizing Inventory Policies for Industrial Network Reliability

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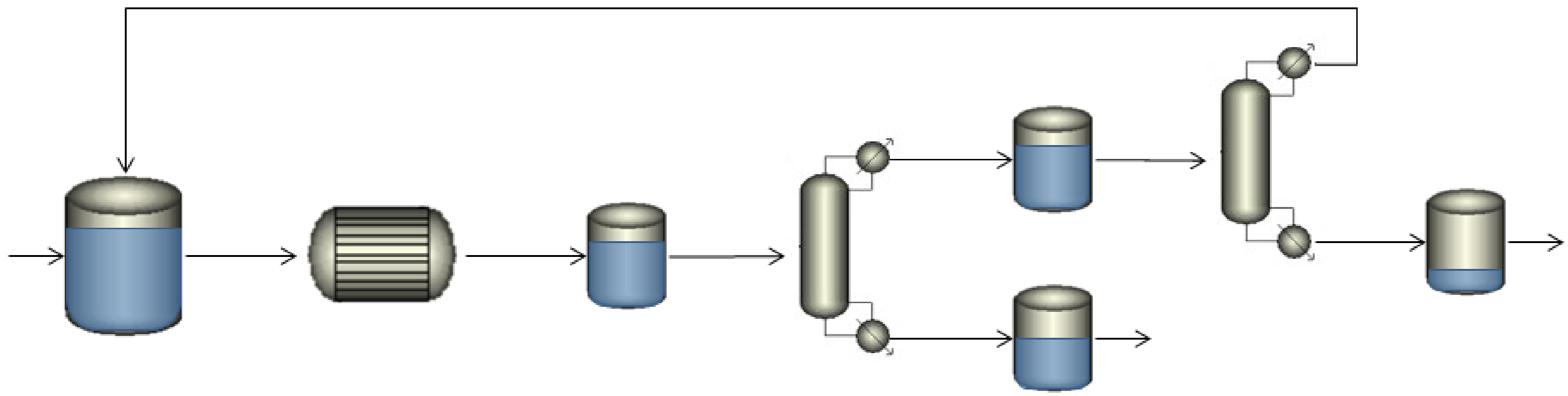
Motivation

Process networks describe the operation of chemical plants

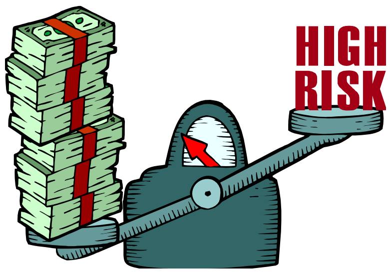


Inventories are necessary because of process uncertainty:

- Raw material storage tanks hedge against supply variability
- Intermediate storage tanks hedge against production rates variability
- Finished product inventories hedge against demand variability

Holding inventory is expensive!

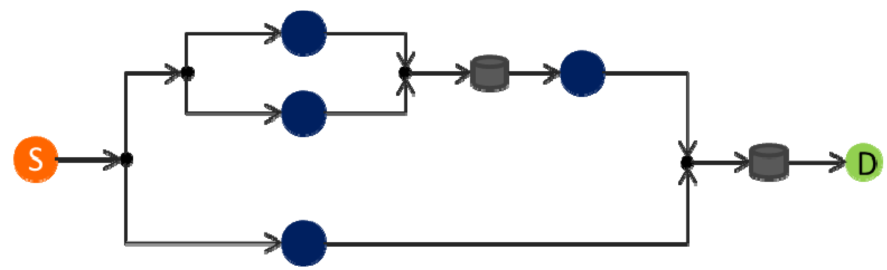
Need to trade-off between inventory and stock-out cost



Problem Statement

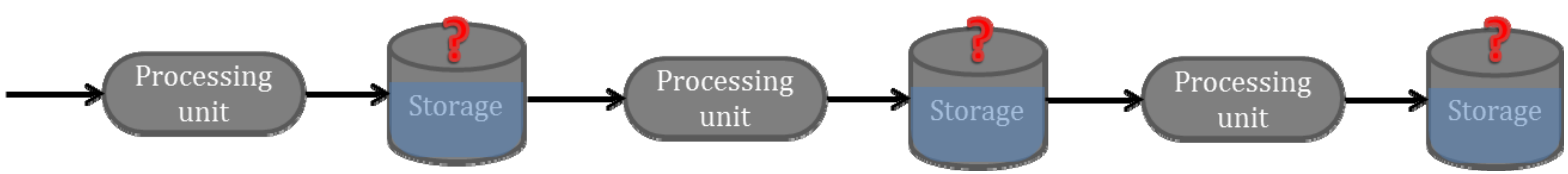
Given:

- A process network with several processing units
- A discrete time finite horizon
- Probabilistic description of supply, processing rates, and demand over the entire horizon
- Storage units



Find the operating plan that minimizes inventory and stockout costs:

- Determine the optimal inventory management strategy in every time period
  - ➢ Location of inventory
  - ➢ Amount of inventory

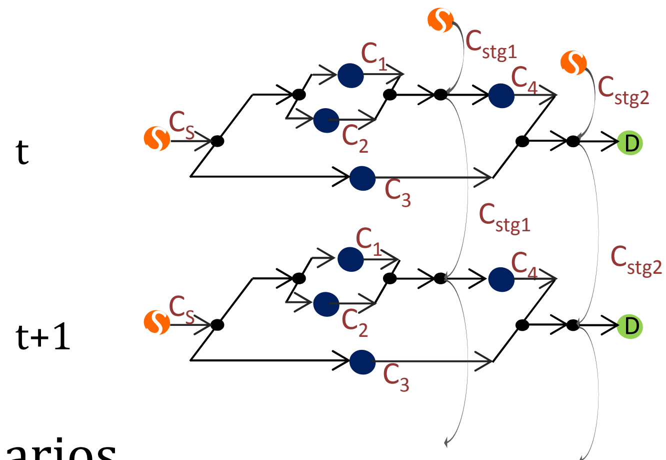


Multistage Formulation (MILP)

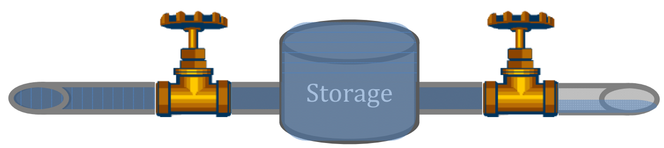
Minimize: expected inventory cost + expected stockout cost

Subject to:

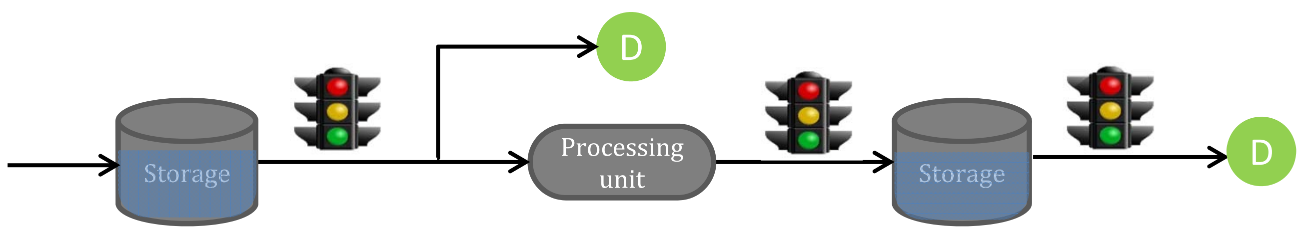
- Mass balances in all scenarios



- Capacity constraints in all scenarios



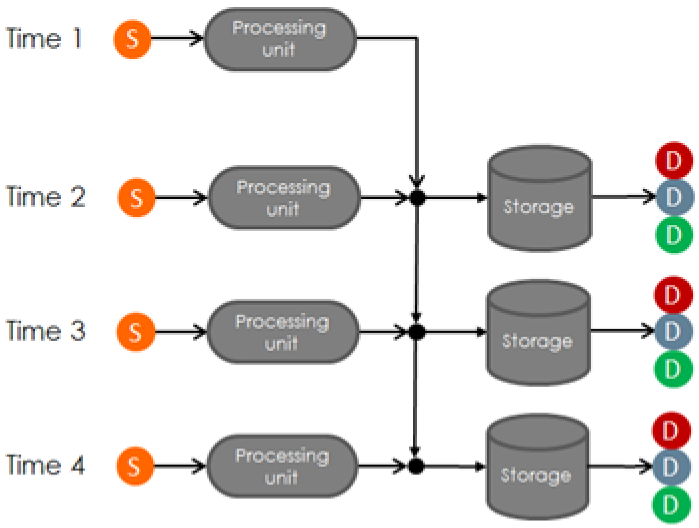
- Single policy for all scenarios in a decision stage

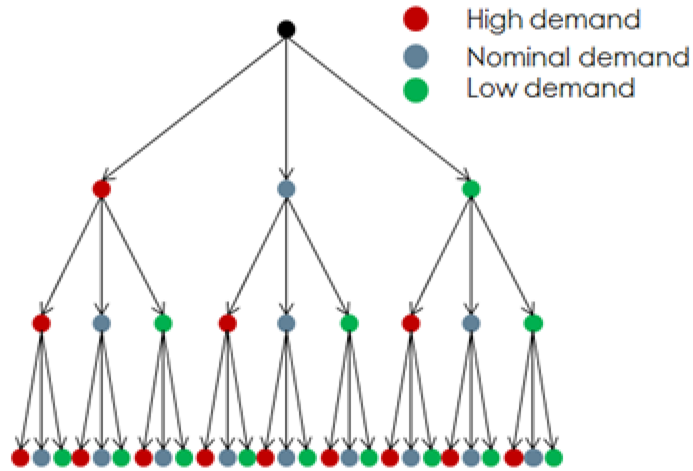


- Bounds

Single-echelon Inventory Policy

Example: Multistage stochastic programming formulation





Alternative: rules to operate the production-inventory system with limited capacity

- Stockouts are allowed if tank is empty:

$$s_t^\xi = \begin{cases} 0, & \text{if } s_t^\xi > 0 \\ D_t^\xi + s_t^\xi - x_{t-1}^\xi - C_t^\xi & \text{if } x_t^\xi = 0 \end{cases} \quad \forall\, t \in T, \xi \in \Xi$$

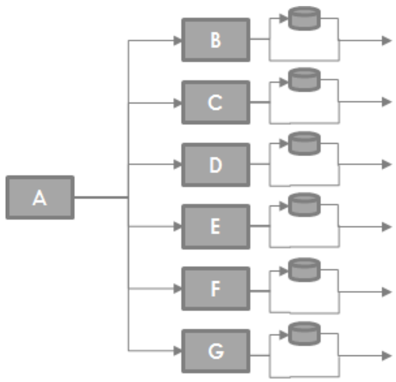
- Capacity underutilization is allowed if tank is at basestock level:

$$u_t^\xi = \begin{cases} 0, & \text{if } x_t^\xi < y_t \\ x_{t-1}^\xi - C_t^\xi - D_t^\xi - s_{t-1}^\xi & \text{if } x_t^\xi = y_t \end{cases} \quad \forall\, t \in T, \xi \in \Xi$$

Sets
Ξ: set of scenarios
T: set of time periods
Parameters
D_t^ξ: Demand
C_t^ξ: Capacity
Variables
x_t^ξ: inventory level
y_t: basestock level
s_t^ξ: stockouts
u_t^ξ: underutilization

Policy for Inventories in Parallel

Establish priorities for replenishment when inventories share the same source



Sets
I: set of production-inventory installations
L: set of priority levels
Variables

$$z_{l,i} = \begin{cases} 1 & \text{if tank } i \text{ is assined priority } l \\ 0 & \text{otherwise} \end{cases}$$

$$w_{t,l}^{\xi} = \begin{cases} 1 & \text{if tank with priority } l+1 \text{ can be replenished} \\ 0 & \text{otherwise} \end{cases}$$

$r_{t,i}^{\xi}$: replenishment to tank i

- **Assign unique priorities to each inventory:**

$$\sum_{i \in I} z_{l,i} = 1 \quad \forall\, l \in L\,; \qquad \sum_{l \in L} z_{l,i} = 1 \quad \forall\, i \in I$$

- **Relate $w_{t,l}^{\xi}$ with the inventory level at tank with priority l:**

$$\bigvee_{l \in L} \begin{bmatrix} z_{l,i} = 1 \\ x_{t,i}^{\xi} < y_{t,i} \\ u_{t,i}^{\xi} > 0 \end{bmatrix} \Rightarrow w_{t,l}^{\xi} = 0 \qquad \forall\, t \in T, i \in I, \xi \in \Xi$$

- **Inventory is replenished according to their priorities:**

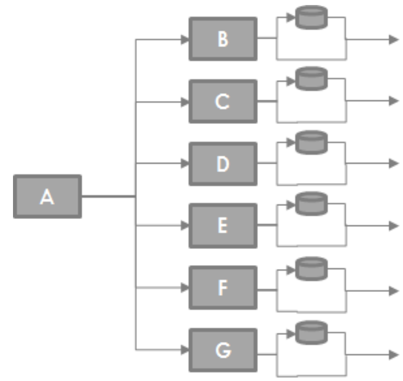
$$w_{t,l}^{\xi} \geq w_{t,l+1}^{\xi} \qquad \forall\, t \in T, l \in L, \xi \in \Xi$$

$$\bigvee_{l \in L} \begin{bmatrix} z_{l,i} = 1 \\ w_{t,l-1}^{\xi} = 0 \end{bmatrix} \Rightarrow r_{t,i}^{\xi} = 0 \qquad \forall\, t \in T, i \in i, \xi \in \Xi$$

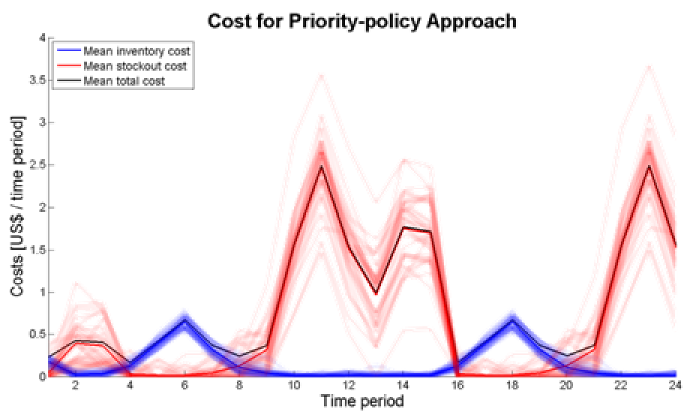
Example for Inventories in Parallel

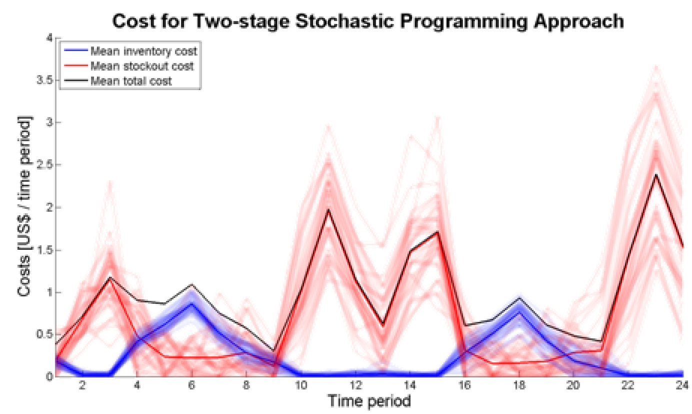
Process A supply processes B to G.

- Deterministic demands for 6 products
- Production capacities are random
- Simulation length: 24 time periods
- Planning horizon: 12 time periods
- Number of replications: 30



Results:





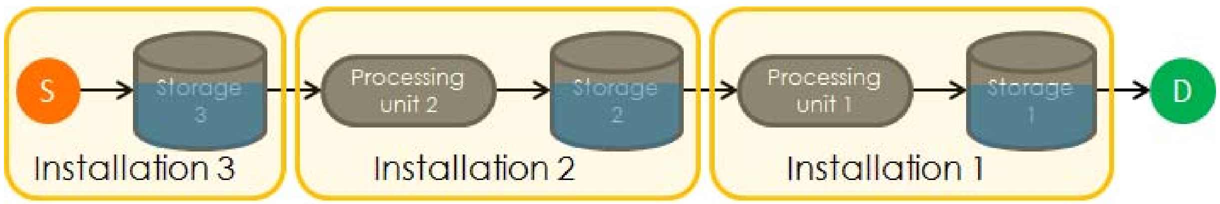
	Priority-policy formulation	Two-stage stochastic program
Expected Inventory cost [$/period]	3.78	5.64
Expected stockout cost [$/period]	17.53	18.28
Expected total cost [$/period]	21.31	23.92

- The formulation with priority policy produces lower inventory and stockout cost
- The total expected savings account for 10.9%
- The priority-policy formulation is effective recognizing the products with higher stockout risk

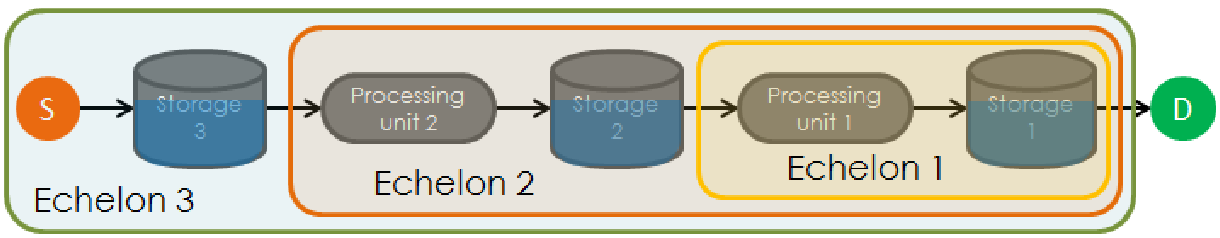
Policy for Inventories in Series

Tanks in series form a multiechelon system for inventory management

- Installations are numbered starting at the tank that is farthest downstream



- Echelon (e) includes installations ($i \in I_e$) that are downstream:



- Basestock levels are defined for echelons:

$$\sum_{i \in I_e} x_{t,i}^{\xi} \leq y_{t,e} \qquad \forall\, t \in T, e \in E, \xi \in \Xi$$

- Upstream underutilization is allowed if echelon inventory is at basestock level :

$$\sum_{i \in I_e} x_{t,i}^{\xi} < y_{t,e} \Rightarrow \max_{\{i|\, i \in I, i \geq e\}} \left[u_{t,i}^{\xi} \right] = 0 \qquad \forall\, t \in T, e \in E, \xi \in \Xi$$

Sets

I: set of installations

E: set of echelons

I_e: set of installations in echelon e

Variables

$x_{t,i}^{\xi}$: inventory level of installation i

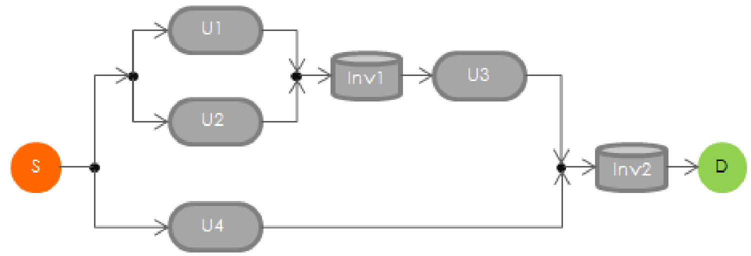
$y_{t,e}$: basestock of echelon e

$u_{t,i}^{\xi}$: underutilization of installation i

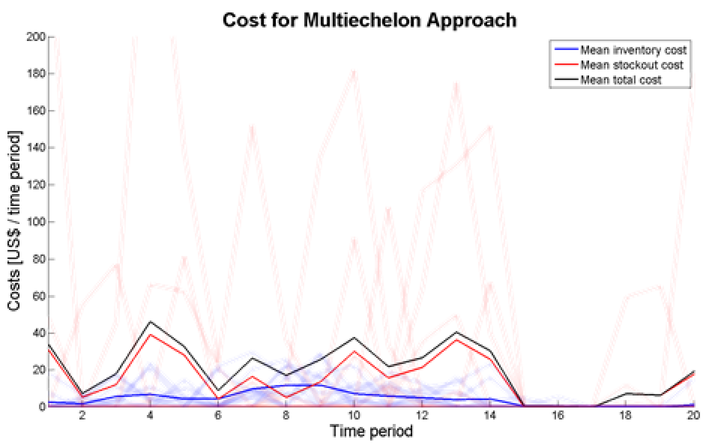
Example for Inventories in Series

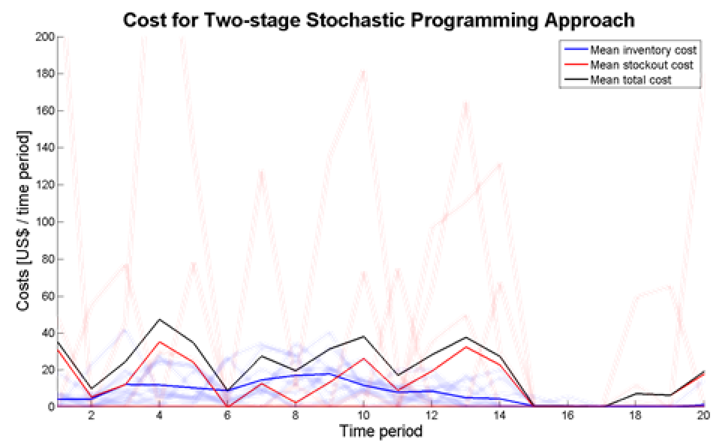
Process network with failures, uncertain supply, and uncertain demand

- Normally distributed supply
- Processing units with random failures
- Normally distributed demand
- Simulation length: 20 time periods
- Planning horizon: 5 time periods
- Number of replications: 10



Results:





	Multiechelon formulation	Two-stage stochastic program
Expected Inventory cost [$/period]	90	144
Expected stockout cost [$/period]	318	280
Expected total cost [$/period]	408	424

- The total savings account for 3.8% of expected cost
- The multiechelon strategy reduces inventory cost significantly but incurs in higher stockouts

Conclusions

Novelty:

- Framework for inventory planning in finite horizons
- Detailed logic for units in parallel and in series
- Methodology to evaluate results
- Performance improvement over two-stage approach

Impact for industrial application:

- Supply and demand forecasts can be used directly for inventory optimization
- Inventory management considering predictable and unpredictable events