Solution Strategies for the Dynamic Warehousing Location under Discrete Transportation Costs

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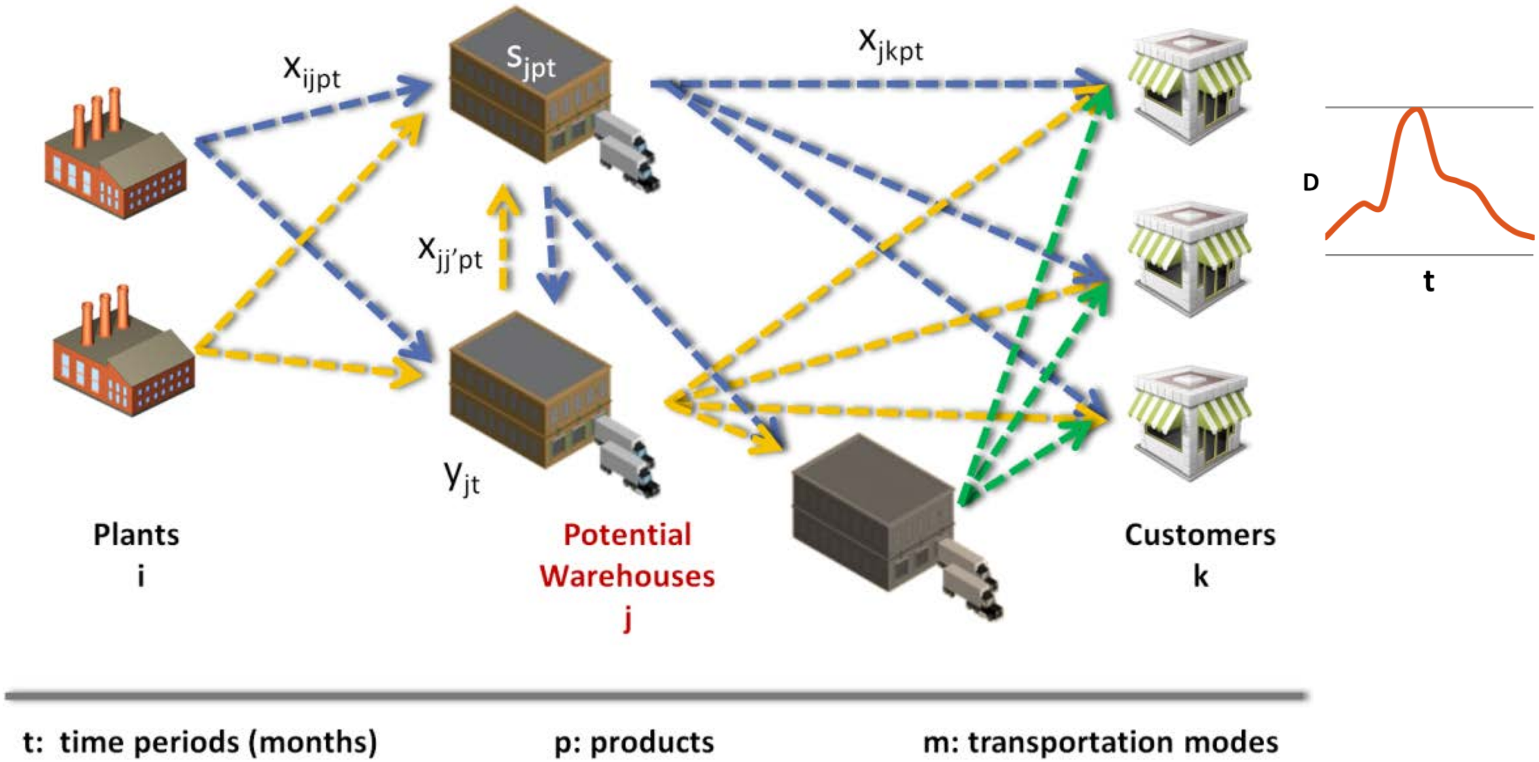
Problem Description



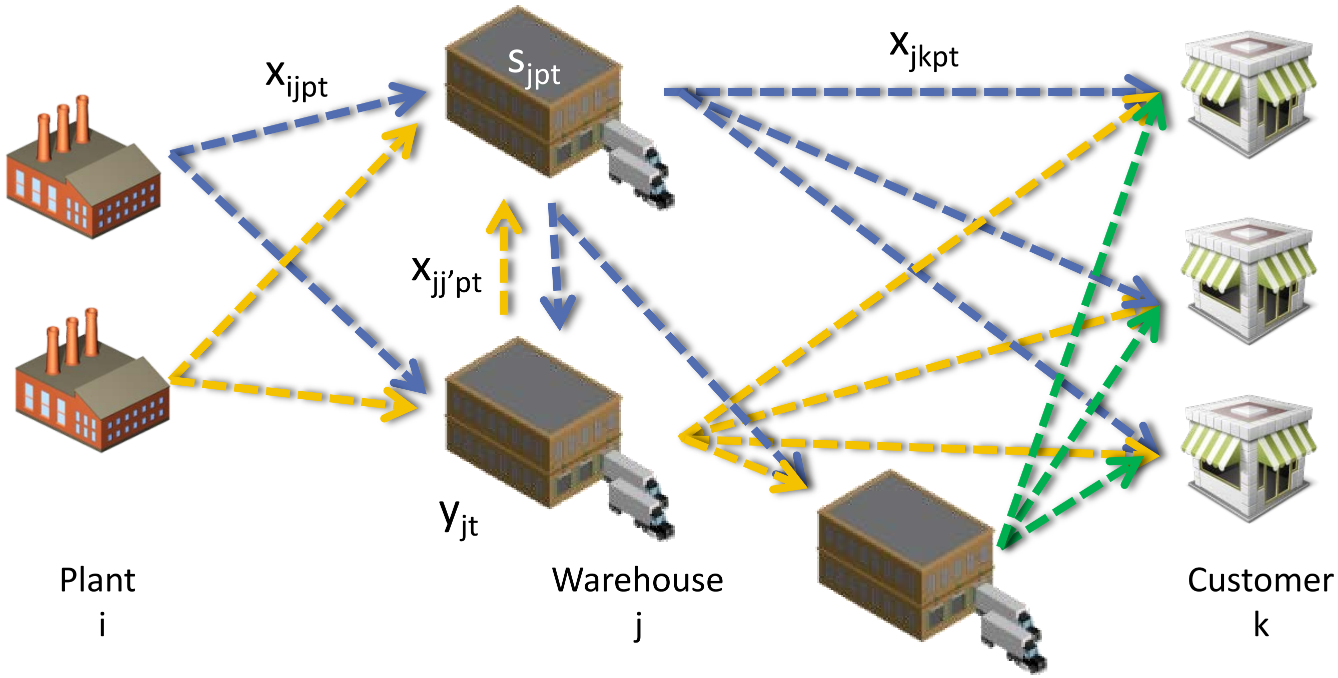
- Decide the number, size, location and contracting length for warehouses
- Dynamic decision of opening/closing warehouses at every period
- Plan the inventory allocation
- Multiple transportation modes, with discrete costs
- 5 year planning horizon
- Seasonal Demand

Novelty

- Find the best approach for discrete transportation costs
  - Integer variables: Manzini and Bindi (2009), Brahimi and Khan (2014), Quttineh and Lidestam (2014)
  - Disjunctive programming: Gao et al. (2010)

- Warehouse contracting length policies
- Inventory flow cost structure
- Deal with safety stock without using probability distributions
- Reformulation and solution strategies

Variables Definition



Decision Variables

- $x_{ijpt}, x_{jj'pt}, x_{jkpt}$ Quantity of product p transported in each link in each time period
- y_{jt} 1 if warehouse j is used in period t
- s_{jpt} Stock of product p at warehouse j in the end of period t

t: time periods (months)

p: products

m: transportation mode

MILP Model

Minimize Cost

Transportation

Inventory

Warehouse Rent

Subject to

Demand Satisfaction

$$\sum_j x_{jkpt} = D_{kpt}$$

Inventory Balance

$$s_{jpt} = \sum_i x_{ijpt} + \sum_{j'} x_{j'jpt} - \sum_{j'} x_{jj'pt} - \sum_k x_{jkpt} + s_{jpt-1}$$

Warehouse Upper Bound

$$s_{jpt} + \sum_k \sum_p x_{jkpt} + \sum_{j'} x_{jj'pt} \leq \mathbb{U}\, y_{jt}$$

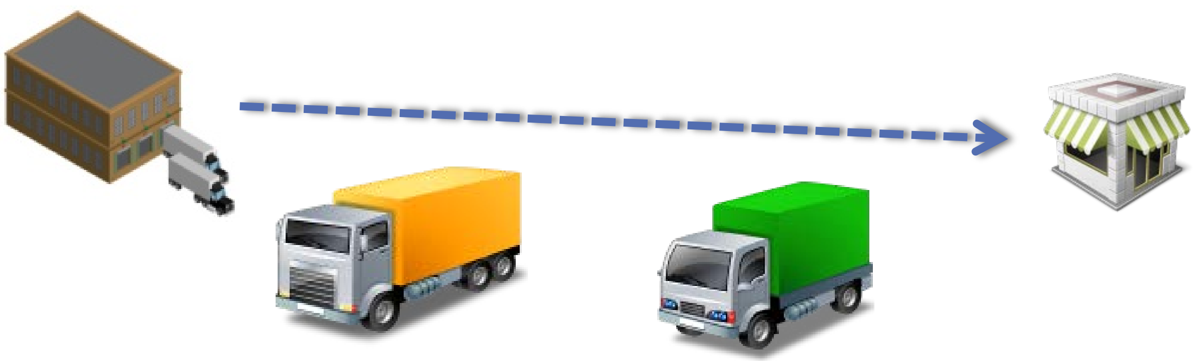
Discrete transportation costs
Dynamic warehouse contracting
Flow cost structure
Safety stock with risk pooling

??

MODELING OF COMPLICATING ISSUES

1. Discrete Transportation Costs

- For a defined transportation link we need to calculate how many **units** of each size (MOT) we need to use
- The cost is paid per transportation unit (i.e.: per truck)



4 Alternatives Considered

- Integer Variables
- SOS2 variables for the piecewise cost function
- Big-M formulation for the disjunction of the piecewise cost funciton
- Convex hull formulation for the disjunction of the piecewise cost funciton

Bravo and Vidal (2013), Freight transportation function in supply chain optimization models: A critical review of recent trends

Out of 97 articles reviewed, only 4 consider discrete transportation costs

1. Discrete Transportation Costs

Modeling Approaches

1. Integer Units

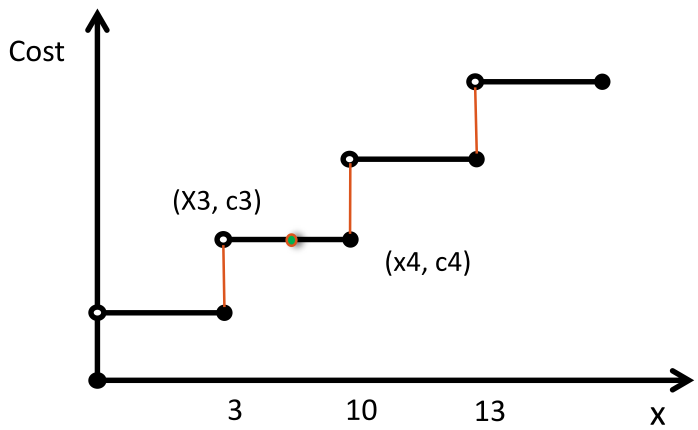
$$\sum_p x_{jkpt} \leq \sum_m TCap_m \, u_{jkmt}$$

u_{jkmt} integer

Manzini and Bindi (2009), Brahimi and Khan (2014), Quttineh and Lidestam (2014)

2. SOS2 Variables

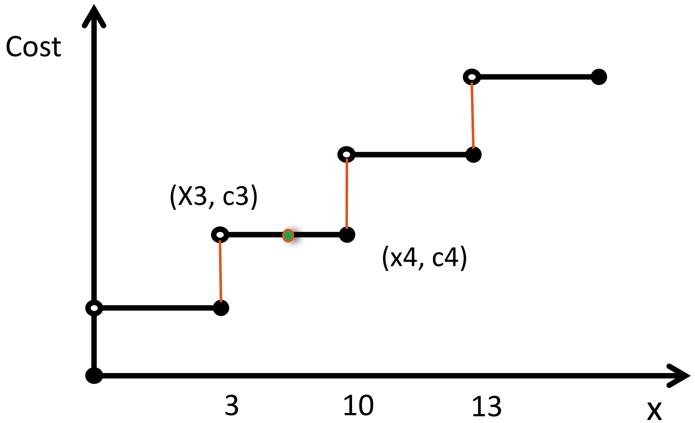
Beale and Tomlin (1970)



$$x = \gamma_3 x_3 + \gamma_4 x_4$$
$$cost = \gamma_3 c_3 + \gamma_4 c_4$$
$$\sum_i \gamma_i = 1$$

1. Discrete Transportation Costs

Modeling Approaches



$$\bigvee_{n \in N} \begin{bmatrix} Y_n \\ L_n \le x \le U_n \\ FreightCost = C_n \end{bmatrix}$$

Raman and Grossmann (1994)

3. Big M

$$L_n y_n \le x \le U_n + M(1 - y_n)$$

$$\sum_n y_n = 1$$

$$y_n \in 0, 1$$

4. Convex Hull

$$x = \sum_n x_n$$

$$L_n y_n \le x_n \le U_n y_n \qquad \forall n$$

$$\sum_n y_n = 1$$

$$y_n \in 0, 1 \qquad \forall n$$

1. Discrete Transportation Costs

Instance
T6C4K1
T6C4K5
T12C8K5
T36C8K5

Problem Size

Best Formulation

Solution time in minutes

*** %GAP after 10 min limit**

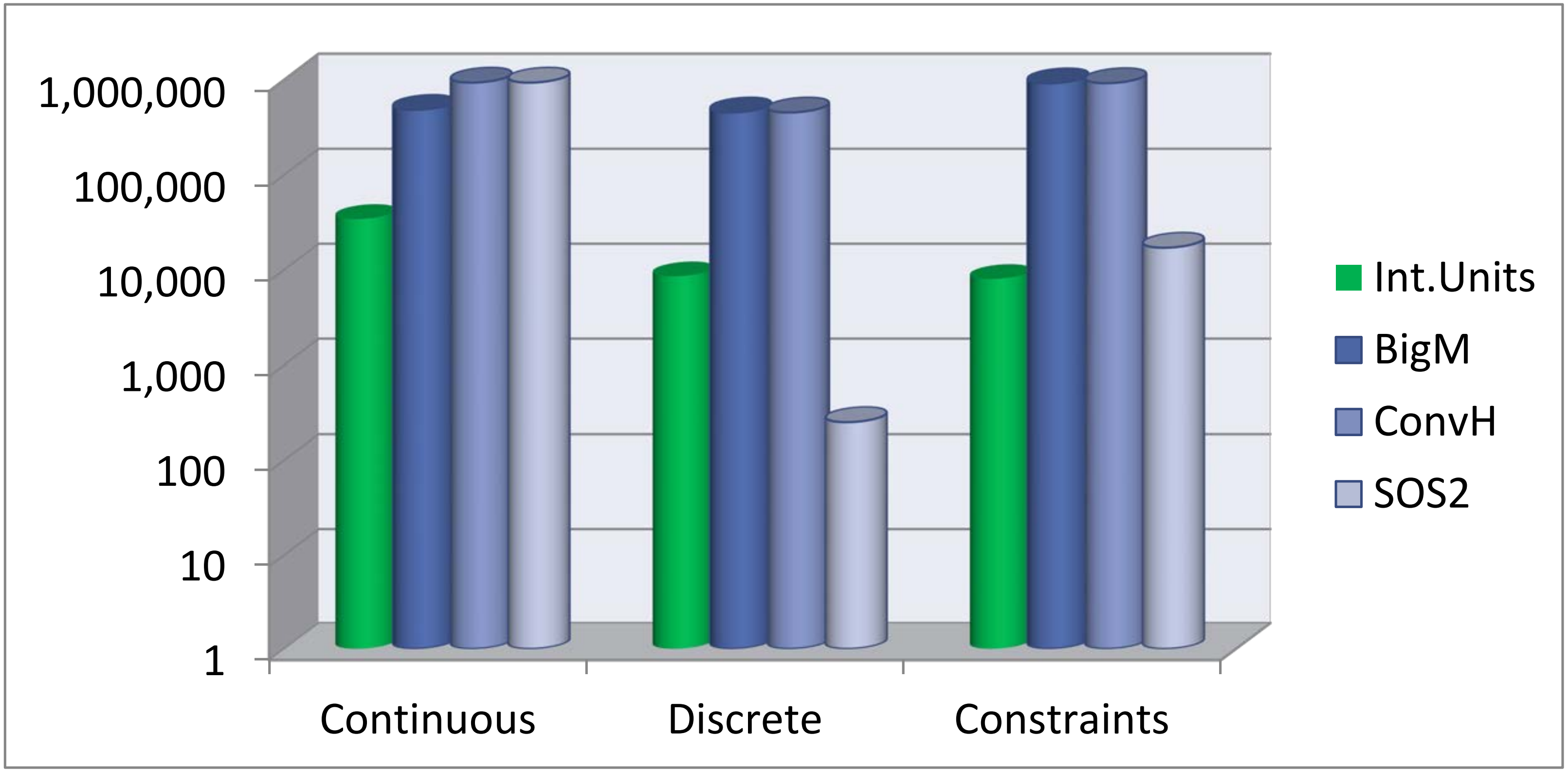
No feasible solution found after 10 min

Int. units solves in the largest problem in 2 min

Solver: Gurobi 6.0.2
through GAMS 24.3
Intel core i7, 4 cores

1. Discrete Transportation Costs

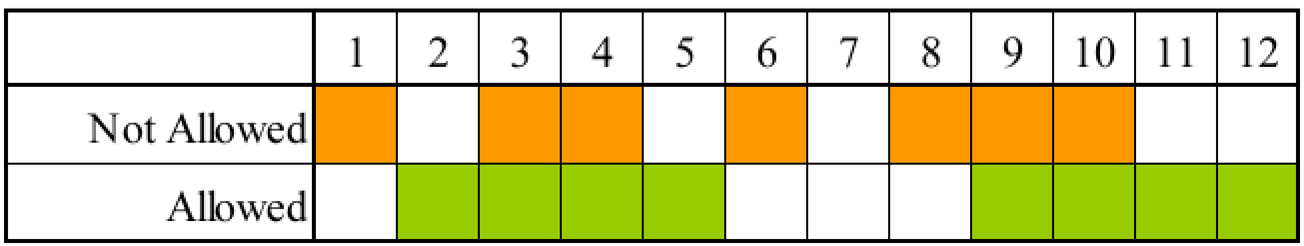
Model Size Comparison



Integer Units model has the least number of continuous variables and constraints

2. Dynamic Contracting Policies

- Constraints to ensure continuity of the warehousing service
  - Min length per contract
  - When a contract is finished, immediately renew or wait at least W months to renew



Ex: min wait = 3, min length = 3

2. Dynamic Contracting Policies

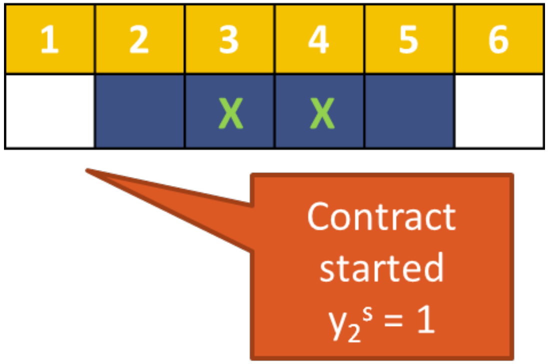
- Define new binary variables and add linear constraints

1. y_t^s: 1 if a contract is started in period t

$$-y_t + y_{t-1} + y_t^s \geq 0$$

2. y_t^f: 1 if a contract is finished in period t

$$-y_t + y_{t+1} + y_t^f \geq 0$$



Minimum wait

$$\sum_{\tau=t+1}^{t+W} y_\tau + W y_t^f \leq W$$

If a contract is finished in period t the warehouse can not be used in the next W periods

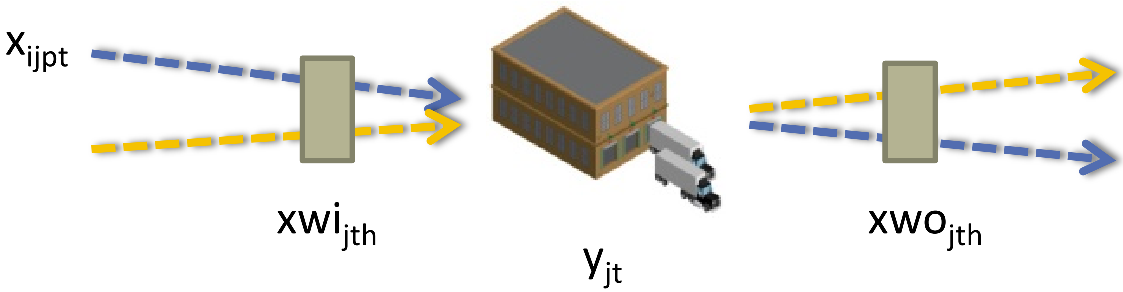
Minimum length

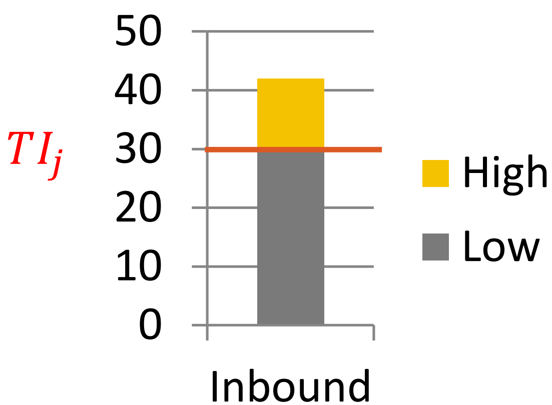
$$\sum_{\tau=t}^{t+L-1} y_\tau \geq L y_t^s$$

If a contract is started in period t the warehouse must be used in the next L periods

3. Flow Cost Structure

- Inbound and outbound unit cost with penalty for high volume





$$\sum_p \sum_i x_{ijpt} = xwi_{jt,low} + xwi_{jt,high}$$

$$xwi_{jt,low} \leq TI_j$$

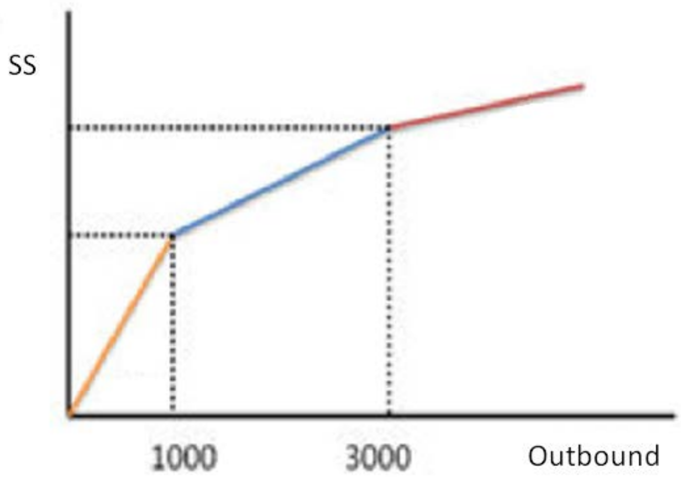
4. Safety Stock with Risk Pooling effect

Safety Stock

- $safety\ stock = z\,\sigma\sqrt{L} \quad \approx \quad ss = \beta x$

Risk Pooling

- Stock out risk decreases with number of customers served



$$\sum_k x_{jkpt} = \sum_n \lambda_{jptn}\, SS_{xn} MY\, D_p$$

$$ss_{jpt} = \sum_n \lambda_{jptn} SS_{xn} SS_{yn}\, MY\, D_p$$

$$\sum_n \lambda_{jptn} = 1$$

TIGHTENING AND REFORMULATION OPTIONS

Valid Inequalities

1. Plants must supply at least enough to meet demand

$$\sum_{i \in I_p} \sum_j x_{ijpt} \geq \sum_k D_{kpt} - \sum_j (s_{jpt} + ss_{jpt}) \qquad \forall p, t$$

2. It is not possible to ship to/from a closed warehouse

$$\sum_i \sum_p x_{ijpt} \leq UB\, y_{jt} \qquad \forall j, t$$

3. No transportation units are used in a lane with a closed warehouse

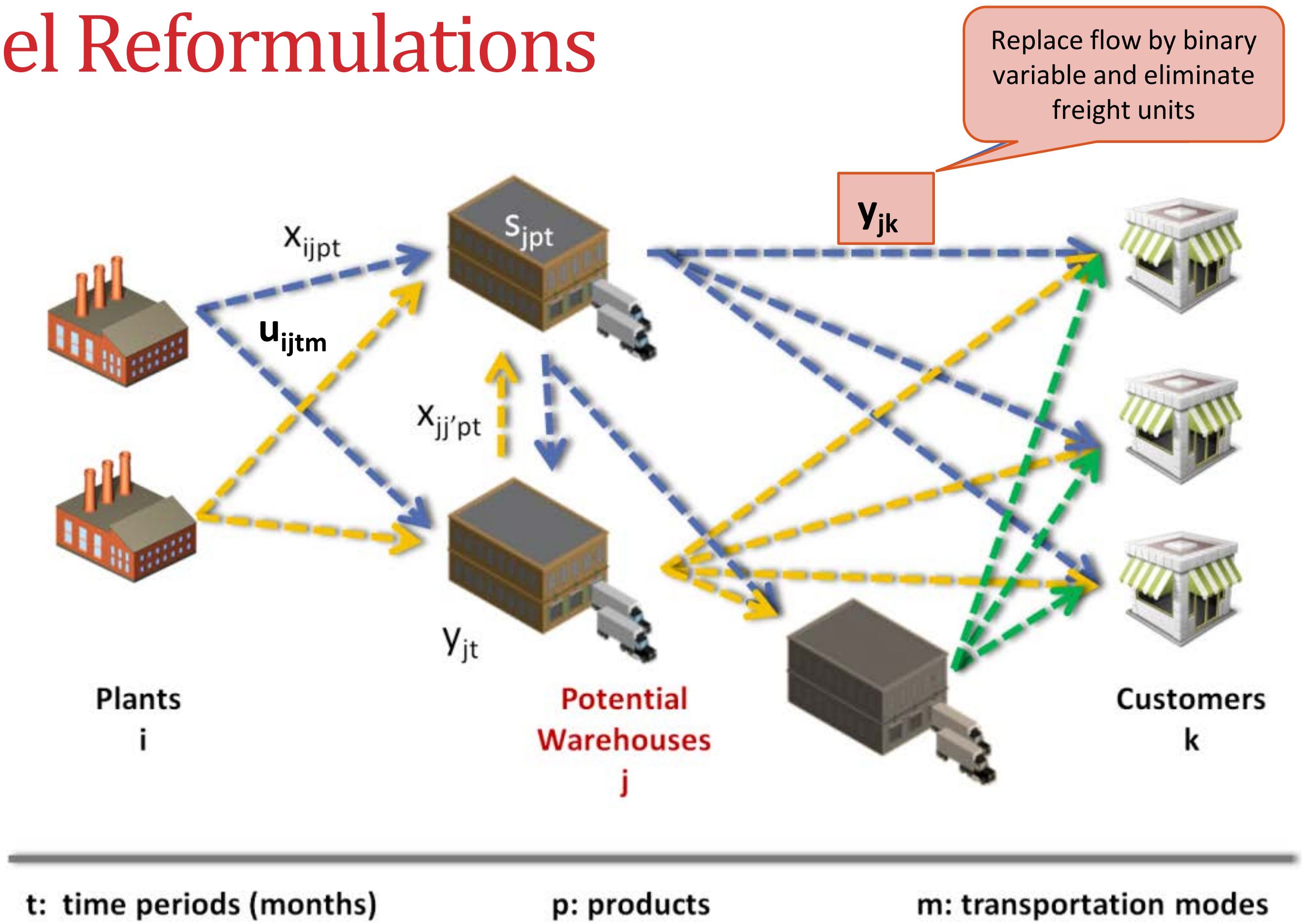
$$u_{jktm} \leq UB\, y_{jt} \qquad \forall j, k, t, m$$

4. Transportation units for a defined MOT are at most the exclusive mode number of units

$$u_{ijrm} \leq \frac{1}{TCap_m} \sum_{p \in P_i} x_{ijpt} + 1 \qquad \forall i, j, m$$

15% time reduction

Model Reformulations



Model Reformulations

Instance: C10P10T12

- 15 Plants, 15 Warehouses, 10 Customers, 10 Products, 12 Months, 16 Modes of Transportation

	Original	JKP	JK
CPU (s)	364	60	26
Objective Value	$ 3.29 M	$ 3,32 M	$ 3,35 M
Objective var	-	+ 0.7%	+ 1.5%

Models:

JKP : A customer receives **each product** from a specific warehouse

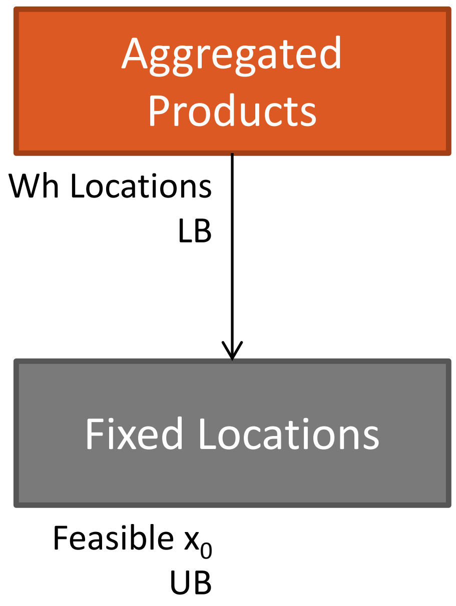
JK : A customer receives **all products** from a exclusive warehouse

Observations:

1. Each of the reformulations obtains the same network
2. If the assumption is reasonable for the application it pays off to use it. It reduces the solution time and allows to solve larger problems

SOLUTION STRATEGIES

2-Stage Heuristic



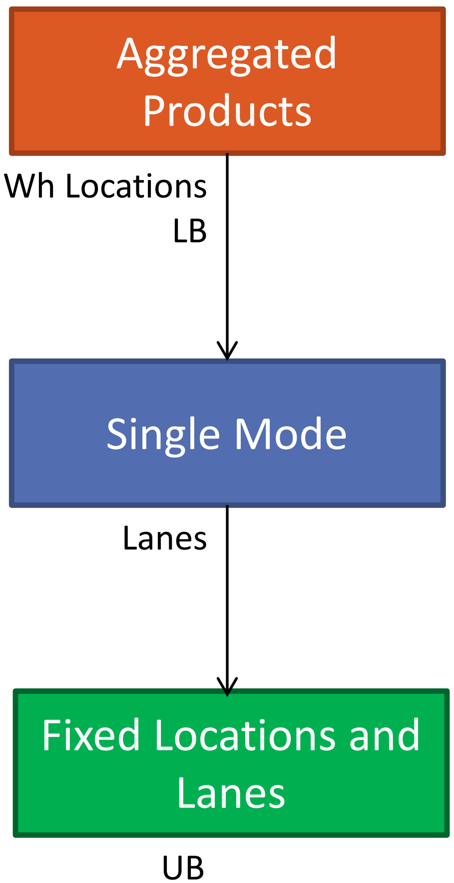
Idea:

1. Group products to solve a simpler problem and provide a LB
2. Solve the full problem with the warehouses selected

Observations

1. Single Product delivers a LB within 8% of the optimal
2. Tested in more than 300 instances, reaching the optimal in 95% of the problems
3. Not able to solve the 5-year horizon problem

3-Stage Heuristic



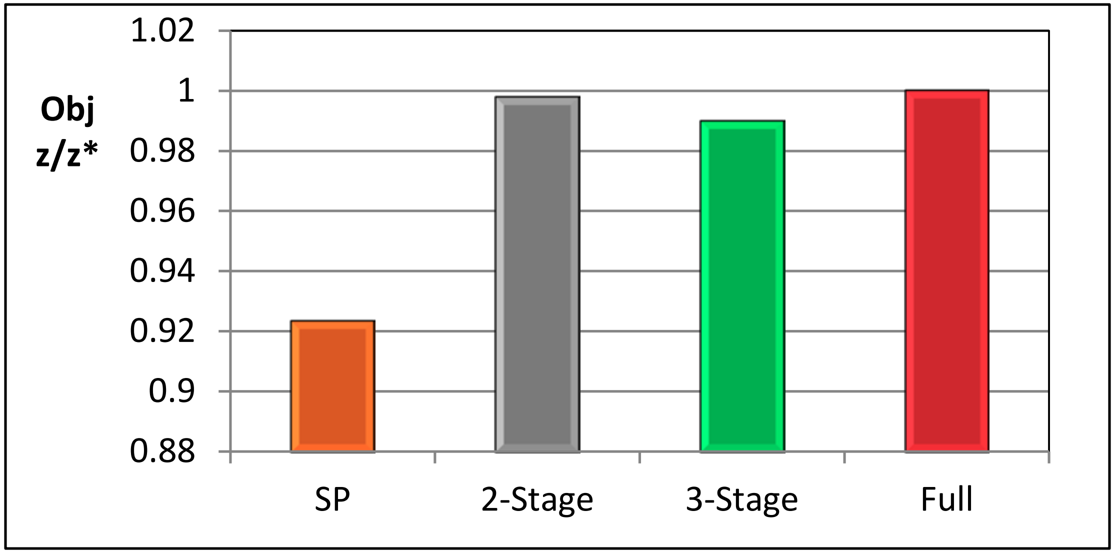
Idea:

1. Group all products to solve a simpler problem
2. Solve a single transportation mode to estimate used lanes
3. Solve full problem with fixed locations and lanes

Observations

1. Can handle very large problems in under 1 hour
2. The upper bound obtained is estimated to be at 1% from the optimal

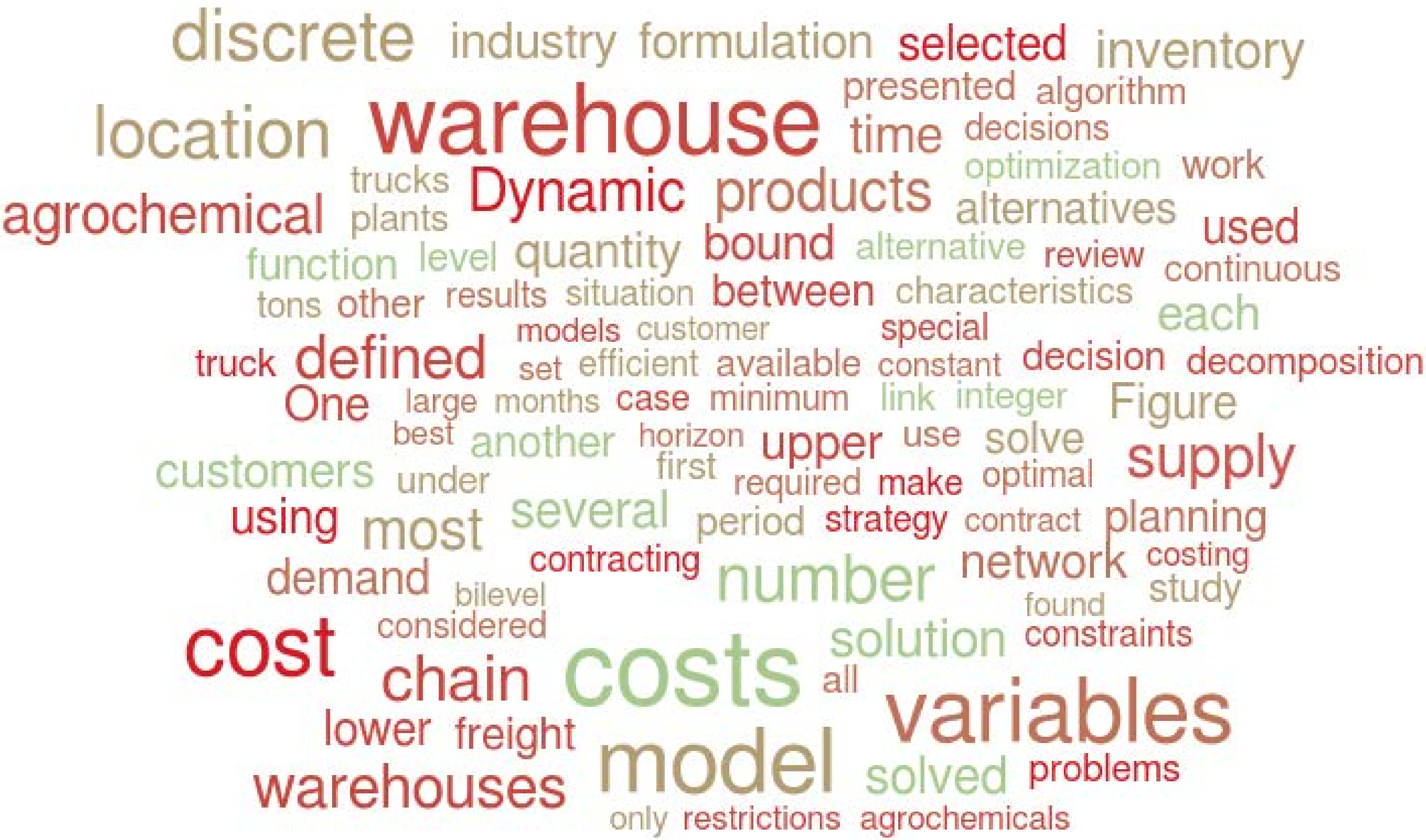
Algorithms Comparison



	SP	2-Stage	3-Stage	Full
CPU time	25 s	3 h	30 s	>24 h

Conclusions

1. A difficult supply chain problem was effectively modeled
    1. Integer variables is the most efficient way of modeling transportation cost for this problem
    2. It is possible to capture the behavior of safety stock with risk pooling effect using linear constraints
2. Reformulations based on reasonable assumptions were used to reduce the problem complexity
    1. 10-fold speed-ups were obtained while predicting the same network
3. Multistage heuristics were proposed to solve large problems
    1. The 2-stage heuristic has a high probability of obtaining the optimal solution
    2. The 3-stage heuristic can solve a very large problem in under 1 hour, with only 1% deviation from the optimal solution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presented algorithm
location warehouse time decisions
optimization work
trucks Dynamic products
agrochemical plants alternatives
used
function level quantity bound alternative review continuous
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models customer special
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