# Multiperiod Blend Scheduling Problem

*Juan Pablo Ruiz*
*Ignacio E. Grossmann*

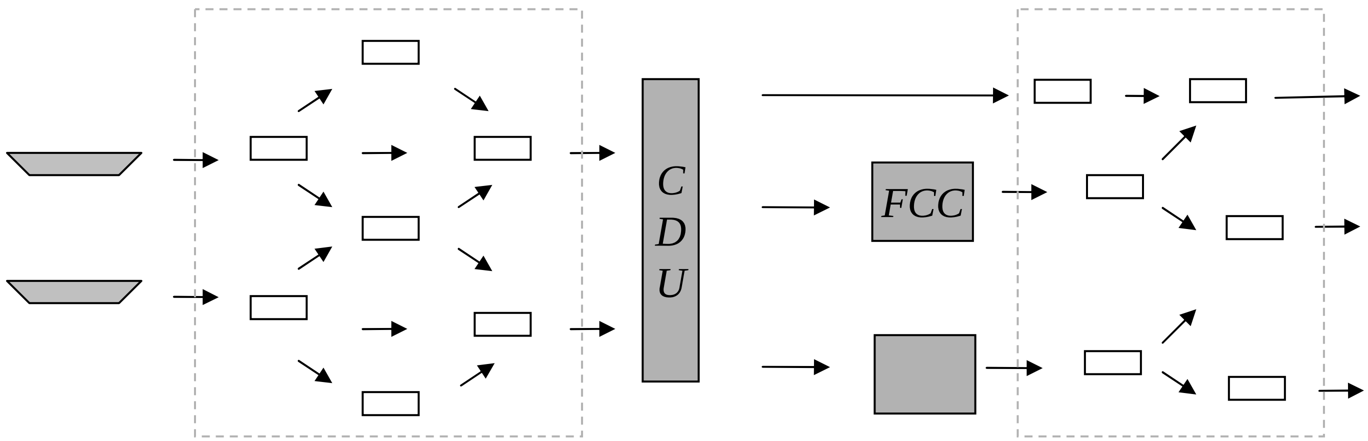
*Department of Chemical Engineering*
*Center for Advanced Process Decision-making*
*Carnegie Mellon University*
*Pittsburgh, PA 15213*

# *Motivation*

- The **scheduling problem of blending operations** arises **frequently** in the petrochemical industry.

- Although simple to represent, these **models are highly nonconvex**, leading to the need of **global optimization** techniques to find the optimal solution.

- The development of **efficient solution methods** that take care of the general case applied to large scale systems **remains as a challenge**…



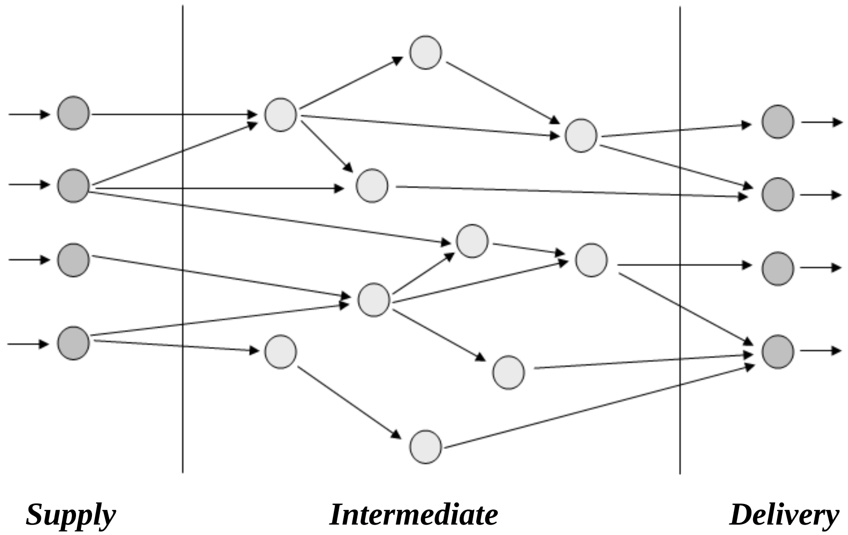

**Blending Upstream the CDU**

**Blending Downstream the CDU**

***Goal:*** *Develop* ***tools and strategies*** *aiming at* ***improving the efficiency*** *of the solution methods for the global optimization of the* ***multiperiod blend scheduling problem***

# General Problem Topology

The **general case** of a blending problem can be represented schematically as follows




**Remarks:**

- Examples of the **supply nodes** are the tanks loaded by ships or the feedstocks downstream the CDU

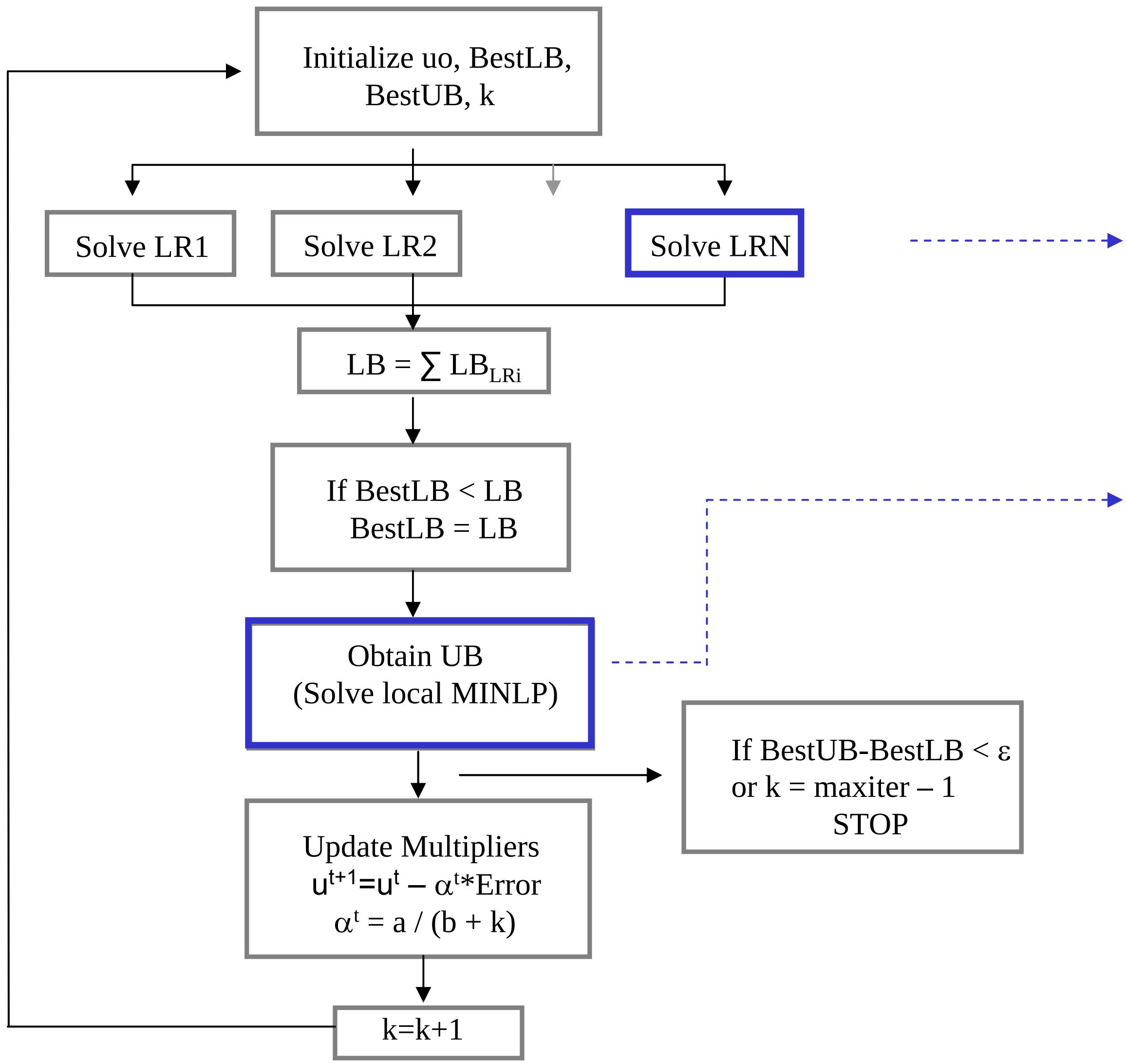
- Examples of the **delivery nodes** are the tanks feeding the CDU or the tanks delivering to final customers

***Important Assumptions:***

- The **quality** of each stream/inventory is **constant** for a given **period**.
- A tank can **receive or deliver** in a given period of time but **not both**.
- **Supply** tanks keep a **constant quality**.
- **Delivery** tanks keep the **quality** within a given **range**.
- Streams **entering delivery** tanks should satisfy a **quality condition.**

# *Observations*

## Numerical tests using Lagrangian Relaxation with temporal decomposition

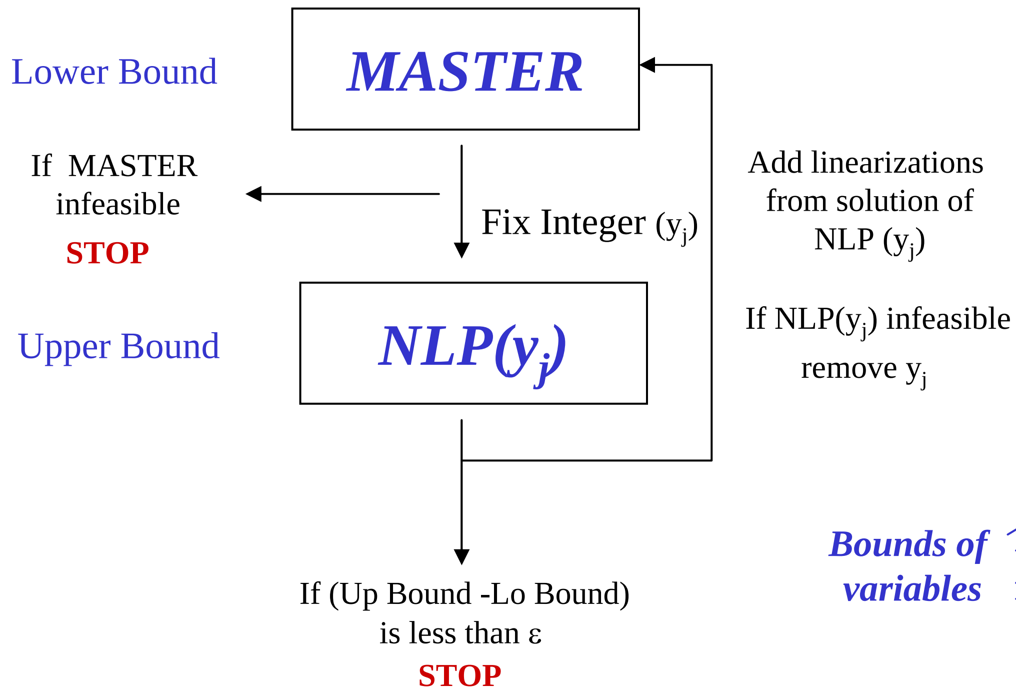



**1- High computational time required in sub-problems (> 5min)**

**2- Difficulties to find local solutions**

***How do we tackle these issues?***

# *Finding Local Solutions*
## Outer Approximation Method




***The MASTER problem can be tightened by adding McCormick Convex envelopes for the bilinear terms***

$$f_{qijt} \le F_{ijt} C^{UP}_{qit-1} + C_{qit-1} F^{LO}_{ij} - C^{UP}_{qit-1} F^{LO}_{ij}$$

$$f_{qijt} \le F_{ijt} C^{LO}_{qit-1} + C_{qit-1} F^{UP}_{ij} - C^{LO}_{qit-1} F^{UP}_{ij}$$

$$f_{qijt} \ge F_{ijt} C^{UP}_{qit-1} + C_{qit-1} F^{UP}_{ij} - C^{UP}_{qit-1} F^{UP}_{ij}$$

$$f_{qijt} \ge F_{ijt} C^{LO}_{qit-1} + C_{qit-1} F^{LO}_{ij} - C^{LO}_{qit-1} F^{LO}_{ij}$$

***Bounds of variables***

*Similarly for terms* $I_{qit} C_{qit}$

## *Remarks*

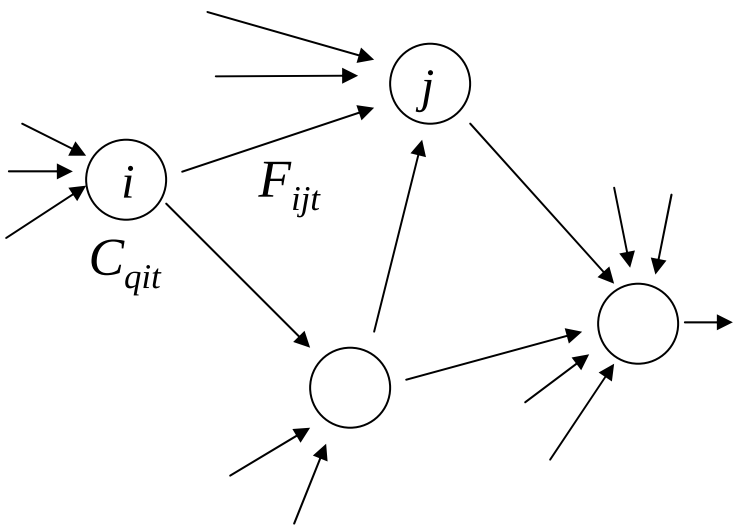
- **Reducing the number of bilinear** terms in NLP($y_j$) leads to a more **robust formulation**
- Having **good bounds** for the variables is of main importance to find **tight relaxations**

# *Finding Local Solutions*

## Tighter bounds for variables (I)

### *Observation*

If **two streams** are **mixed** together, the **concentration** of any given component in the mixture is always **higher/lower** than the **minimum/maximum** concentration in the streams




*Mathematical Representation*

$$C_{qit} \geq \min_{(i,j)\in E}(C_{qjt-1}) \qquad \forall q,i,t$$

$$C_{qit} \leq \max_{(i,j)\in E}(C_{qjt-1}) \qquad \forall q,i,t$$

***How can we use it to infer bounds for the compositions?***

# *Finding Local Solutions*

## Tighter bounds for variables (II)

### *Lower Bounds*

$$C_{qit}^{LO} = C_{qi0} \qquad \forall q,i,t=1$$

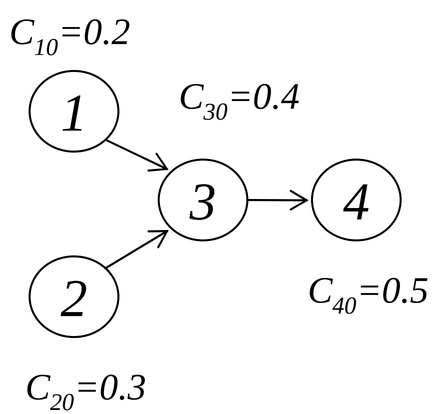
$$C_{qit}^{LO} \geq \min_{(i,j)\in E}(C_{qjt-1}^{LO}) \qquad \forall q,i,t>1$$

### *Upper Bounds*

$$C_{qit}^{UP} = C_{qi0} \qquad \forall q,i,t=1$$

$$C_{qit}^{UP} \leq \max_{(i,j)\in E}(C_{qjt-1}^{UP}) \qquad \forall q,i,t>1$$

### *Illustrative Example*



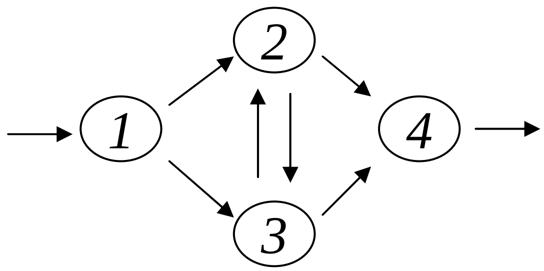

| | t=0 | | t=1 | | t=2 | |
|---|---|---|---|---|---|---|
| | LO | UP | LO | UP | LO | UP |
| Node 1 | 0.2 | 0.2 | 0.2 | 0.2 | 0.2 | 0.2 |
| Node 2 | 0.3 | 0.3 | 0.3 | 0.3 | 0.3 | 0.3 |
| Node 3 | 0.4 | 0.4 | 0.2 | 0.4 | 0.2 | 0.4 |
| Node 4 | 0.5 | 0.5 | 0.4 | 0.5 | 0.2 | 0.5 |

***Lower and upper bound tightening can be achieved in the preprocessing step***

# *Finding Local Solutions*

## Tighter bounds for variables (III)

### *Performance Analysis*




*Predicted lower bounds at first **MASTER** problem*

| | *Global Optimum* | *Using original bounds* | *Using inferred bounds* |
|---|---|---|---|
| *Instance 1* | -2900 | -4958 | -4083 |
| *Instance 2* | -10900 | -20958 | -14650 |

### *Remark*

**Improvements** in the bounds prediction can be obtained if lower/upper bounds of **flows and inventory** levels are **considered**

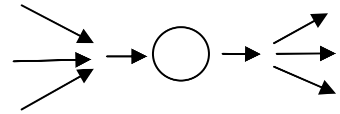
$$C_{qit} \ge \min\left(\frac{I_{qit-1}C_{qit-1} + \sum_{(i,j)\in E} F_{jit}C_{qjt-1}}{I_{qit}}\right) \ge \frac{I^{LO}_{qit-1}C^{LO}_{qit-1} + \sum_{(i,j)\in E} F^{LO}_{jit}C^{LO}_{qjt-1}}{I^{UP}_{qit}} \qquad \forall q,i,t$$

$$C_{qit} \le \max\left(\frac{I_{qit-1}C_{qit-1} + \sum_{(i,j)\in E} F_{jit}C_{qjt-1}}{I_{qit}}\right) \le \frac{I^{UP}_{qit-1}C^{UP}_{qit-1} + \sum_{(i,j)\in E} F^{UP}_{jit}C^{UP}_{qjt-1}}{I^{LO}_{qit}} \qquad \forall q,i,t$$

# *Finding Local Solutions*

## Reduced number of bilinear terms

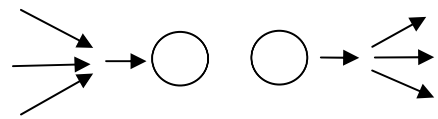
***Traditional MINLP Formulation***



***One general state***

$$C^{B}_{qjt} I_{jt} = C^{B}_{qjt-1} I_{jt-1} + \sum_{\substack{j' \in J^{P} \\ (j',j) \in E}} C^{P}_{qj'} F_{j'jt} + \sum_{\substack{j' \in J^{B} \\ (j',j) \in E}} C^{B}_{qjt-1'} F_{j'jt} - \sum_{\substack{j' \\ (j',j) \in E}} C^{B}_{qjt-1'} F_{j'jt} \quad \forall q \in Q, j \in J^{B}, t \in T$$

***Proposed GDP Formulation***



***Two states***

$$\begin{bmatrix} Y_{jt} \\ C^{B}_{qjt} I_{jt} = C^{B}_{qjt-1} I_{jt-1} + \sum_{\substack{j' \in J^{P} \\ (j',j) \in E}} C^{P}_{qj'} F_{j'jt} + \\ \sum_{\substack{j' \in J^{B} \\ (j',j) \in E}} C^{B}_{qjt-1'} F_{j'jt} \quad \forall q \in Q, j \in J^{B}, t \in T \end{bmatrix} \vee \begin{bmatrix} \neg Y_{jt} \\ C^{B}_{qjt} = C^{B}_{qjt-1'} \quad \forall q \in Q, j \in J^{B}, t \in T \end{bmatrix}$$

***By exploiting the underlying logic structure of the problem, a reduction of the number of bilinear terms can be achieved***

# *Finding Local Solutions*

## Numerical Results

### *Performance Analysis*

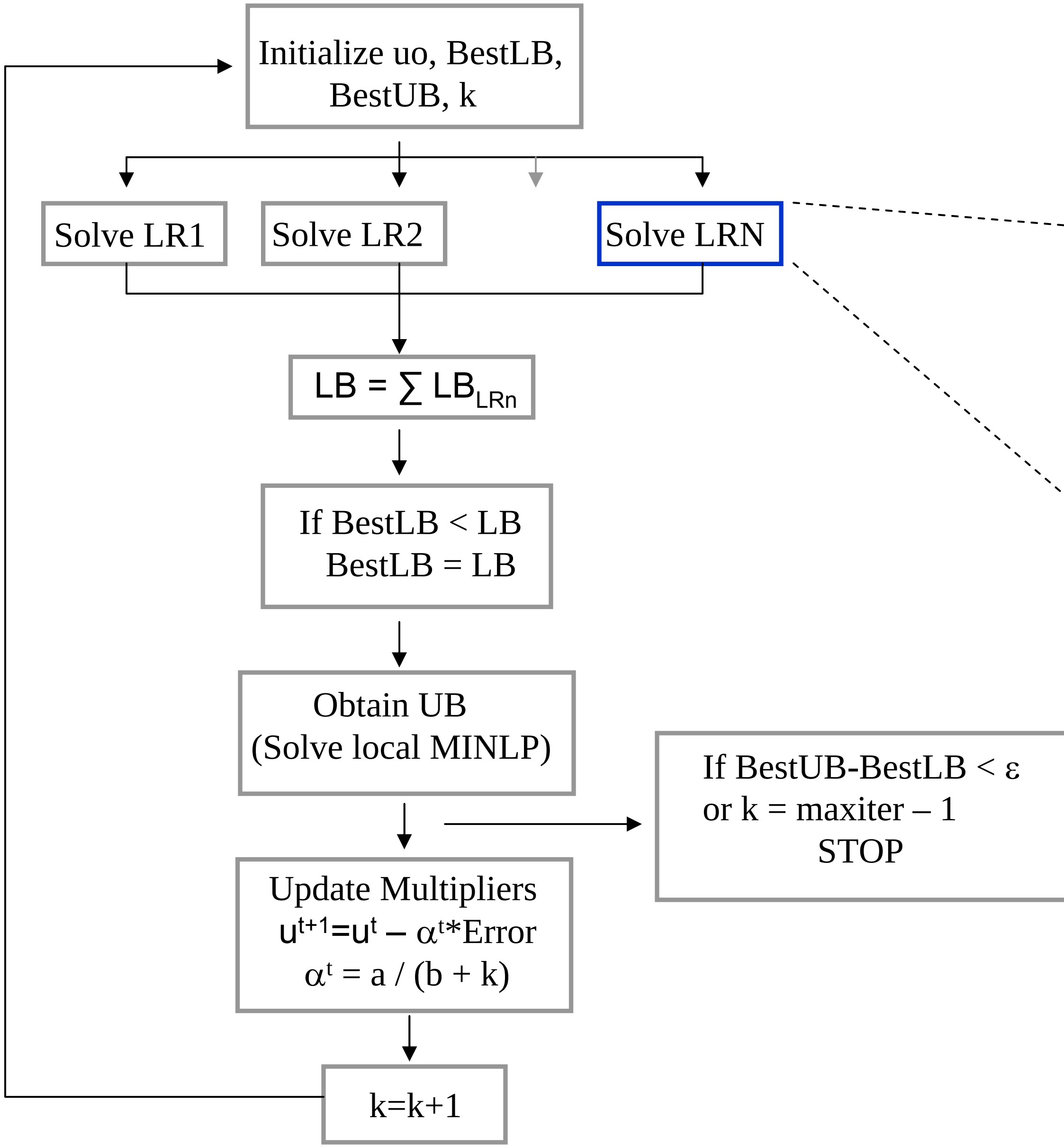
- 11 **random instances**
- Outer approximation solver **DICOPT(GAMS)**
- Three different formulations (all using McCormick envelopes):
  - **1**- Original MINLP
  - **2**- Formulation with reduced number of bilinear terms
  - **3**- Formulation with reduced number of bilinear terms plus bound tightening
- Forced to **stop after 10 iterations or 30 minutes**

### *Remarks*

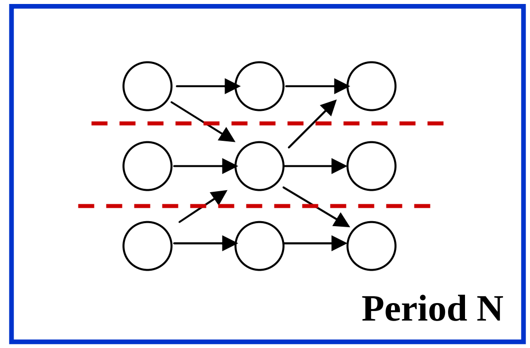
- Formulation (**2**) and (**3**) found **feasible solutions** in more than **70%** of instances
- Formulation (**3**) **outperformed** Formulation (**2**) in **20%** of the instances
- Formulation (**1**) led to a large number of **"false" infeasible** problems

# *Solution of $LR_i$ sub-problems*

## Spatial Decomposition




*Spatial Decomposition*




***How do we decompose it spatially?***

# Solution of LRi sub-problems

## Minimal cut-edge with fixed nodes

*Objective:* *Minimize the edges that cross the boundaries of each subset*

**Incidence Matrix** ← $A_{ij}$

$$\min \sum_{ijk} A_{ij}(y_{ik} - z_{ijk})$$

$$s.t. \quad \sum_{k} y_{ik} = 1 \qquad \forall i$$

$$\sum_{i} y_{ik} = \alpha_k \qquad \forall k$$

**Number of nodes in disjoint subsets** ← $\alpha_k$

**If $y_{ik}$ = 1 then the node i belongs to the subset k**

$$1 - y_{ik} - y_{jk} + z_{ijk} \geq 0 \qquad \forall ijk$$

$$y_{ik} \geq z_{ijk} \qquad \forall ijk$$

$$y_{jk} \geq z_{ijk} \qquad \forall ijk$$

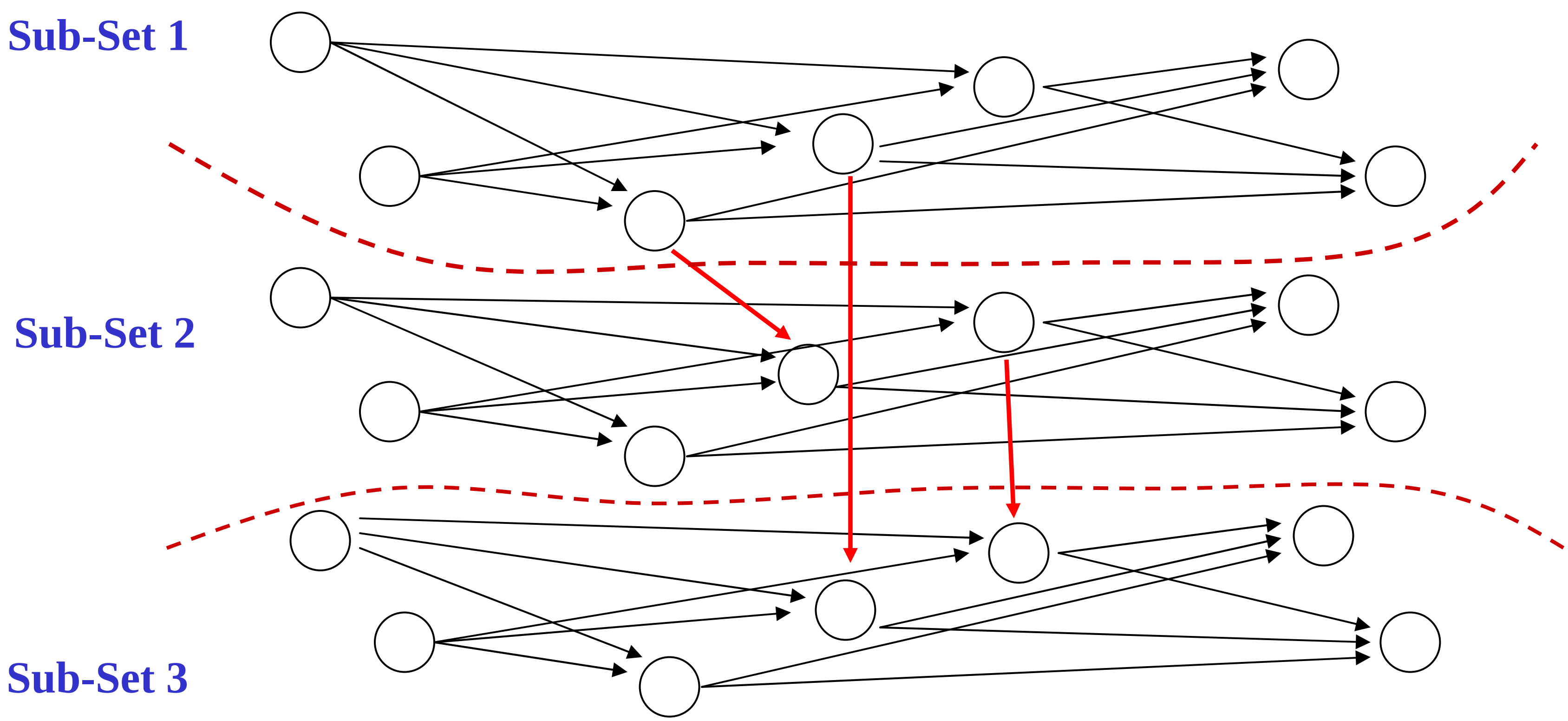
→ $z_{ijk} = y_{ik} y_{jk}$

$$\sum_{ij} z_{ijk} = \alpha_k^2 \qquad \forall k$$

→ **cut!**

# *Solution of LRi sub-problems*

## Minimal cut-edge with fixed nodes example




Dualized constraints necessary: **3(n+1)**
(*n: number of properties considered*)

## *Solution of $LR_i$ sub-problems*
### Numerical Results

-Baron takes 347 seconds **(~6min)** to solve the problem with a solution of **20954.8**

-The spatial decomposition solves the problem in **1 iteration**:

| | | |
|---|---|---|
| MIP separation problem: | | 5 seconds |
| Sub-problem 1: | (sol: 6096.0) | 1.6 seconds |
| Sub-problem 2: | (sol: 11451.8) | 1.4 seconds |
| Sub-problem 3: | (sol: 3407.0) | 1.5 seconds |
| TOTAL: | **(sol: 20954.8)** | **9.5 seconds** |

***Remarks:***

- Even though it is not expected for general problems to converge in one iteration, **even with 15 iterations, the time necessary would be ~1 min**

# *Remarks*

- Proposed a **new formulation and bounding procedure** to find **local solutions** more easily.

- Explored the **spatial decomposition** of each LR sub-problem to speed up convergence. A **systematic decomposition** approach was proposed.

## *Future work*

- Implement **spatial decomposition** of the sub-problems within the **global** optimization **framework**.
- Add **cuts to strengthen relaxation for LR** (from Vector Space Analysis ?)