Multi-Echelon Inventory Optimization under the Threat of Disruptions

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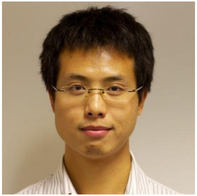
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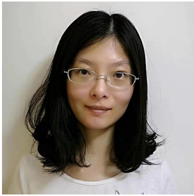
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Ying Rong
(SJTU)


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(McKinsey)

Outline

Outline

Supply Chain Disruptions Are as Old as Supply Chains

East India Company

Supply Chain Disruptions Are as Old as Supply Chains

Wells Fargo

Why the Recent Interest?

1. Recent high-profile disruptions
    - West-coast port lockout (2002)
    - Icelandic volcano (Eyjafjallajökull) eruption (2010)
    - Japan earthquake (2011)
    - Hurricane Sandy (2012)
    - COVID-19 pandemic (2020–??)

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    - aka just-in-time (JIT), etc.
    - Systems contain very little slack
    - Efficient, but fragile
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   - aka just-in-time (JIT), etc.
   - Systems contain very little slack
   - Efficient, but fragile
   - **There is value to having slack in a system**
3. Increasingly global supply chains
   - A single supply chain may span the globe
   - Firms are less vertically integrated

Mitigation Strategies

- Sourcing
- Business interruption insurance
- Rerouting
- Demand management
- Inventory

Stockpiling: Petroleum

Northeast Home Heating Oil Reserve

(photo: energy.gov)

Stockpiling: Helium

U.S. Federal Helium Reserve

(photo: redorbit.com)

Stockpiling: ???

Stockpiling: Maple Syrup

Global Strategic Maple Syrup Reserve

(photo: `theglobeandmail.com`)

Stockpiling: Twinkies

Hostess Bankruptcy

(photo: `money.msn.com`)

Literature: Single-Stage Systems with Disruptions

- Classical models + disruptions:
  - EOQ: Parlar and Berkin (1991), Berk and Arreola-Risa (1994), LVS (2014)
  - EOQ + safety stock: Parlar and Perry (1995, 1996), Heimann and Waage (2007)
  - Stochastic demand: Gupta (1996), Parlar (1997), Mohebbi (2003, 2004), Schmitt, LVS, and Shen (2010).

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  - Stochastic demand: Gupta (1996), Parlar (1997), Mohebbi (2003, 2004), Schmitt, LVS, and Shen (2010).
- Strategic questions:
  - Optimal strategy: Tomlin (2006)
  - Supplier flexibility: Tomlin and Wang (2005), Saghafian and van Oyen (2012, 2014)
  - Value of advanced information: LVS and Tomlin (2007), Wang and Tomlin (2009)
  - "Bundling" disruptions and yield uncertainty: Chopra et al. (2006), Schmitt and LVS (2006)

Literature: Single-Stage Systems with Disruptions

- Related areas:
  - Yield/quality uncertainty: Anupindi and Akella (1993), Dada et al. (2007), Federgruen and Yang (2009), Wang et al. (2010), Wang (2013), Li, Li, and Saghafian (2013)
  - Capacity uncertainty: Ciarallo et al. (1994), Wang et al. (2010)
  - Lead-time uncertainty: Nahmias (1979), Wang and Tomlin (2009)
- Survey papers: Vakharia and Yeniparzarli (2008), Atan and LVS (2012, 2014), LVS et al. (2016)

Literature: Multi-Echelon Systems with Disruptions

- Yield uncertainty in 3-echelon supply chain: Kim et al. (2005)
- Simulation studies: Deleris and Erhun (2005), LVS and Shen (2006), Schmitt and Singh (2009, 2011)
- Network analysis: Wu and Blackhurst (2005), Wu et al. (2007)
- Inventory and capacity in assembly systems: Hopp and Yin (2006)
- Service levels in general systems: Schmitt (2011)
- Inventory optimization for assembly systems: DeCroix (2013)

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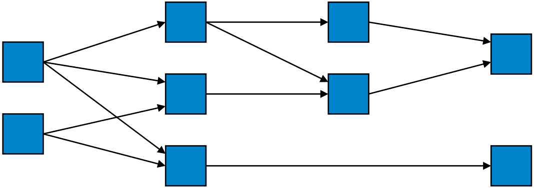
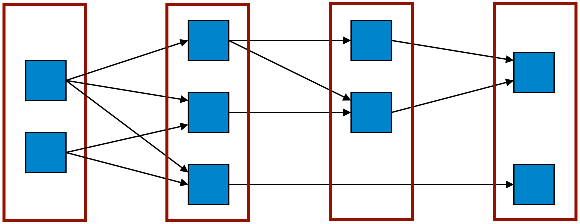
Very little research on multi-echelon inventory optimization with disruptions

Network Topology



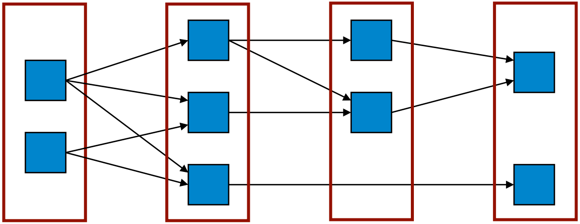
- System is composed of **stages**

Network Topology



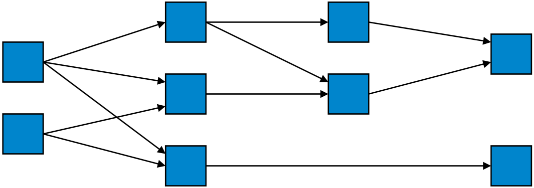
- System is composed of **stages**
- Stages are grouped into **echelons**

Network Topology



- System is composed of **stages**
- Stages are grouped into **echelons**
- Stages can represent:
  - Physical locations
  - Items in BOM
  - Processing activities

Terminology



- Stages to the left are **upstream**
- Stages to the right are **downstream**
- Downstream stages face customer demand
- Upstream stages receive outside supply

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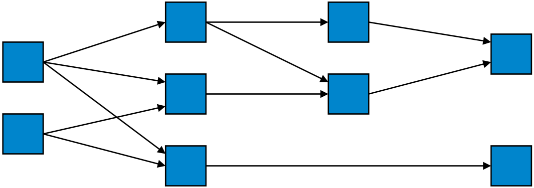

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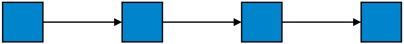
- Upstream stages receive outside supply
- Network topologies, in increasing order of difficulty:

Serial (Series) System

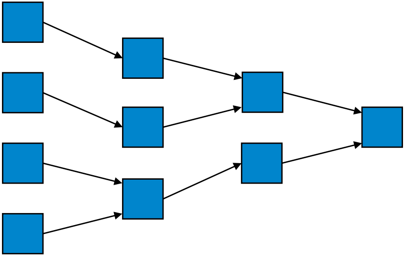
- Each stage has at most one predecessor and at most one successor



- **Optimal Replenishment Policy:** Echelon base-stock policy
- **Algorithm:** Decompose into single-variable, convex optimization problems—one per stage
  - Clark and Scarf (1960), Chen and Zheng (1994)
- **Heuristic:** Newsvendor heuristic (Shang and Song 2003)

Assembly System

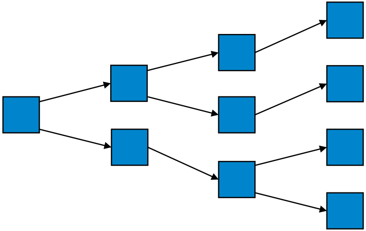
- Each stage has at most one successor



- **Optimal Replenishment Policy:** Balanced echelon base-stock policy
- **Algorithm:** Reduce to equivalent serial system; solve using serial system algorithm (Rosling 1989)
- **Heuristics:** Various heuristic policies

Distribution System

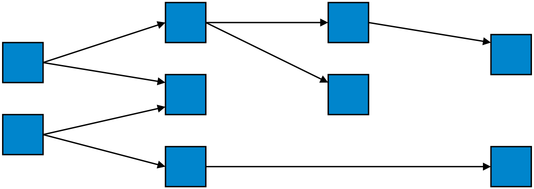
- Each stage has at most one predecessor



- **Optimal Replenishment Policy:** ???
- **Optimal Allocation Policy:** ???
- **Algorithm:** Projection algorithm (Graves 1985)
- **Heuristics:** METRIC (Sherbrooke 1968), two-moment approximation (Graves 1985), restriction–decomposition (Gallego, et al. 2007), decomposition and aggregation (Özer and Xiong 2008; Rong, Atan, and LVS 2017), recursive optimization (Rong, Atan, and LVS 2017)

Tree System

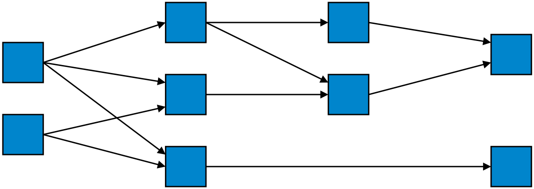
- No restrictions on neighbors, but no cycles



- Usually modeled using **guaranteed-service** approach
  - "Strategic safety stock placement"
  - Graves (1988), Graves and Willems (2000)
  - Dynamic programming algorithm

General System

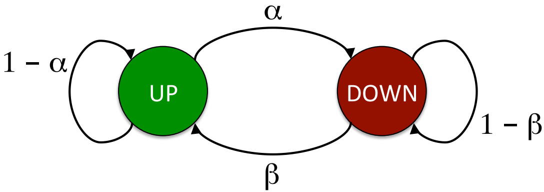
- No restrictions on cycles



- Guaranteed-service approach
  - Magnanti, et al. (2006)
  - Commercial IP solver

Modeling Disruptions

- Disruption process follows 2-state discrete-time [continuous-time] Markov process
  - Disruption probability [rate] α
  - Recovery probability [rate] β
  - Capacity $= \infty$ when UP, 0 when DOWN



- Disruption at node j prevents j from placing replenishment orders
- Node j may serve demand from on-hand inventory during disruption

Expected Cost Function

- Minimize long-run expected cost per unit time:

$$C(\mathbf{S}) = \sum_{i \in V} h_i \mathbb{E}[I_i] + \sum_{i \in L} p_i \mathbb{E}[B_i],$$

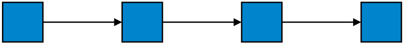
  where

  - $\mathbf{S}$ = vector of base-stock levels
  - V = set of nodes
  - L = set of "leaf" nodes (demand-facing nodes)
  - h_i, p_i = holding, stockout costs at i
  - I_i, B_i = on-hand inventory, backorders at i

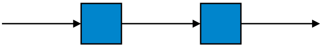
- $\mathbb{E}[\cdot]$ may be wrt supply, demand, or both
- I_i and B_i are typically complex functions of $\mathbf{S}$

Outline

Serial Systems

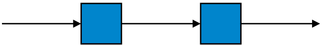


Serial Systems



- Consider 2-node system
  - Can extend result to N nodes

Serial Systems



- Consider 2-node system
    - Can extend result to N nodes
- Assumptions:
    - Discrete time, infinite horizon
    - General iid demand distribution
    - Disruptions at either node
    - (Clark–Scarf + disruptions)

Optimality of Base-Stock Policy

Theorem (Atan, Rong, and LVS 2009, Atan and LVS 2012)

An echelon base-stock policy is optimal at stage j, $j = 1, \ldots, N$.
S_j^* depends only on disruption state.

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3. Characteristics of S^*:
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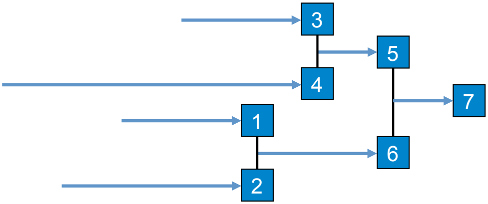
4. For finite horizon, solve as DP (large state space)

Outline

Echelon Inventory and Long-Run Balance



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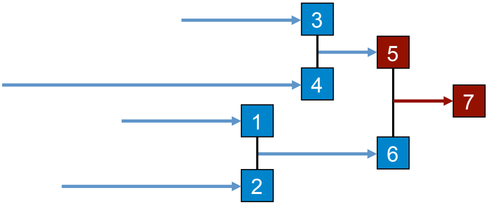

- IN_j = echelon inventory of item j

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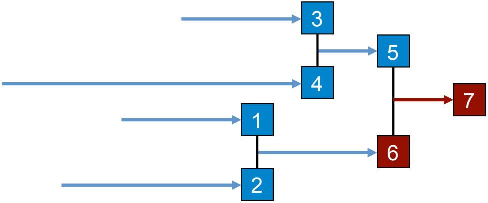

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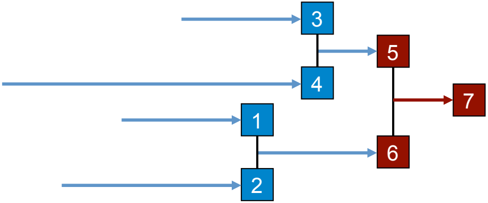

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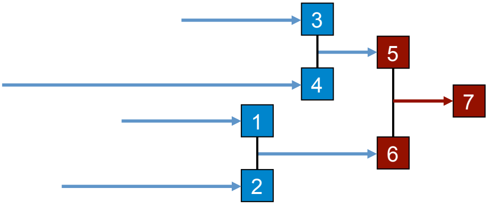

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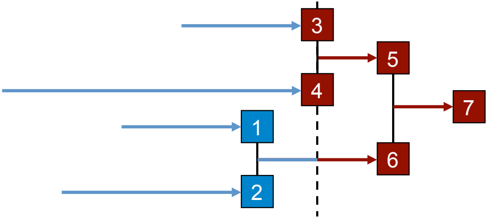

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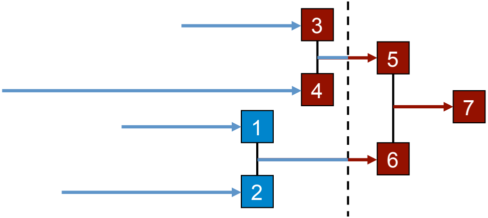

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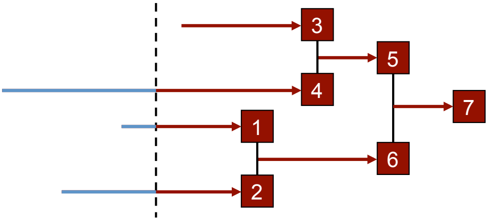

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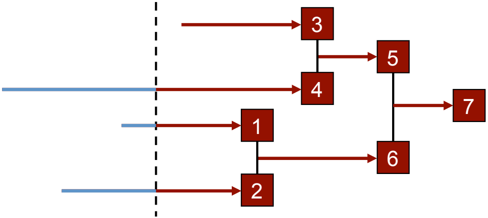
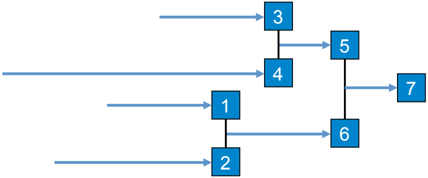


- IN_j = echelon inventory of item j
- Want $IN_i = IN_j$ if i and j have common successor
- Need pipeline inventories to be $\leq$ if total lead-times are $\leq$
- Called **long-run balance**

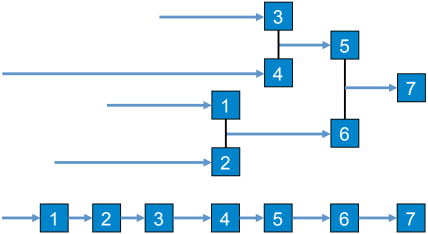
Long-Run Balance and Series Reduction



Proposition (Rosling 1989)

In an assembly system without disruptions, it is optimal for the system to be in long-run balance at all times.

Long-Run Balance and Series Reduction



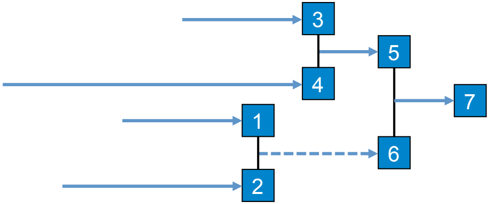
Proposition (Rosling 1989)

In an assembly system without disruptions, it is optimal for the system to be in long-run balance at all times.

Theorem (Rosling 1989)

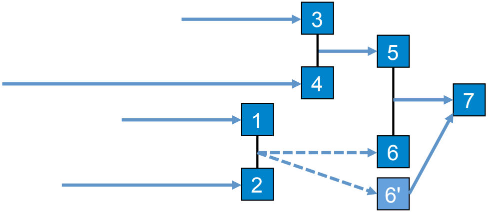
An assembly system without disruptions can be reduced to an equivalent serial system.

Disruptions Destroy Long-Run Balance



- If stage 6 may be disrupted, may want $IN_6 > IN_5$
- DeCroix (2013):
  - Conditions under which **item-specific long-run balance** is optimal
  - Reduction to *partial series* system
  - Heuristic for base-stock levels based on Chen–Zheng (1994)

Our Proposed Policy



- Work in progress (He, LVS, DeCroix, Li 2020):
  - Allow disruption-prone stages to hold **disruption stock**
  - Optimized separately from regular inventory at stage

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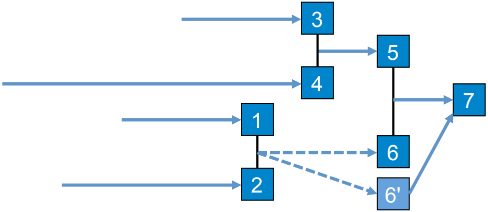

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Proposition

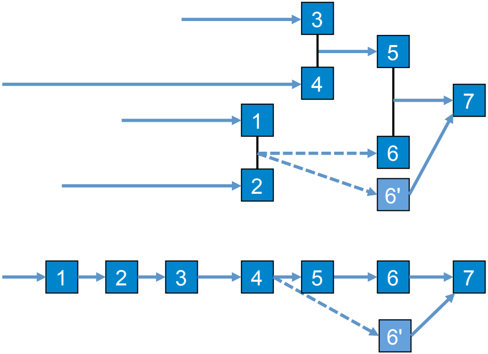
It is optimal to hold disruption stock at stage i iff

[condition involving supply and demand distributions, costs, and a constant].

Moreover, it is optimal for disruption stock to follow a base-stock policy.

- Unfortunately, it is difficult to determine the constant explicitly

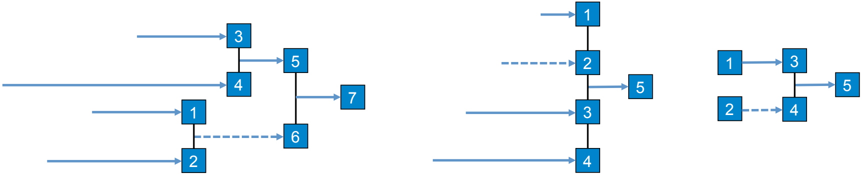
Partial Series Reduction with Disruption Stock



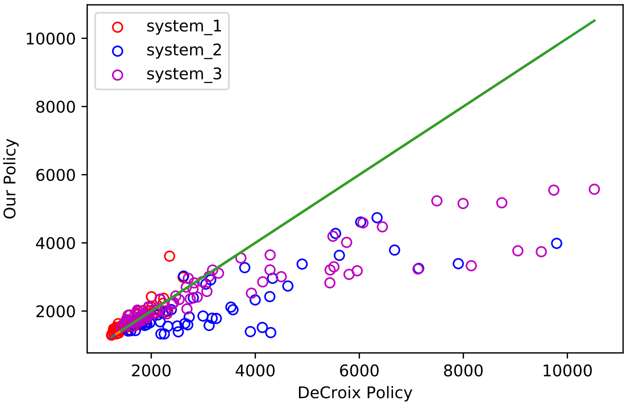
- Approximate reduction to equivalent series system plus disruption stock
- Heuristic for base-stock levels

Numerical Results

- Test on 3 network structures
- Various values of costs, disruption parameters
- Normally distributed demand
- Expected cost via simulation



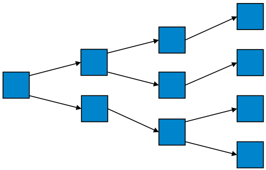
Comparison to DeCroix's Solution



- Both heuristics are fast (seconds)

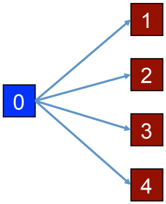
Outline

Distribution Systems



- Must consider both replenishment policy and allocation policy
- Optimal policy is not known for either
- Typically choose plausible policies—e.g., base-stock and FCFS—and then optimize parameters
- But parameter optimization is also difficult

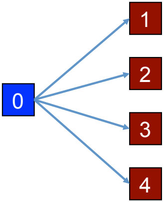
Risk Pooling and Risk Diversification



- **One-warehouse, multiple retailer (OWMR)** system
- Periodic review
- Inventory allowed at warehouse or retailers (not both), using base-stock policy
  - Centralization vs. decentralization

The Risk-Pooling Effect

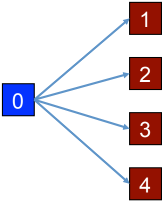
- If demand is stochastic, centralization minimizes expected cost
  - The risk-pooling effect (Eppen 1979)



	Expected Cost	Variance of Cost
Stochastic Demand	$\mathbb{E}[C_C] < \mathbb{E}[C_D]$	
Stochastic Supply		

The Risk-Pooling Effect

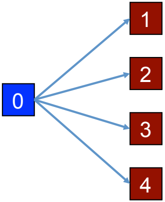
- If demand is stochastic, centralization minimizes expected cost
  - The risk-pooling effect (Eppen 1979)
- Cost variances are equal (Schmitt, Sun, LVS, Shen 2015)



	Expected Cost	Variance of Cost
Stochastic Demand	$\mathbb{E}[C_C] < \mathbb{E}[C_D]$	$\mathbb{V}[C_C] = \mathbb{V}[C_D]$
Stochastic Supply		

The Risk-Diversification Effect

- Now assume supply can be disrupted
  - Same disruption process at all sites
- Demand is deterministic



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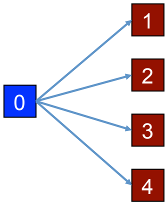

	Expected Cost	Variance of Cost
Stochastic Demand	$\mathbb{E}[C_C] < \mathbb{E}[C_D]$	$\mathbb{V}[C_C] = \mathbb{V}[C_D]$
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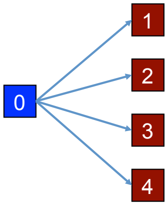
The Risk-Diversification Effect

- Now assume supply can be disrupted
  - Same disruption process at all sites
- Demand is deterministic
- Then decentralization minimizes cost variance
- But expected cost is equal under centralization and decentralization
  - The risk-diversification effect
- (Schmitt, Sun, LVS, Shen 2015)



	Expected Cost	Variance of Cost
Stochastic Demand	$\mathbb{E}[C_C] < \mathbb{E}[C_D]$	$\mathbb{V}[C_C] = \mathbb{V}[C_D]$
Stochastic Supply	$\mathbb{E}[C_C] = \mathbb{E}[C_D]$	$\mathbb{V}[C_C] > \mathbb{V}[C_D]$

Which Effect Dominates?

- Suppose demand and supply are both stochastic
- Is centralization or decentralization preferable?

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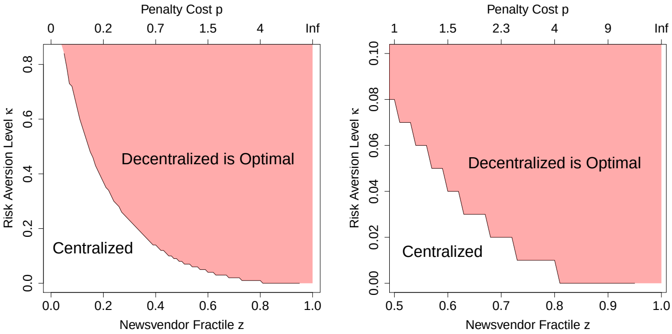
Which Effect Dominates?

- Suppose demand and supply are both stochastic
- Is centralization or decentralization preferable?
- Fully risk-neutral decision maker prefers centralization
- Fully risk-averse decision maker prefers decentralization
- What about in between?

$$(1-\kappa)\mathbb{E}[C] + \kappa\sqrt{\mathbb{V}[C]}$$

Risk-Diversification Effect Nearly Always Dominates



Decentralization is typically preferred, unless:

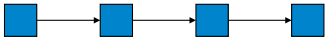
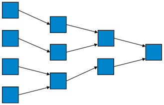
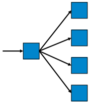
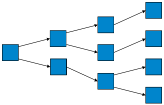
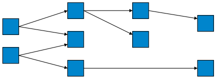
- Service level (newsvendor fractile) is very small
- κ is very small
- Disruptions are very infrequent or short

Other Work

- Inventory at warehouse *and* retailers (Atan and LVS 2012)
- General distribution systems: approximate cost function (He dissertation 2014)

Outline

Conclusions and Future Research

Problem	Analytical Results	Heuristic	Exact Alg.
			
			
			
			
			

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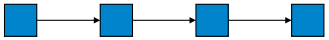
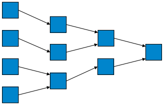
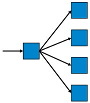
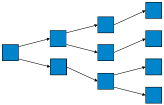
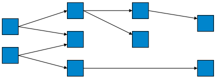
	Optimal policy	✓	✓
			
			
			
			

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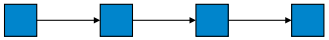
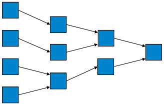
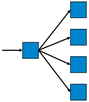
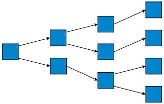
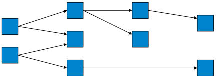
	Optimal policy	✓	✓
	Long-run balance	✓	✗
			
			
			

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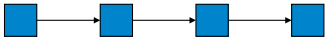
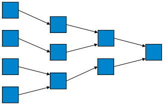
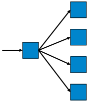
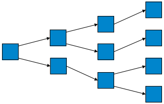
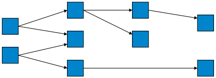
	Optimal policy	✓	✓
	Long-run balance	✓	✗
	Closed-form solutions Risk diversification (Determ. demand)	✓	✓ / ✗
			
			

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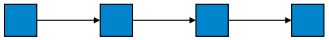
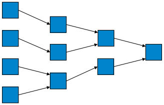
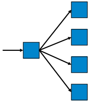
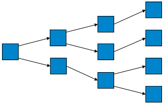
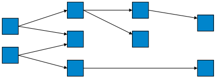
	Optimal policy	✓	✓
	Long-run balance	✓	✗
	Closed-form solutions Risk diversification (Determ. demand)	✓	✓ / ✗
	Approx. cost function	✓	✗
			

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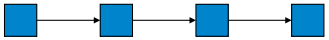
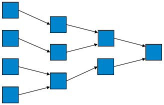
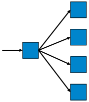
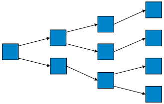
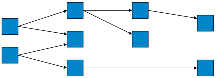
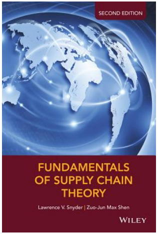
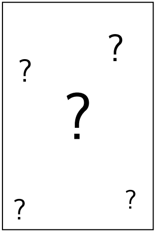
	Optimal policy	✓	✓
	Long-run balance	✓	✗
	Closed-form solutions Risk diversification (Determ. demand)	✓	✓ / ✗
	Approx. cost function	✓	✗
	✗	✗	✗

A Quick, Shameless Plug



Snyder and Shen, *Fundamentals of Supply Chain Theory*, 2nd edition, Wiley, 2019.



Snyder, Smilowitz, and Shen, *Supply Chain Modeling and Optimization*, Dynamic Ideas, 2021.

Questions?

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Thank You!

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