Production Planning for Batch Operations

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Motivation and Introduction

- ✓ Goal: To develop production planning models
  - ✓ Determine the available production capacity accurately
  - ✓ Accounting for sequence-dependent changeovers

- ✓ Real world problem:
  - ✓ Specialty Chemicals and Plastics business within Dow Chemical

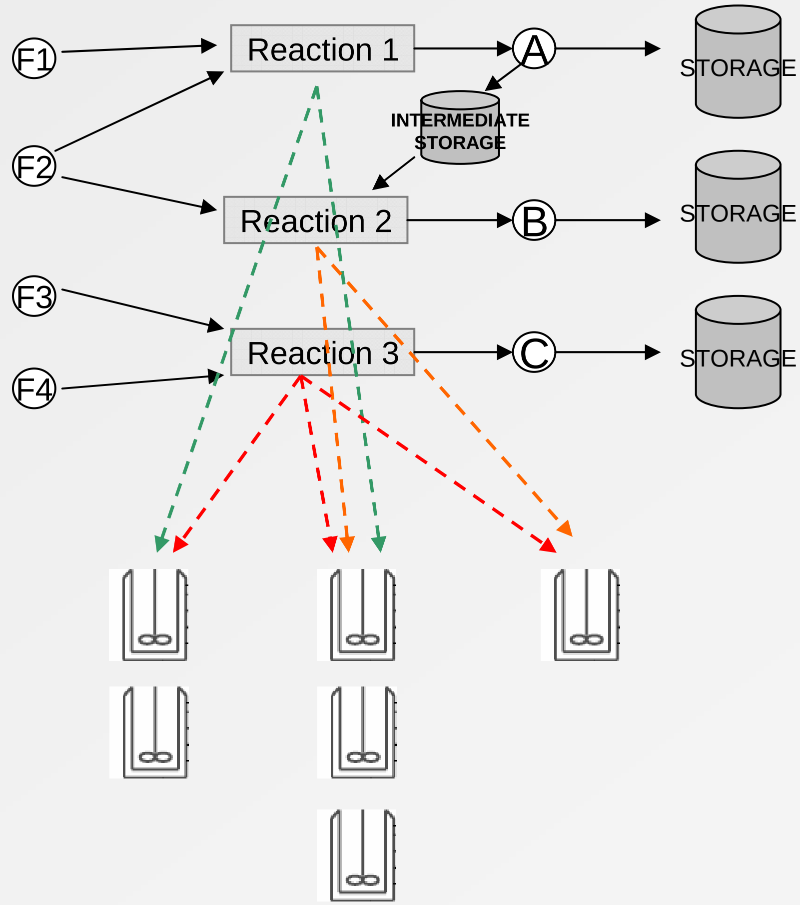
- ✓ Business challenges
  - ✓ Introduction of new products
  - ✓ Cost pressures
- ✓ Flexibility increases the complexity in the planning process
- ✓ Accurate production capability offers significant competitive advantage

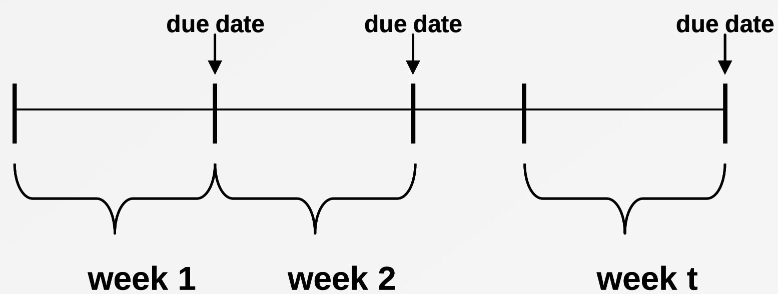
Problem Statement

Materials:
- ➢Raw materials, Intermediates, Finished products
- ➢Unit ratios (lbs of needed material per lb of material produced)

Production Site:
- ✓ Raw material availability and Raw material costs
- ✓ Storage tanks with associated capacity
- ✓ Reactors:
  - ✓ Materials it can produce
  - ✓ Batch sizes (lbs) for each material it can produce
  - ✓ Operating costs ($/hr) for each material
  - ✓ **Sequence dependent change-over times** (hrs per transition for each material pair)
  - ✓ Time the reactor is available during a given month (hrs)

Customers:
- ➢Monthly forecasted demands for desired products
- ➢Price paid for each product





Problem Statement

DETERMINE THE PRODUCTION PLAN:

- ✓ Production quantities
- ✓ Inventory levels
- ✓ Number of batches of each product
- ✓ Assignments of products to available processing equipment
- ✓ *Sequence of production* in each processing equipment

OBJECTIVE:

To Maximize **Profit**.

Profit = Sales – Costs

Costs=Operating Costs – Inventory Costs –Transition Costs

Proposed MILP Planning Models

OPTION A. (Relaxed Planning Model-RP)

Constraints that **underestimate the sequence-dependent** changeover times

=> Weak upper bounds

OPTION B. (RP*)

Simplification of Relaxed Planning Model

Tightening transition constraints are neglected

OPTION C. (Detailed Planning Model-DP)

Sequencing constraints for accounting for changeovers rigorously

=> Tight upper bounds

OPTION D. (Relaxed Detailed Planning Model-DP*)

Relaxation of DP

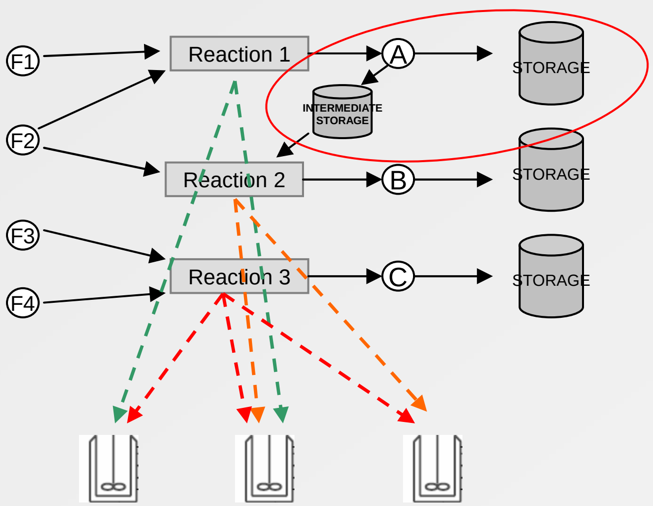
Number of batches are treated as continuous variables

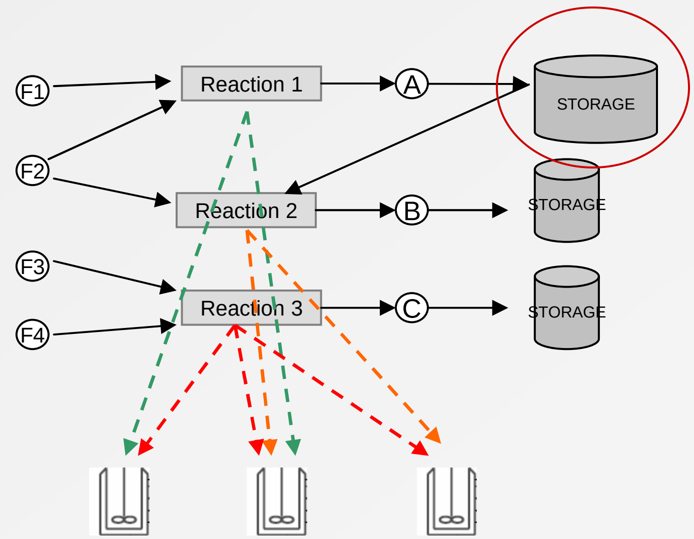
OPTION E. (Rolling Horizon Approach- RH)

Forward rolling horizon algorithm

Simplification

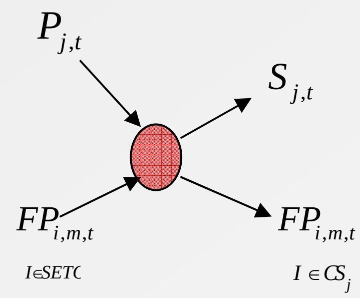
✓Intermediate storage tank and the dedicated storage tank **aggregated** into a single tank
✓**Aggregate mass balances** for the intermediates



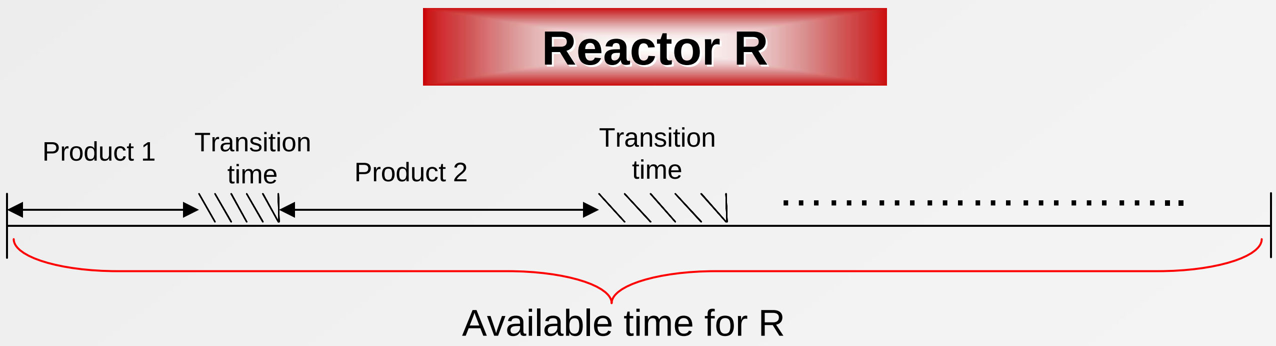


RELAXED PLANNING MODEL (RP)

- ✓ Mass Balances on State Nodes



- ✓ Time Balance Constraints on Equipment



- ✓ Objective Function

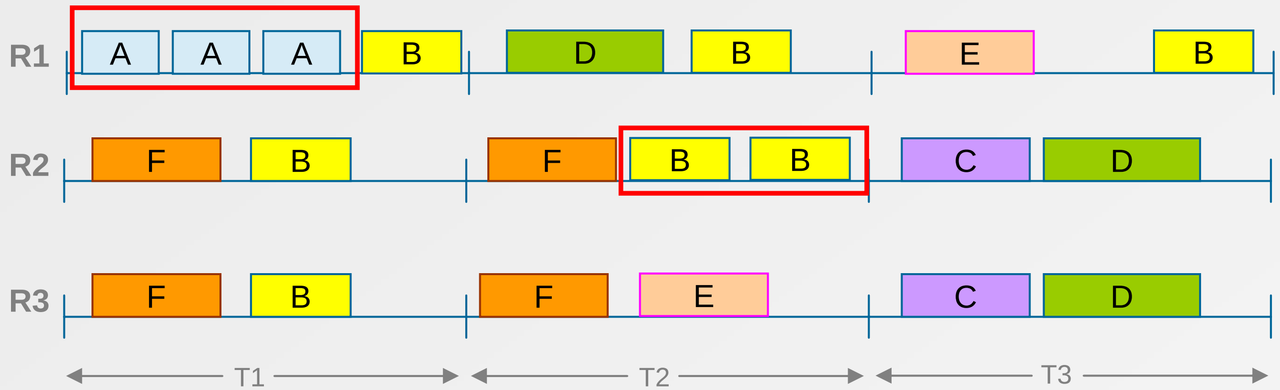
Key Variables for the Model (RP)

$YP_{i,m,t}$:the assignment of products to units at each time period

NB_{imt} :number of each batches of each product on each unit at each period

FP_{imt} :amount of material processed by each task

Products: A, B, C, D, E, F → Reactor 1 or Reactor 2 or Reactor 3



$$YP_{A,reactor1,time1} = 1$$

$$NB_{A,reactor1,time1} = 3$$

$$YP_{B,reactor2,time2} = 1$$

$$NB_{B,reactor2,time2} = 2$$

Proposed Planning Model RP

Mass Balance and Assignment Constraints:

$$Bound_{imt} = \underbrace{\frac{H_t}{BT_{im}}}_{\text{Largest number that the task can be repeated}} \cdot \underbrace{Q_{im}}_{\text{Maximum capacity}}$$

Maximum lbs of product i produced on unit m at time period t if product i is assigned throughout the time period

$$FP_{imt} \le Bound_{imt} \cdot YP_{imt}$$

$$FP_{imt} \ge Q_{im} \cdot YP_{imt}$$

- Bound on production levels of product i on unit m at time period t.
- Sets production level to zero if product i is not assigned on unit m at time period t.

$$NB_{i,m,t} = FP_{i,m,t} / Q_{i,m}$$

Number of batches of product i in unit m at time t *(INTEGER)*

Mass balance on each state node:

$$\underbrace{P_{jt}}_{\text{purchases}} + \underbrace{\sum_{i \in PS_j} \rho_{ji} \sum_{m \in MT_i} FP_{imt}}_{\text{production}} = \underbrace{S_{jt}}_{\text{sales}} + \underbrace{\sum_{i \in CS_j} \bar{\rho}_{ji} \sum_{m \in MT_i} FP_{imt}}_{\text{consumption}} + \underbrace{INV_{jt} - INV_{jt-1}}_{\text{change in inventory}}$$

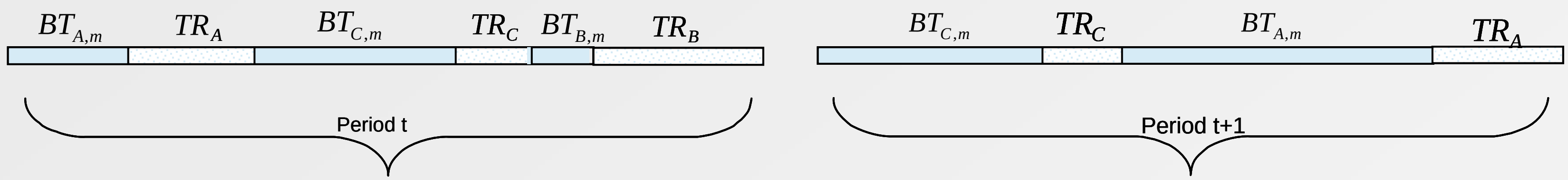
Transitions and Time Balance Constraints for RP

- ✓ Lower bounds for changeovers
- ✓ Sequencing constraints are neglected
- ✓ Introduce a minimum transition time for each assigned product

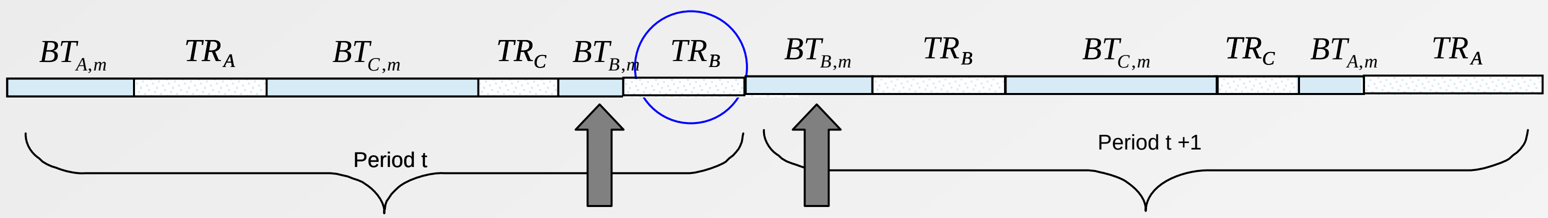
Parameter:

$$TR_i = \underset{i' \neq i}{Min}\{\tau_{i,i'}\}$$

$\tau_{i,i'}$	A	B	C
A	0	5	10
B	8	0	22
C	4	12	0



$$\sum_i NB_{i,m,t} \cdot BT_{i,m} + \sum_i TR_i \cdot YP_{i,m,t} \leq H_t \qquad \forall m,t$$



- ✓ Over estimation of changeovers
- ✓ Can not model transitions across adjacent periods

$$\sum_i NB_{i,m,t} \cdot BT_{i,m} + \sum_i TR_i \cdot YP_{i,m,t} - U_{m,t} \leq H_t \qquad \forall m,t$$

Transitions and Time Balance Constraints for RP

where $U_{m,t}$:

$$U_{m,t} \geq TR_{i,m} \cdot YP_{i,m,t} \quad \forall i \in I_m, m, t$$

$$U_{m,t} \leq \underset{i \in I_m}{Max}\left\{TR_{i,m}\right\} \quad \forall m, t$$

$$U_{m,t} \leq \sum_{i \in I_m} TR_i \cdot YP_{i,m,t} \quad \forall m, t \text{ (*)}$$

(*) Redundant, tightens the formulation, neglecting may result in overestimation of the available production time.

Transition Costs:

$$Transition\ Cost = \sum_t \sum_m \left(\sum_i TRC_i \cdot YP_{i,m,t}\right) - UT_{m,t}$$

where:

$$TRC_i = \underset{i' \neq i}{Min}\left\{C_{trans_{i,i'}}\right\}$$

$$UT_{m,t} \geq TRC_{i,m} \cdot YP_{i,m,t} \quad \forall i \in I_m, m, t$$

$$UT_{m,t} \leq \underset{i \in I_m}{Max}\left\{TRC_{i,m}\right\} \quad \forall m, t$$

$$UT_{m,t} \leq \sum_{i \in I_m} (TRC_{i,m} \cdot YP_{i,m,t}) \quad \forall m, t \text{ (**)}$$

(**) Redundant, tightens the formulation, neglecting may result in underestimation of the transition cost.

Maximize PROFIT:

$$Z^p = \underbrace{\sum_j \sum_t cp_{jt} \cdot S_{jt}}_{\text{Sales}} - \underbrace{\sum_j \sum_t c_{jt}^{inv} \cdot INV_{jt}}_{\text{Inventory costs}} - \underbrace{\sum_i \sum_m \sum_t c_{it}^{oper} \cdot FP_{imt}}_{\text{Variable operating costs}} - \underbrace{\sum_t \sum_m \left(\sum_i TRC_i \cdot YP_{i,m,t}\right) + UT_{m,t}}_{\text{Transition costs}}$$

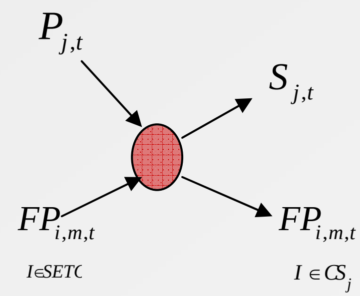
Generic Form of Model (RP*)- Option B

PLANNING MODEL (RP*)

- ✓ RP* same as RP but constraints (*) and (**) are neglected.
- ✓ Provides a valid but a weaker upper bound compared to RP.
- ✓ It can lead to overestimation of the available production time and underestimation of transition costs.

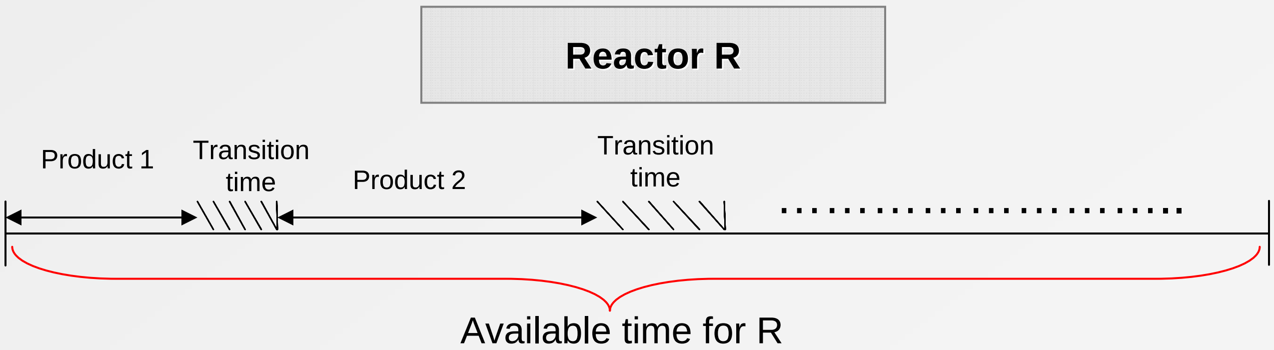
DETAILED PLANNING MODEL (DP)

- ✓ Mass Balances on State Nodes



- ✓ *Sequencing Constraints*
  - ✓ *Sequence dependent changeovers determined*
  - ✓ *Detailed timings of operations neglected*

- ✓ ***Time Balance Constraints on Equipment***



- ✓ ***Objective Function***

Sequencing Constraints for DP

Sequence dependent changeovers:

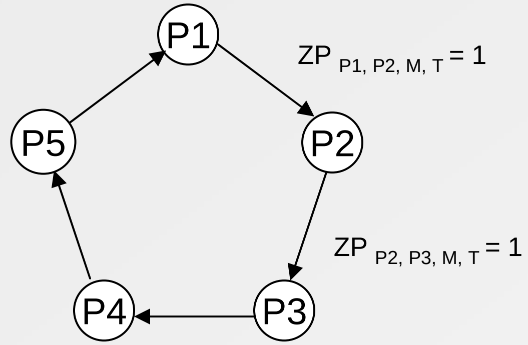
✓ **Sequence dependent changeovers within each time period:**

1. **Generate a cyclic schedule where total transition time is minimized.**

KEY VARIABLE:

$ZP_{ii'mt}$:becomes 1 if product i is after product i’ on unit m at time period t, zero otherwise

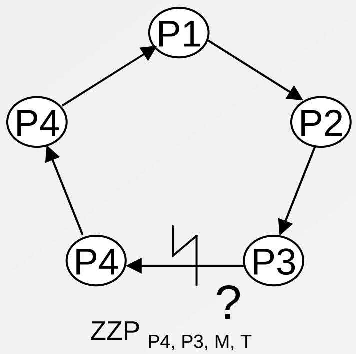
P1, P2, P3, P4, P5



2. **Break the cycle at the pair with the maximum transition time to obtain the sequence.**

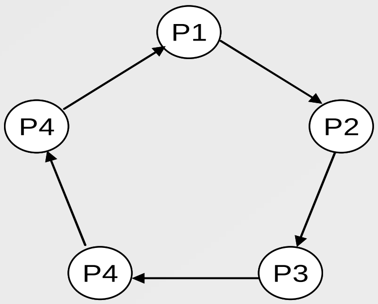
KEY VARIABLE:

$ZZP_{ii'mt}$:becomes 1 if the link between products i and i’ is to be broken, zero otherwise



Changeovers within each period for DP

According to the location of the link to be broken:



P2, P3, P4, P5, P1 ⟶ $ZZP_{P1, P2, M, T} = 1$

P3, P4, P5, P1, P2 ⟶ $ZZP_{P2, P3, M, T} = 1$

P4, P5, P1, P2, P3 ⟶ $ZZP_{P3, P4, M, T} = 1$

P5, P1, P2, P3, P4 ⟶ $ZZP_{P4, P5, M, T} = 1$

P1, P2, P3, P4, P5 ⟶ $ZZP_{P5, P1, M, T} = 1$

The sequence with the minimum total transition time is the **optimal sequence** within time period t.

$$YP_{imt} = \sum_{i'} ZP_{ii'mt} \qquad \forall i,m,t$$

$$YP_{i'mt} = \sum_{i} ZP_{ii'mt} \qquad \forall i',m,t$$

$$YP_{imt} \wedge \left[\bigwedge_{i'\neq i} \neg YP_{i'mt}\right] \Leftrightarrow ZP_{iimt} \qquad \forall i,m,t$$

$$YP_{imt} \geq ZP_{i,i,m,t} \qquad \forall i,m,t$$

$$ZP_{i,i,m,t} + YP_{i',m,t} \leq 1 \qquad \forall i,i'\neq i,m,t$$

$$ZP_{i,i,m,t} \geq YP_{i,m,t} - \sum_{i'\neq i} YP_{i',m,t} \qquad \forall i,m,t$$

$$\sum_{i}\sum_{i'} ZZP_{ii'mt} = 1 \qquad \forall m,t$$

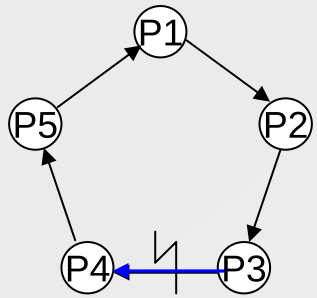
$$ZZP_{ii'mt} \leq ZP_{ii'mt} \qquad \forall i,i',m,t$$

Generate the cycle and break the cycle to find the optimum sequence where transition times are minimized.

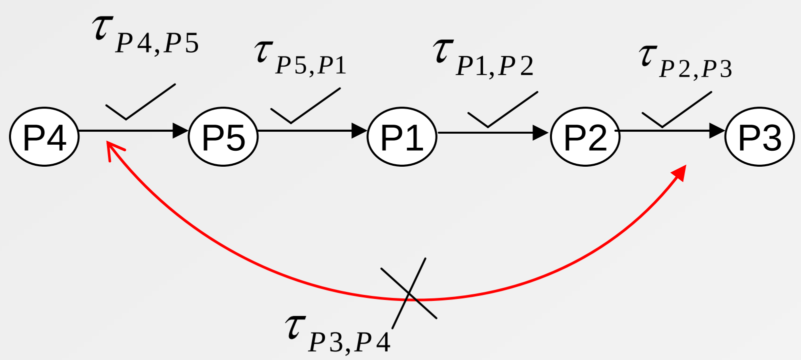
Having determining the sequence, we can determine **the total transition time** within each week.

Changeovers within each period for DP

1) generate the cycle



2) break the cycle to obtain the sequence



$$TRNP_{m,t} = \tau_{P4,P5} + \tau_{P5,P1} + \tau_{P1,P2} + \tau_{P2,P3} + \tau_{P3,P4} - \tau_{P3,P4}$$

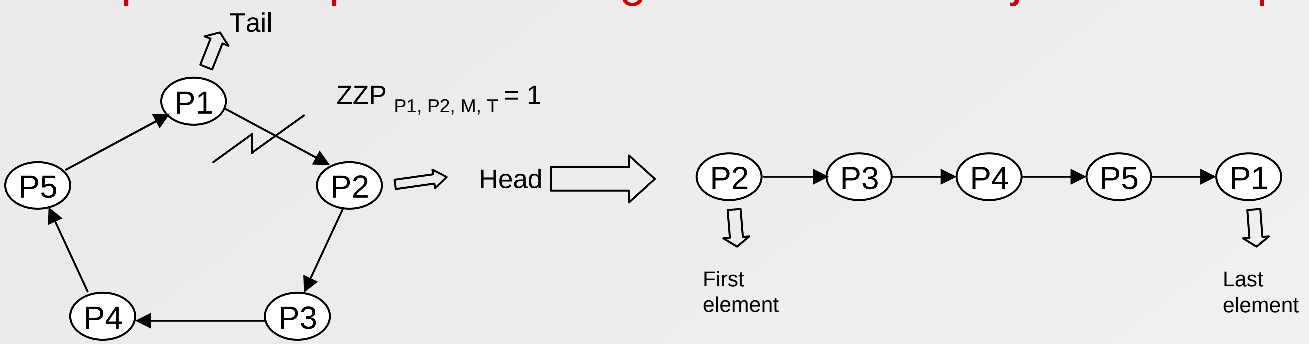
Total transition time within period t on unit m

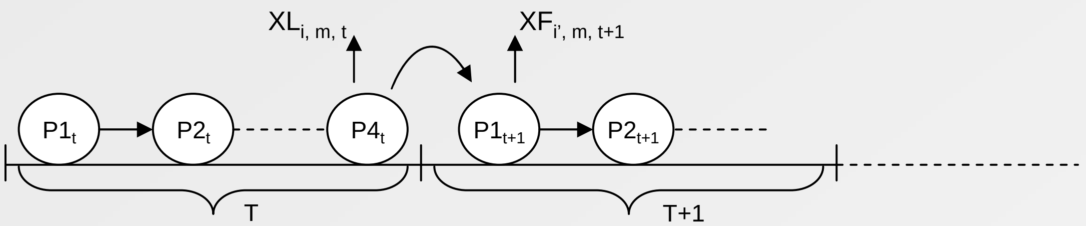
Transition time required to change the operation from P1 to P2

$$TRNP_{m,t} = \sum_{i}\sum_{i'} \tau_{i,i'} \cdot ZP_{i,i',m,t} - \sum_{i}\sum_{i'} \tau_{i,i'} \cdot ZZP_{i,i',m,t} \qquad \forall m,t$$

Changeovers across adjacent periods for DP

✓ Sequence dependent changeovers across adjacent time periods:





$$ZZZ_{i,i',m,t} \geq XL_{i,m,t} + XF_{i',m,t+1} - 1 \quad \forall i,i',m,t$$

Transitions across adjacent weeks

Time Balance and Objective Function for DP

The time balance constraint:

$$\underbrace{\sum_i NB_{i,m,t} \cdot BT_{i,m}}_{\text{summation of batch times of assigned products to unit m at period t.}} + \underbrace{TRNP_{m,t}}_{\text{total transition time within time period t}} + \underbrace{\sum_i \sum_{i'} ZZZ_{i,i',m,t} \cdot \tau_{ii'}}_{\text{transition time between period t and period t+1}} \le \underbrace{H_t}_{\text{total available time for unit m}} \qquad \forall m,t$$

summation of batch times of assigned products to unit m at period t.

total transition time within time period t

transition time between period t and period t+1

total available time for unit m

Objective Function:

Maximize PROFIT:

$$Z^p = \underbrace{\sum_j \sum_t cp_{jt} \cdot S_{jt}}_{\text{Sales}} - \underbrace{\sum_j \sum_t c_{jt}^{inv} \cdot INV_{jt}}_{\text{Inventory costs}} - \underbrace{\sum_i \sum_m \sum_t c_{it}^{oper} \cdot FP_{imt}}_{\text{Variable operating costs}} \underbrace{- \sum_i \sum_{i'} \sum_m \sum_t c_{i,i'}^{trans} \cdot ZP_{i,i',m,t} + \sum_i \sum_{i'} \sum_m \sum_t c_{i,i'}^{trans} \cdot ZZP_{i,i',m,t} - \sum_i \sum_{i'} \sum_m \sum_t c_{i,i'}^{trans} \cdot ZZZ_{i,i',m,t}}_{\text{Transition costs}}$$

Sales

Inventory costs

Variable operating costs

Transition costs

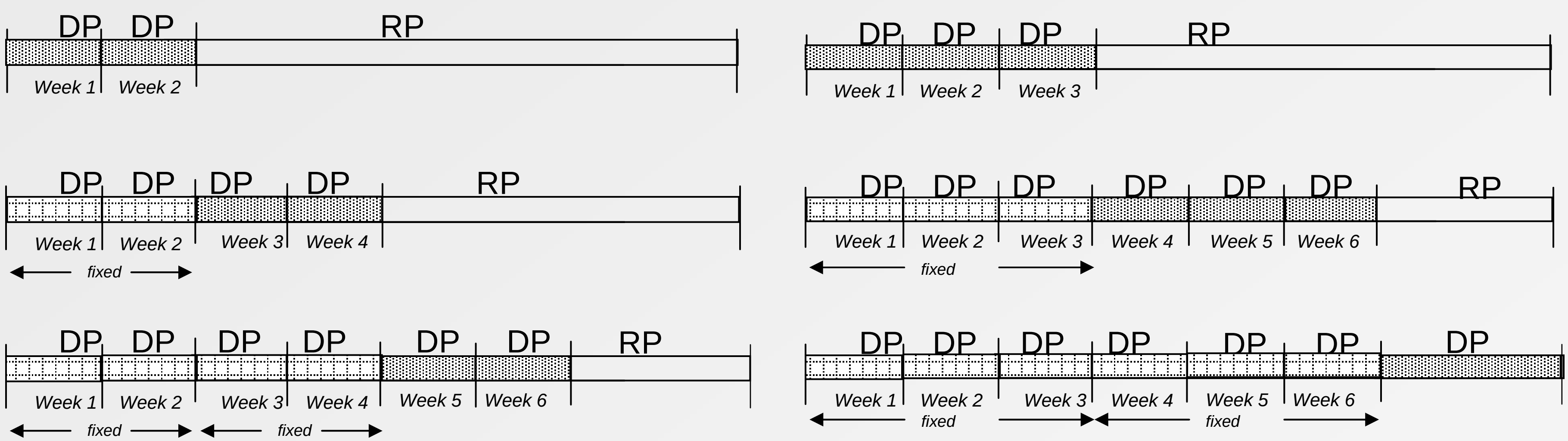
Generic Form of Model (DP*)- Option D

RELAXED DETAILED PLANNING MODEL (DP*)

- ✓ DP* same as DP but *number of batches* are treated as continuous variables.
- ✓ Provides a valid upper bound on the profit.
- ✓ Significantly reduces the computational expense.

Rolling Horizon Approach – Option E

- ✓ The **proposed planning models** may still be to very expensive to solve.
- ✓ Sequence of sub-problems that are solved recursively.
- ✓ Provides a lower bound on the profit.



- ✓ The detailed planning period (DP) moves as the model is solved in time.
- ✓ Future planning periods include only underestimations for transition times (RP*).
- ✓ In each iteration we fix the binary variables for assignment and sequencing variables.

REMARKS

- ✓ Relaxed Planning Model (RP) is adequate if:
  - ✓ Demand rates are low
  - ✓ Transition times show low variance

  RP could lead to significant overestimation of the available production capacity if the above conditions are not true.

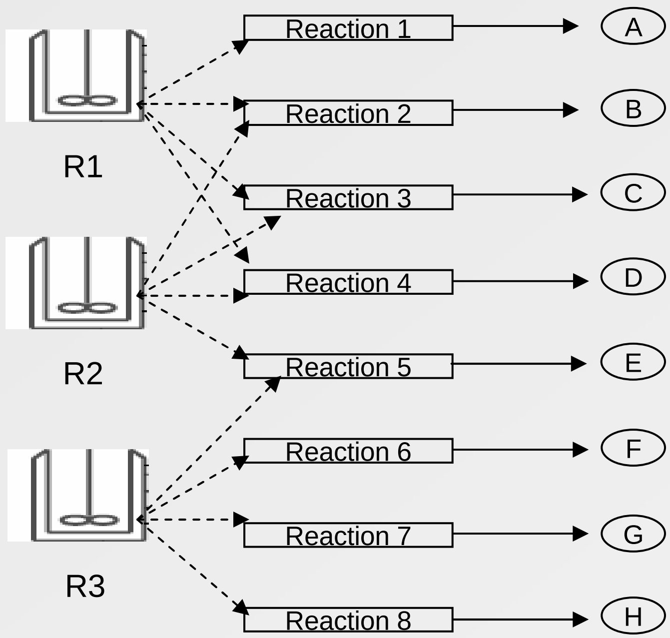
- ✓ Detailed Planning Model (DP) very powerful:
  - ✓ Since the sequencing constraints are explicitly accounted for, it yields very tight upper bounds.
  - ✓ In the absence of subcycles and for single stage production, it produces the identical solution as a detailed scheduling model would.
  - ✓ Among all the instances we have solved so far, only one exhibited a subcycle.

  Trade-off between the extent of scheduling decisions incorporated and the size and the computational effort of the resulting problem.

Examples

EXAMPLE 1 - 8 Products, 3 Reactors

✓ Determine the **plan** for **8** products, **3** reactors plant so as to maximize **profit.**

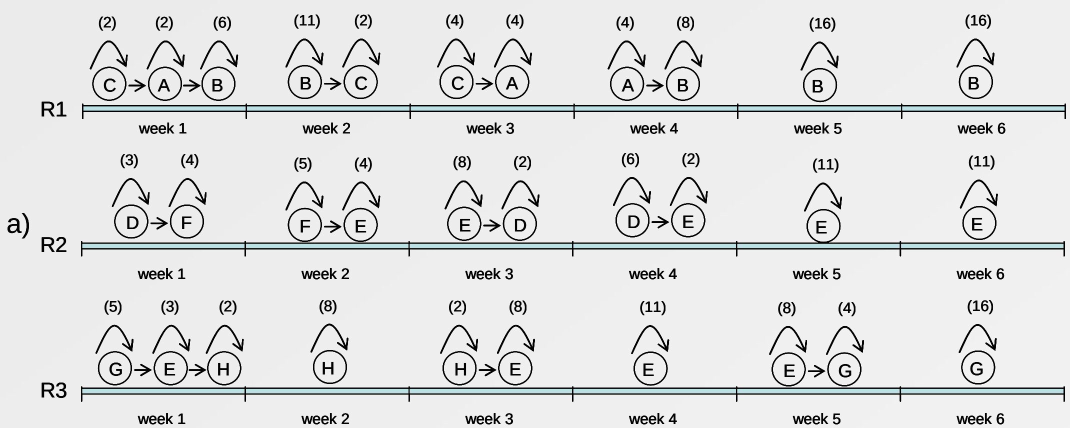


- 8 Products, A,B,C,D,E,F,G,H
- All produced in a single stage.
- 3 Reactors, R1,R2,R3
- End time of the week is defined as due dates
- Demands are lower bounds

EXAMPLE 1 - 8 Products, 3 Reactors

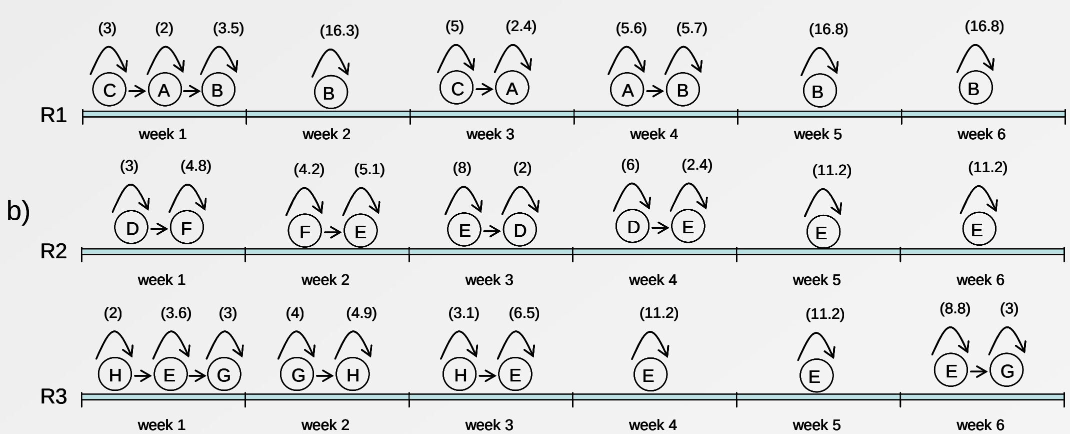
For a planning horizon of 6 weeks:

method	number of binary variables	number of continuous variables	number of equations	time (CPU s)	solution ($)
detailed planning (DP)	864	1327	1483	1667	11,819
detailed planning (DP*)	792	1327	1483	96	12,211
relaxed planning (RP)	144	349	553	1.75	13,460
rolling horizon (RH)	624	1,291	1,483	322	11,377



Schedule and Number of Batches obtained by **DP**

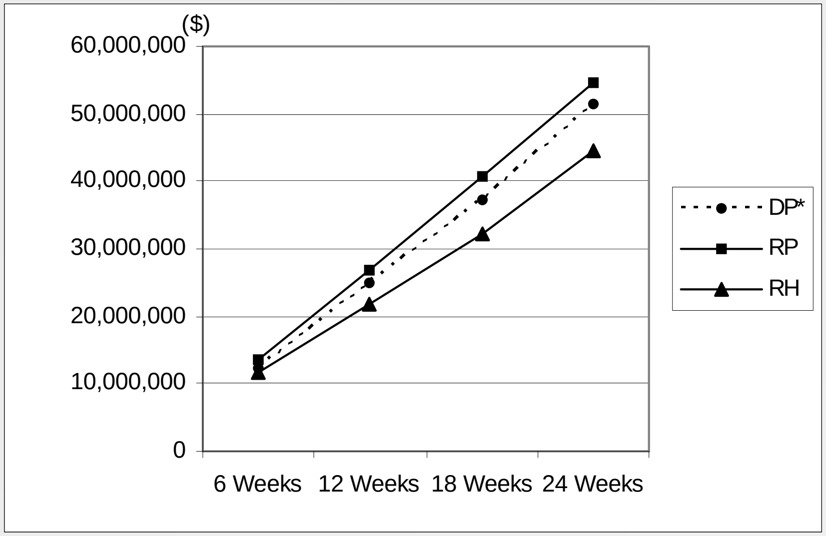
Exact schedule!

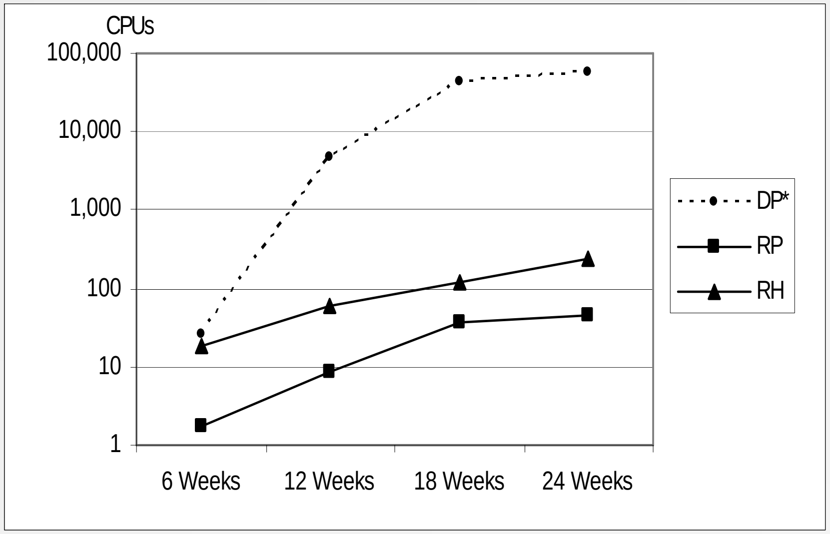


Schedule and Number of Batches obtained by **DP***

EXAMPLE 1 - 8 Products, 3 Reactors

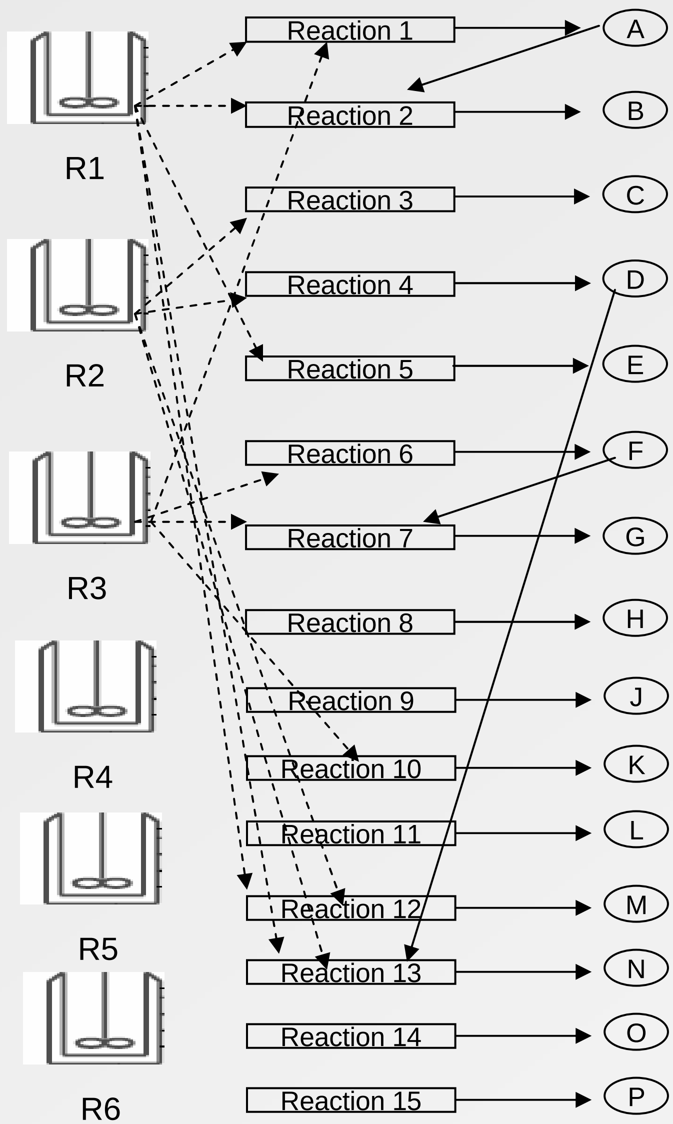
Comparison of Models for Planning Horizons of 6 to 24 Weeks for 5% Optimality Tolerance





EXAMPLE 2 - 15 Products, 6 Reactors, 48 Weeks

✓ Determine the **plan** for **15** products, **6** reactors plant so as to maximize **profit.**



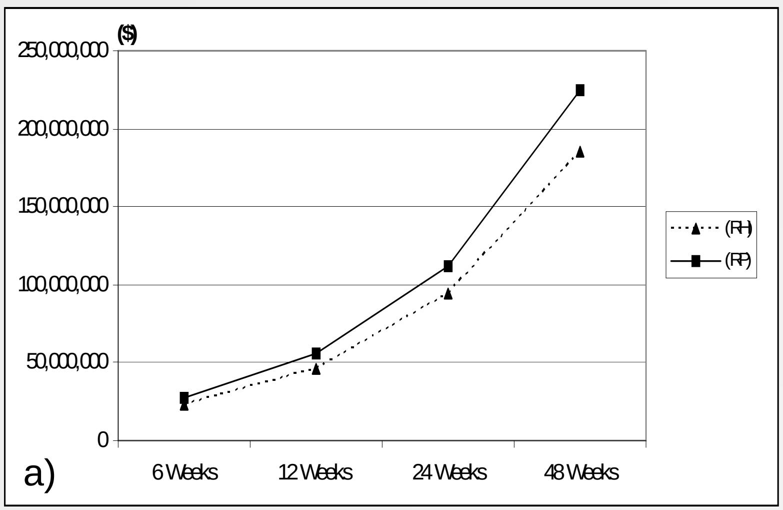
- 15 Products, A,B,C,D,E,F,G,H,J,K,L,M,N,O,P
- B, G and N are produced in 2 stages.
- 6 Reactors, R1,R2,R3,R4,R5,R6
- End time of the week is defined as due dates
- Demands are lower bounds

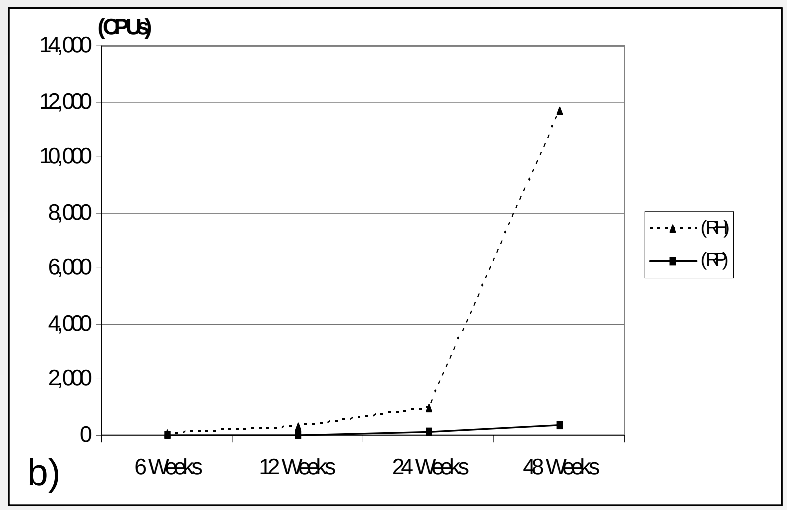
EXAMPLE 2 - 15 Products, 6 Reactors, 48 Weeks

For a planning horizon of 48 weeks for 6% optimality tolerance :

method	number of binary variables	number of continuous variables	number of equations	time (CPU s)	solution ($)
relaxed planning (RP)	2,592	5,905	9,361	362	224,731,683
rolling horizon (RH)	10,092	25,798	28,171	11,656	184,765,965
rolling horizon (RH**)	1,950	25,798	28,171	4,554	182,169,267

Variation of results from 6 to 48 weeks for 6% optimality tolerance :





Concluding Remarks and Summary

- ✓ Relaxed Planning Model (RP):
  - ✓ Underestimates sequence-dependent changeover times and costs.
  - ✓ Overestimates sales and profit.
- ✓ Detailed Planning Model (DP):
  - ✓ Explicitly accounts for scheduling via sequencing variables and constraints.
  - ✓ Very accurate production plans
- ✓ DP yields more realistic plans compared to RP but at the expense of increasing size and computational effort.
- ✓ For large problems and long time horizons without giving up the solution quality:
  - ✓ Rolling Horizon Algorithm (RH): yields a lower bound on profit
  - ✓ Relaxed Detailed Planning Model (DP*): yields an upper bound on profit