Mixed Integer Non-linear Optimization for Crude Oil Scheduling

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Enterprise-wide Optimization Project

Introduction

- Purchasing and Scheduling of crude oil in refineries involves billions of dollars every year

- Simulation tools are used by refinery schedulers to determine feasible schedules
  - Interfaced with "Black-box" optimization tools
  - Suffer from poor run-times

GOAL:

Derive benefits from use of optimization to obtain crude movement schedules in refinery front-end

Motivation

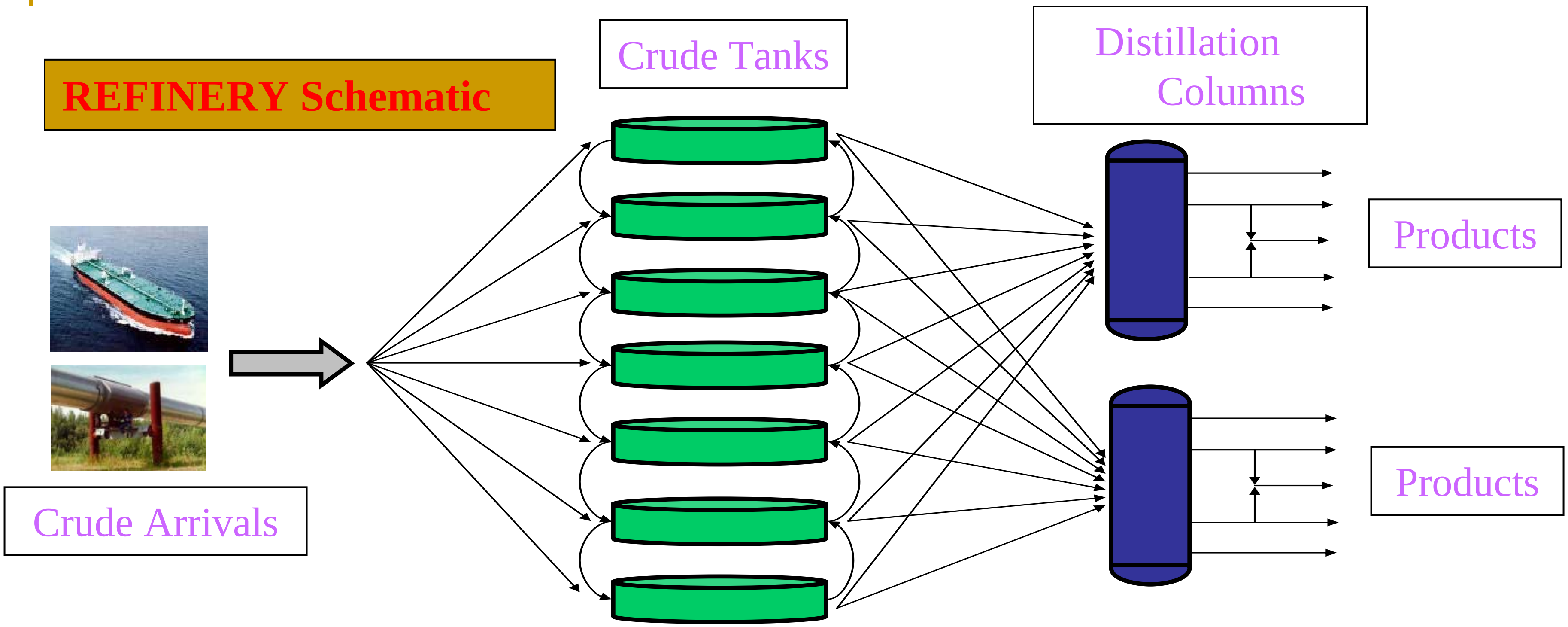
Economic goals

- *Significant reduction in current costs of scheduling and inventory management* is practically possible
- *Reduced costs* in crude-movement scheduling
- *Meet product quality targets and demands* more accurately

Technical goals

- *Improve optimization capabilities* inside refinery simulators
- *Predicting yields*, crude qualities and tank inventory levels over the whole horizon
- Integration with *Marine Transportation* models

Problem Outline



Given:

1. Scheduling Horizon
2. Tank inventory (min, max, initial levels)
3. Available crude types and their properties
4. Product property specifications and demands
5. Bounds on crude and product flows

Determine:

1. When to order crudes
2. How much of each crude to order
3. Operating flows of crude between tanks
4. Charges to Pipestills
5. How much of each product to produce

Simulation

Schedulers consider multiple scenarios and perform simulations

SIMULATION

+

OPTIMIZATION

- Start with optimal schedule for simulation
- Perform "what-if" analyses for system perturbations

Optimization

Optimization

Defined as getting the best possible solution with respect to an objective and subject to some constraints

Mathematical program

$$
\begin{aligned}
\min \quad & z = f(x) + c^T y \\
s.t. \quad & g(x) \le 0 \\
& h(x) = 0 \\
& \mathrm{A}x + \mathrm{B}y \le d \\
& \mathrm{E}y \le e \\
& x \ge 0,\; y \in \{0,1\}
\end{aligned}
$$

Continuous variables (x) Discrete variables (y)

Solution procedure

Modeling Laguages: ***GAMS, AIMMS***

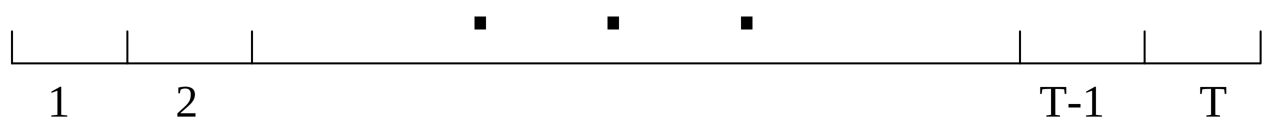
Solvers:

- ***CPLEX*** ⇒ **LP and MILP**
- ***CONOPT*** ⇒ **NLP**
- ***DICOPT, BARON*** ⇒ **MINLP**

Model Development

- **Rigorous multiperiod formulation** to model the important aspects of scheduling (*Discretized time domain*)

Time Horizon



Optimization model

Minimize *cost objective* OR *product property deviation*

s.t.
- Tank constraints
- Crude arrival constraints
- Pipestill constraints
- Variable bounds

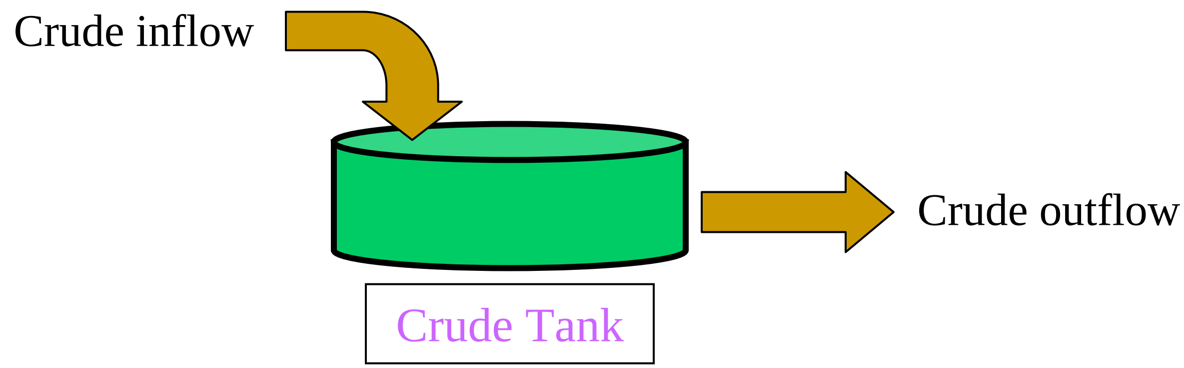
Variables in the model

- **Tank inventories**
- **Crude tank transfers**
- **Crude shipments**
- **Pipestill charges**
- **Cuts' flows**
- **Product flows**
- **Cut properties**
- **Product Properties**
- **Product boiling points**
- **Existence of flow in streams**

Binary Variables → **Existence of flow in streams**

Model Constraints

Tank Constraints



- **Inventory balances**

$$I^{tot}_{b,t-1} + \sum_{a \in A_b} V^{tot}_{abt} = I^{tot}_{bt} + \sum_{c \in C_b} V^{tot}_{bct} \qquad \forall t \in T, \forall b \in B$$

$$I^{comp}_{jb,t-1} + \sum_{a \in A_b} V_{jabt} = I^{comp}_{jbt} + \sum_{c \in C_b} V_{jbct} \qquad \forall j \in J, \forall t \in T, \forall b \in B$$

- **Crude composition in tanks**

$$I^{comp}_{jbt} = f_{jbt} I^{tot}_{bt} \qquad \forall j \in J, \forall t \in T, \forall b \in B$$

Non-linear equations containing Bilinearities

- **Inflow and Outflow restrictions**

$$w_{abt} + w_{bct} \le 1 \qquad \forall a \in A_b, \forall c \in C_b, \forall b \in B, \forall t \in T$$

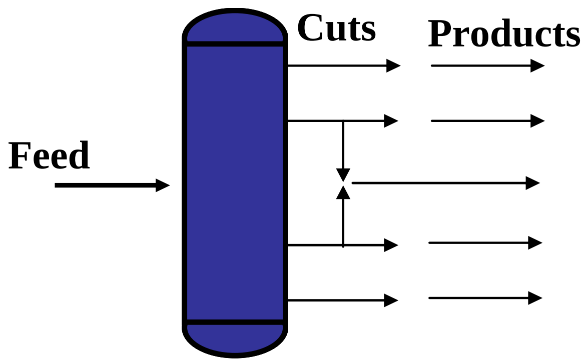
- **Flow balances**

Model Constraints (Contd …)

Crude Arrival Constraints

- **Crude unloading contraints**
  - Only one tank can be fed by one vessel at any instance of time
  - Only one crude type can be unloaded at a point in time

Distillation Column Constraints



- **Inlet flow and property restrictions for Distillation Column**
- **Allocation constraints**
  - At most two charging tanks can charge a pipestill at any time
- **Yield relations** to calculate product flows and properties
- **Product flow and property balances**
- **Product boiling points calculations**

Optimization Model

- Scheduling problem modeled as a
  Multiperiod Mixed Integer Non-linear Program (MINLP)
  - Discrete variables used to determine which flows should exist and when
  - Model is highly non-linear and non-convex

Problem Parameters

- **11-day** horizon, **7** Crude Tanks, **41** Crudes
- **2** Pipestills, **13** Cuts, **6** Final Products

Model statistics

	Base case	**Extreme case**
# Time periods	**3**	**20**
# Properties	**3**	**8**
# Binary variables	**315**	**2,100**
# Continuous variables	**15,522**	**106,560**
# Constraints	**18,153**	**124,540**

- Need specialized algorithm to solve such large-scale models

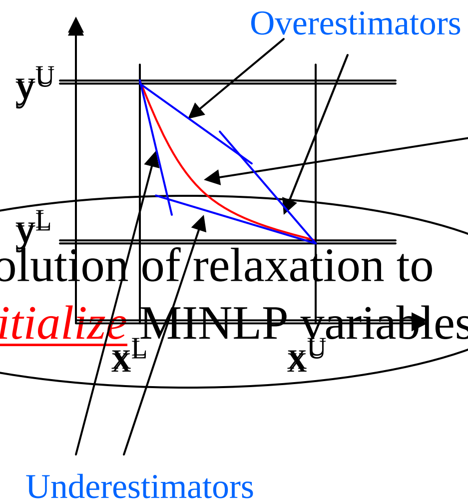
Solution Technique

- Bi-level heuristic algorithm proposed to solve MINLP formulation

Mixed-Integer Linear Programming (MILP) Relaxation

- Linearize non-linear terms and construct convex envelopes

Solve → Get rigorous bound on objective value



Bilinear term $xy = 4$ w = 4

$$w \ge xy^L + x^L y - x^L y^L$$
$$w \ge x^U y + x^U y - x^U y^U$$
$$w \le x^L y + x^U y - x^L y^U$$
$$w \le x^U y + x^L y - x^U y^L$$

Use solution of relaxation to *initialize* MINLP variables

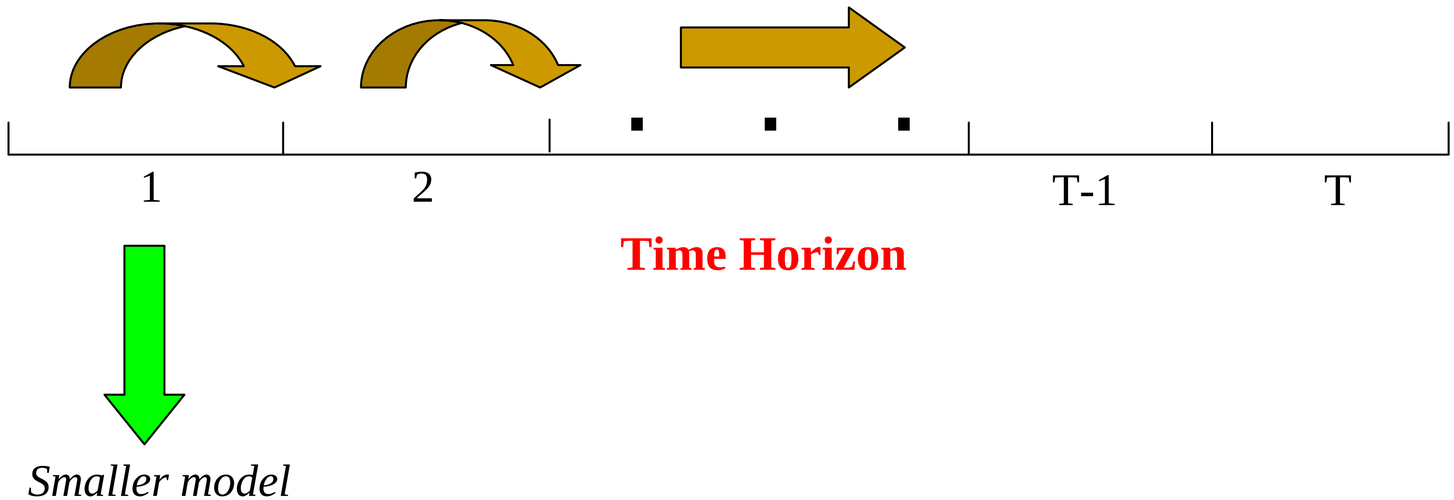
Original MINLP model

Solve → Get locally optimal solution

Alternate Solution Method

- <u>Rolling Horizon Approach</u> to temporally decompose model and solve it

  Solve single time period and fix inventories in tanks for next time period



- Easier to solve temporally decomposed sub-problems
- Results of this method are similar to full space optimization
- Lower likelihood of finding solutions

General Remarks

- Locally optimal schedules obtained for problems where 3 properties are targeted in the products over the entire scheduling horizon
- Cost objective problems easier to solve than problems with product property deviation objectives
- Solution times vary between 500 CPU secs – 5000 CPU secs
- Scaling of data and using good starting points in the optimization are very important → lead to solutions in much reduced solution times

Conclusions

Summary of work

- Development of a rigorous Mixed Integer Non-linear formulation to model crude oil movement in a refinery "front-end"
- Heuristic algorithms to optimize non-linear model to get (locally) optimal schedule
- Method to obtain a bound on the best possible cost objective

Potential future work

- Target more properties in product specifications
- Enhance existing model by including downstream processing and crude vessel routing
- Robust non-linear solver development for solving large-scale refinery scheduling MINLPs

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