Production Planning and Scheduling for Batch Operations

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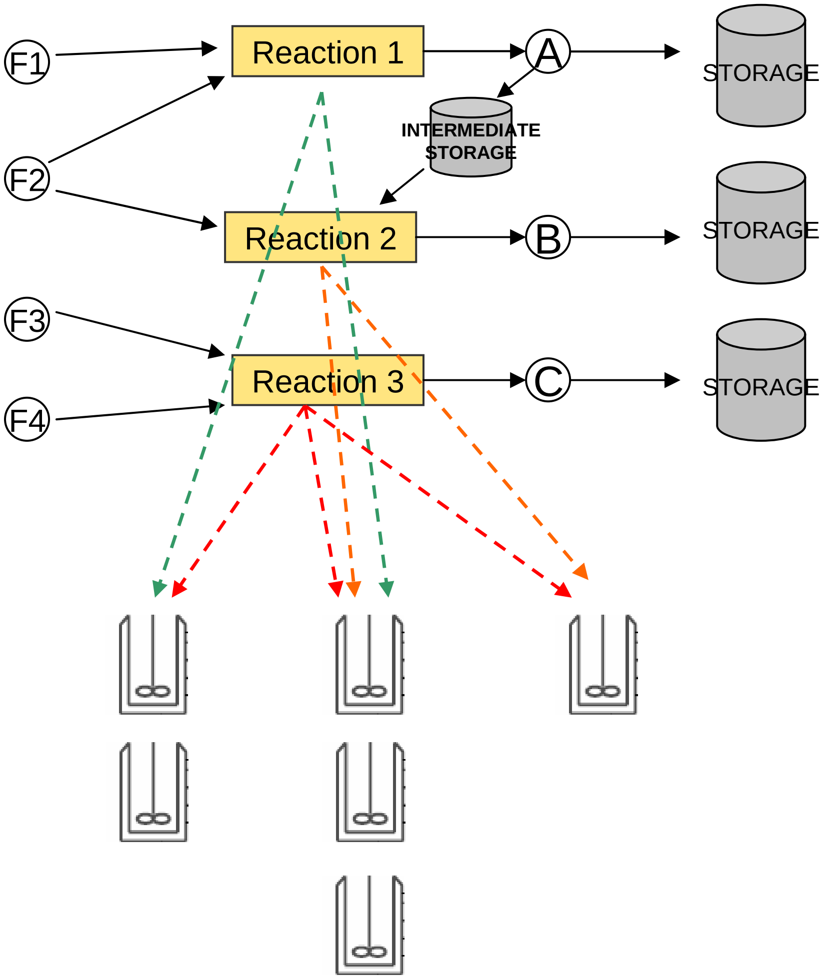
Problem Statement

Materials:
➢Raw materials, Intermediates, End products
➢Unit ratios (lbs of needed material per lb of material produced)

Production Site:
➢ Raw material availability and Raw material costs
➢ Storage tanks with associated capacity
➢ Transportation costs to each customer
➢ Reactors:
- Materials it can produce
- Batch sizes (lbs) for each material it can produce
- Operating costs ($/hr) for each material
- **Sequence dependent change-over times** (hrs per transition for each material pair)
- Time the reactor is available during a given month (hrs)

Customers:
➢Monthly forecasted demands for desired products
➢Price paid for each product

➢Fixed batch times and batch sizes



Problem Statement

DETERMINE PLAN and SCHEDULE:

- Production quantities
- Inventory levels
- Number of batches of each product
- Assignments of products to available processing equipment
- Detailed timing of operations
- Sequence of production in each processing equipment

OBJECTIVE:

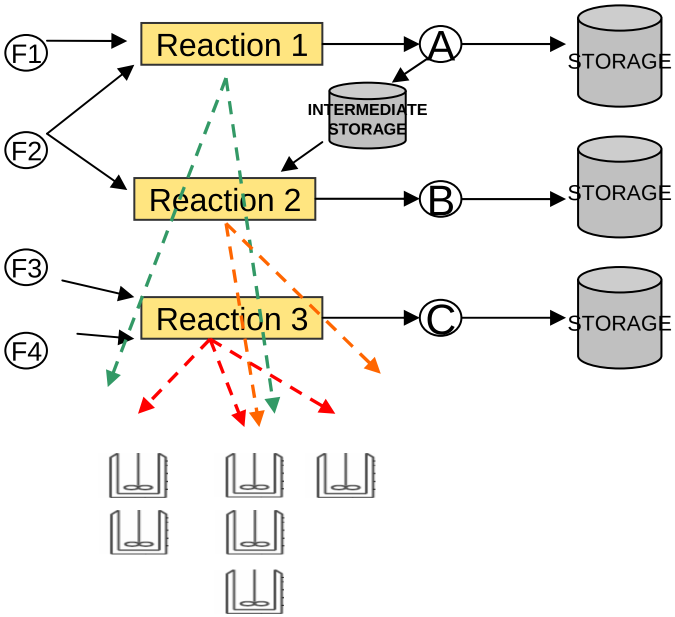
To Maximize **Profit**.

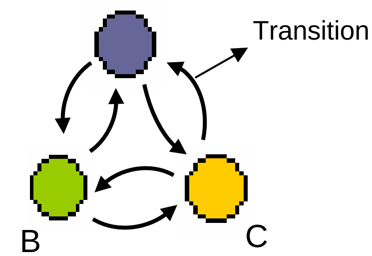
Profit = Sales – Costs

Costs=Operating Costs – Inventory Costs –Transition Costs

Challenges: *How to develop an accurate Planning Model that can anticipate effect of sequence dependent changeovers?*
How to develop an efficient scheduling model that can handle mass balances and changeovers in continuous time?

Proposed Detailed Scheduling Model (MILP)



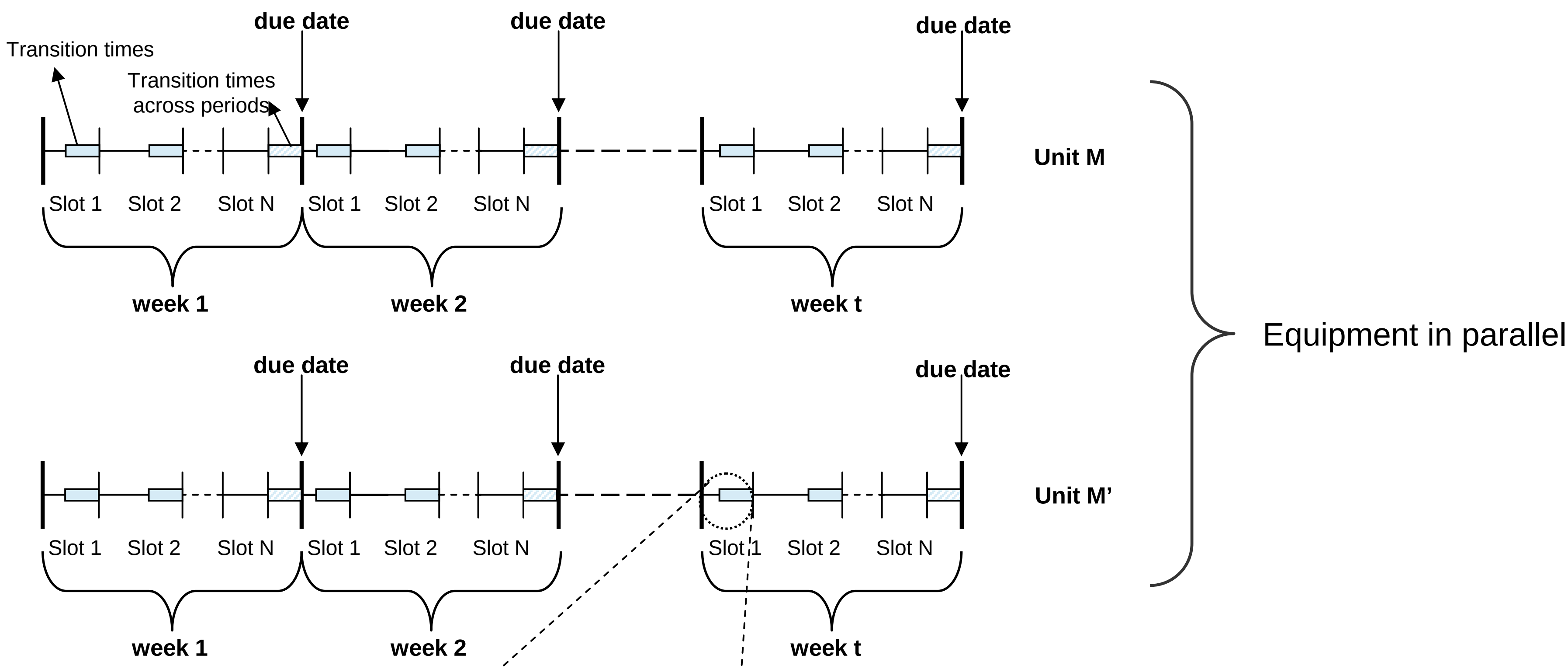


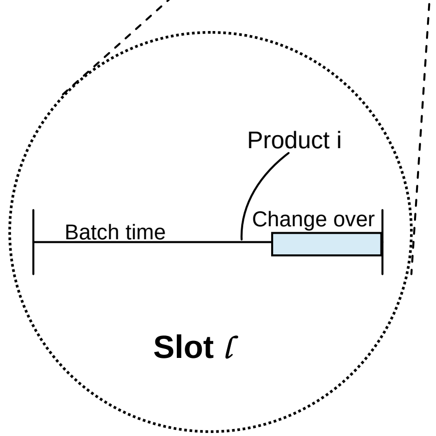
- 2 stage production
- Intermediate storage
- Mass balances

- Sequence dependent change-over times
- Sequence dependent change-over costs

- Continuous time domain representation
- Based on time slots
- Sequence dependent change-over times handled rigorously
- Incorporates mass balances and intermediate storage

Problem Formulation





Key variables for assignments:

$W_{im\ell t}$=1: Product *i* is produced in slot ℓ of unit *m* of time period *t*

binary variable

$Y_{m\ell t}$=1: slot ℓ of unit *m* of time period *t* is occupied.

binary variable

Generic Form of the Proposed MILP Detailed Scheduling Model

- ✓ Objective Function
- ✓ Assignments and Processing Times Constraints
- ✓ Detailed Timing Relations and Sequence Dependent Changeovers
- ✓ Mass Balances and Inventory Balances
  - ✓ Intermediates
  - ✓ End Products Produced in 2 Stages
  - ✓ End Products Produced in 1 Stage

Proposed MILP Detailed Scheduling Model

Objective Function:

$$Profit = \sum_i \sum_t CP_{i,t} \cdot S_{i,t} - \sum_i \sum_m \sum_l \sum_t COP_{i,t} \cdot XB_{i,m,l,t} - \sum_i \sum_t CINV_{i,t} \cdot (INV_{i,t} + INVFIN_{i,t} + INVINT_{i,t}) - \sum_i \sum_k \sum_m \sum_l \sum_t CTRA_{i,k} \cdot \left(Z_{i,k,m,l,t} + \tilde{Z}_{i,k,m,l,t} + \hat{Z}_{i,k,m,l,t} \right)$$

Assignment constraints and Processing times:

$$\sum_i W_{i,m,l,t} \le 1 \quad i \in IM(m), \forall\, l \in (L(m) \cap L(t)), \forall m, \forall t$$

$$\sum_{i \in IM(m)} W_{i,m,l,t} \ge \sum_{i \in IM(m)} W_{i,m,l,t} \quad \forall l \in (L(m) \cap L(t)), l \ne N(t), \forall m, \forall t$$

$$PT_{i,m,l,t} = BT_{i,m} \cdot W_{i,m,l,t} \quad \forall i \in IM(m), \forall l \in (L(m) \cap L(t)), \forall m, \forall t$$

$$X_{i,m,l,t} = R_{i,m} \cdot BT_{i,m} \cdot W_{i,m,l,t} \quad \forall i \in IM(m), \forall l \in (L(m) \cap L(t)), \forall m, \forall t$$

Detailed timing constraints and sequence dependent change :

$$Z_{i,i',m,l,t} \ge W_{i,m,l,t} + W_{i',m,l+1,t} - 1 \quad \forall i \in IM(m), \forall i' \in IM(m), i' \ne i, \forall l \in (L(m) \cap L(t)), l \ne Nt, \forall m, \forall t$$

$$TR_{m,l,t} = \sum_i \sum_{i'} \tau_{i,i'} \cdot Z_{i,i',m,l,t} \quad \forall l \in (L(m) \cap L(t)), l \ne Nt, \forall m, \forall t$$

$$\tilde{Z}_{i,i',m,l,t} \ge W_{i,m,l,t} + W_{i',m,'1',t+1} - 1 \quad \forall i, i' \in IM(m), i' \ne i, \forall l \in (L(m) \cap L(t)), \forall m, \forall t, t \ne H_t$$

$$Te_{m,l,t} = Ts_{m,l,t} + \sum_i PT_{i,m,l,t} + \sum_i \sum_k \tau_{i,k} \cdot Z_{i,k,m,l,t} + TRT_{m,l,t} - TX_{m,l,t} + \left(\sum_i \sum_{i'} \tau_{i,i'} \cdot \hat{Z}_{i,i',m,l,t}\right) \quad \forall m, l, t$$

$$TRT_{m,l,t} = TRT1_{m,l,t} + TRT2_{m,l,t} \quad \forall m, l, t$$

$$TX_{m,l,t} = TRT1_{m,l,t} \quad \forall m, l, t$$

$$TRT1_{m,l,t} \le UPPER \cdot Y_{m,l+1,t} \quad \forall m, l, t$$

$$TRT2_{m,l,t} \le UPPER \cdot (1 - Y_{m,l+1,t}) \quad \forall m, l, t$$

Proposed MILP Detailed Scheduling Model

Mass and Inventory Balances:

$$X_{i,m,l,t} = INVP^{FIN}_{i,m,l,t} + INVINT^{TRA}_{i,m,l,t} \qquad i \in IFINT_i, \forall l \in (L(m) \cap L(t)), \forall m, \forall t$$

$$INVINT^{TRA}_{i,m,l,t} = INVP^{INT}_{i,m,l,t} + \sum_{l'>l, l' \in L(m)} AA_{i,m,l,m,l',t} + \sum_{m' \neq m} \sum_{l' \in L(m')} AA_{i,m,l,m',l',t} \qquad i \in IFINT_i, \forall l \in (L(m) \cap L(t)), \forall l' \in L(t), \forall m, \forall m' \neq m, \forall t$$

$$X_{i,m,l,t} = INVP^{FIN}_{i,m,l,t} + INVP^{INT}_{i,m,l,t} + \sum_{l'>l, l' \in L(m)} AA_{i,m,l,m,l',t} + \sum_{m' \neq m} \sum_{l' \in L(m')} AA_{i,m,l,m',l',t} \qquad i \in IFINT_i, \forall l \in (L(m) \cap L(t)), \forall l' \in L(t), \forall m, \forall m' \neq m, \forall t$$

$$Ts_{m,l,t} + \sum_i PT_{i,m,l,t} \leq Ts_{m',l',t} + BigW_t \cdot (1 - YY_{l,l',m,m',t}) \qquad \forall l \in (L(m) \cap L(t)), \forall l' \in (L(m') \cap L(t)), \forall m, \forall m' \neq m, \forall t$$

$$Ts_{m',l',t} \leq Ts_{m,l,t} + \sum_i PT_{i,m,l,t} + BigW_t \cdot (YY_{l,l',m,m',t}) \qquad \forall l \in (L(m) \cap L(t)), \forall l' \in (L(m') \cap L(t)), \forall m, \forall m' \neq m, \forall t$$

$$AA_{i,m,l,m',l',t} \leq UBOUND_i \cdot (YY_{l,l',m,m',t}) \qquad i \in IFINT_i, \forall l \in (L(m) \cap L(t)), \forall l' \in (L(m') \cap L(t)), \forall m, \forall m' \neq m, \forall t$$

$$\sum_{m' \neq m} \sum_{l' \in (L(m') \cap L(t))} AA_{i,m,l,m',l',t} \leq UBOUND_i \cdot (W_{i,m,l,t}) \qquad i \in IFINT_i, \forall l \in (L(m) \cap L(t)), \forall m, \forall t$$

$$\sum_{l' \in (L(m) \cap L(t))} AA_{i,m,l,m',l',t} \leq UBOUND_i \cdot (W_{i,m,l,t}) \qquad i \in IFINT_i, \forall l \in (L(m) \cap L(t)), \forall m, \forall t$$

$$AA_{i,m,l,m',l',t} \leq UPBOUND_{i,m} \cdot \sum_{i' \in ENDINT(i',i)} (W_{i',m,l,m',l',t}) \qquad i \in IFINT_i, \forall l \in (L(m) \cap L(t)), \forall l' \in (L(m') \cap L(t)), \forall m, \forall m' \neq m, \forall t$$

$$INV^{INT}_{i,t-1} + \sum_m \sum_{l \in (L(m) \cap L(t))} INVP^{INT}_{i,m,l,t} = \sum_m \sum_{l \in (L(m) \cap L(t))} INVC_{i,m,l,t} + INV^{INT}_{i,t} \qquad i \in IFINT_i, \forall t$$

$$INV^{FIN}_{i,t-1} + \sum_m \sum_{l \in (L(m) \cap L(t))} INVP^{FIN}_{i,m,l,t} = S_{i,t} + INV^{FIN}_{i,t} \qquad i \in IFINT_i, \forall t$$

$$X_{i,m,l,t} = \sum_{i'} \alpha_{i,i'} \cdot (INVC_{i',m,l,t} + \sum_{\substack{l''<l \\ l' \in (L(m) \cap L(t))}} AA_{i,m,l'',m,l,t} + \sum_{m' \neq m} \sum_{l' \in (L(m') \cap L(t))} AA_{i,m',l',m,l,t}) \qquad i \in IE_i, \forall l \in (L(m) \cap L(t)), \forall m, \forall t$$

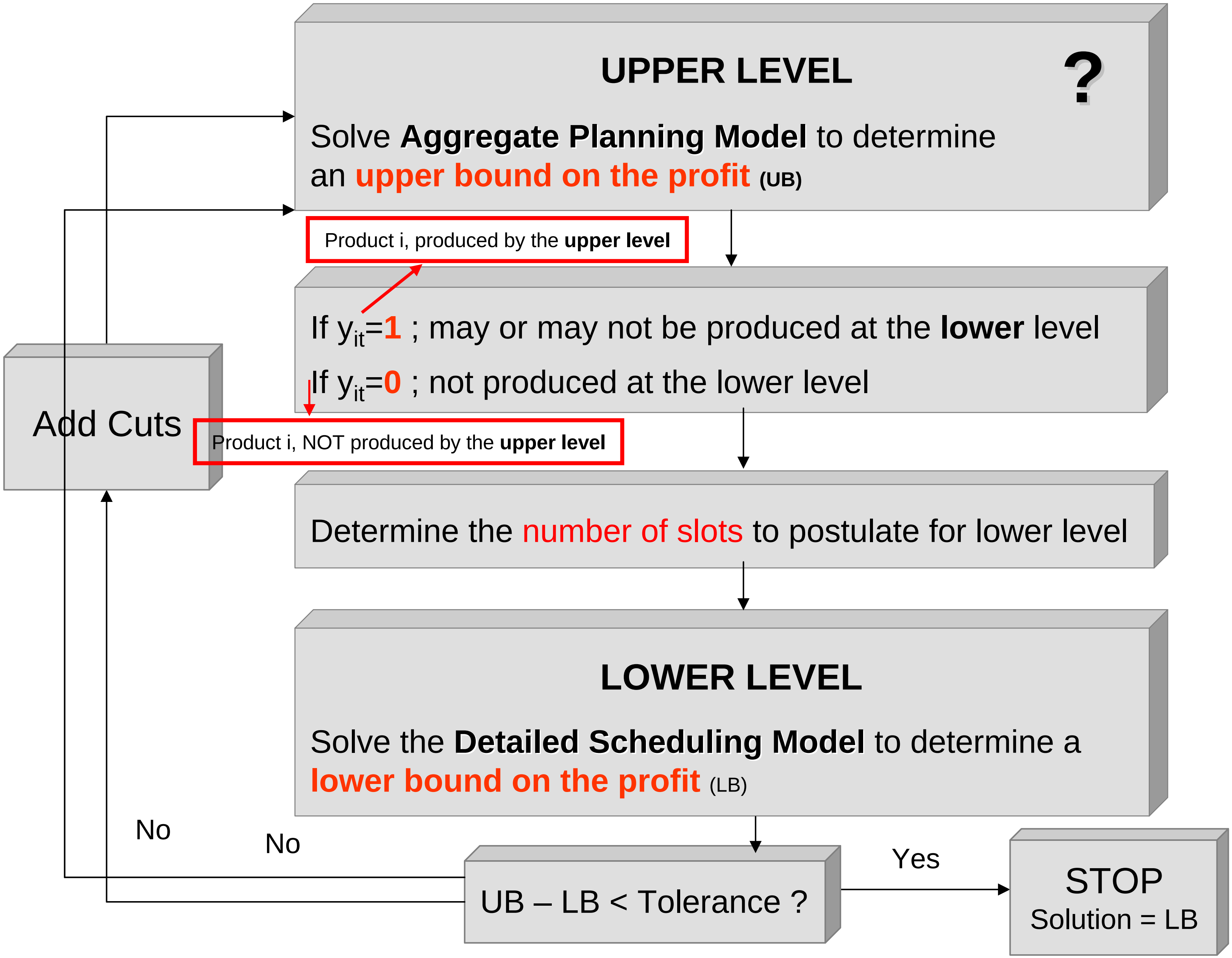
$$INV^{FIN}_{i,t-1} + \sum_m \sum_{l \in (L(m) \cap L(t))} X_{i,m,l,t} = S_{i,t} + INV^{FIN}_{i,t} \qquad i \in (IE_i \cup IF_i), \forall t$$

Remarks

- ✓ Very large scale model
- ✓ Solution times quickly intractable
  - ✓ Number of slots unknown prior to solving the model
  - ✓ Handling mass balances for products produced in 2 stage
  - ✓ Detailed timing constraints introduced for handling sequence dependent change-overs

- ✓ To overcome these difficulties, we introduce a bi-level decomposition scheme

Proposed Decomposition Algorithm

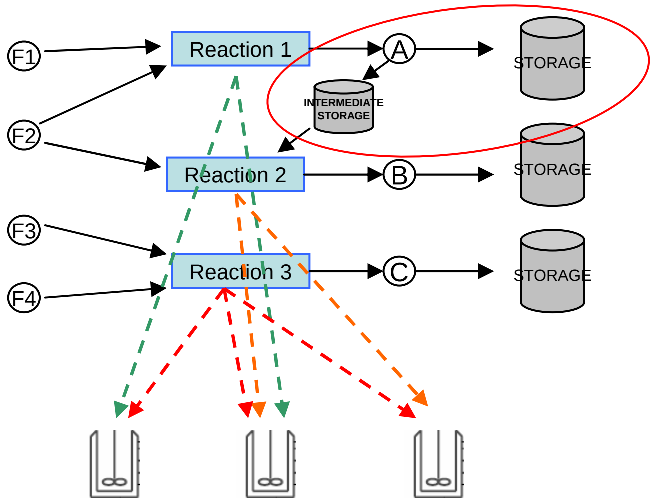


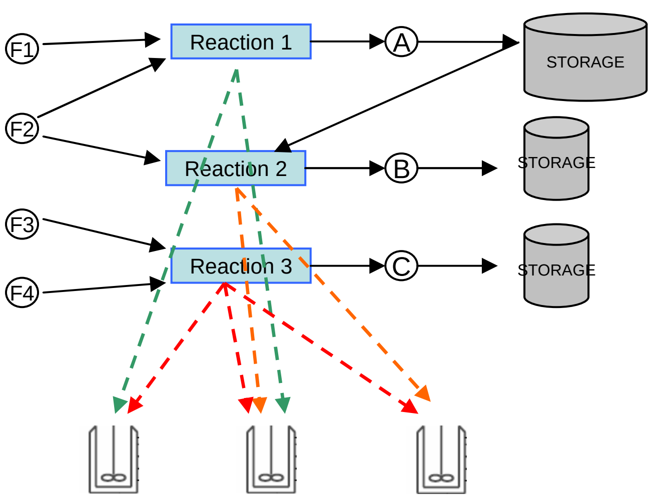
Proposed MILP Planning Model

✓ **Relaxation of the detailed scheduling model**

I.

✓Intermediate storage tank and the dedicated storage tank **aggregated** into a single tank
✓**Aggregate mass balances** for the intermediates





II.

Replace the detailed timing constraints by:

OPTION A.

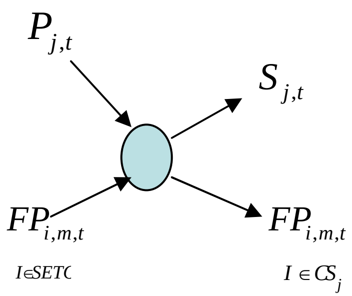
- ✓ Constraints that underestimate the sequence dependent changeover times
- ✓ **Weak** upper bounds

OPTION B.

- ✓ Sequencing constraints for accounting for transitions rigorously.
- ✓ **Tight** upper bounds

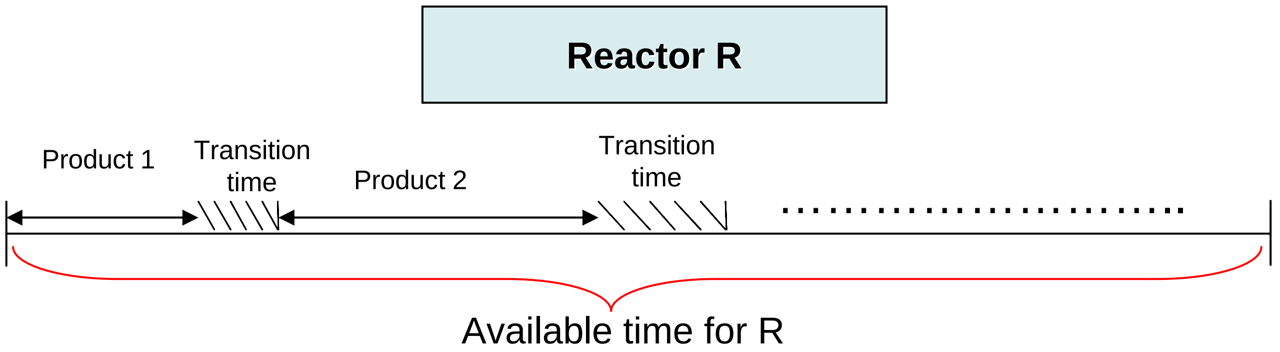
Generic Form of the Proposed MILP Planning Model

- Mass Balances on State Nodes



- Sequencing Constraints
  - *Sequence dependent changeovers determined*
  - *Detailed timings of operations neglected*

- Time Balance Constraints on Equipment



- Objective Function

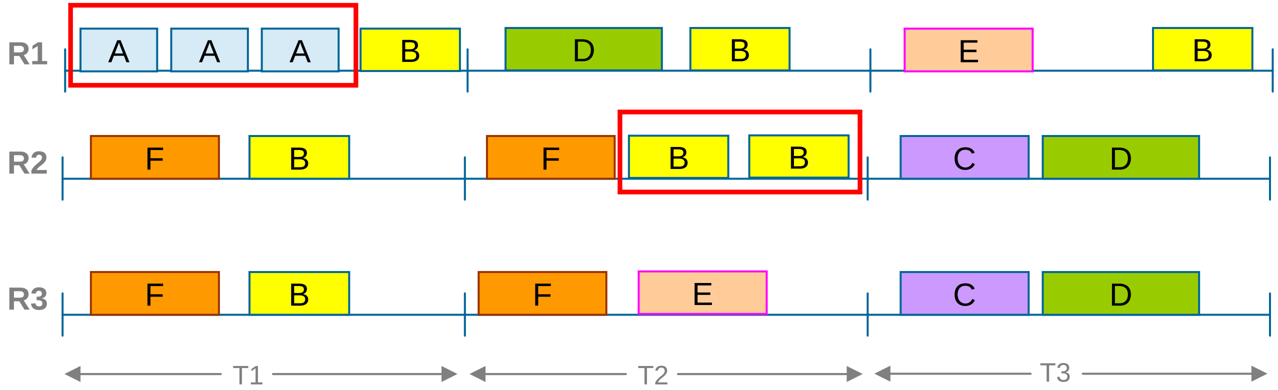
Key Variables for the Model

$YP_{i,m,t}$:the assignment of products to units at each time period

NB_{imt} :number of each batches of each product on each unit at each period

FP_{imt} :amount of material processed by each task

Products: **A,B,C,D,E,F** → Reactor 1 or Reactor 2 or Reactor 3



$$YP_{A,reactor1,time1} = 1$$
$$NB_{A,reactor1,time1} = 3$$

$$YP_{B,reactor2,time2} = 1$$
$$NB_{B,reactor2,time2} = 2$$

Proposed Planning Model

1. Mass Balance and Assignment Constraints:

$$Bound_{imt} = \underbrace{\frac{H_t}{BT_{im}}}_{\text{Largest number that the task can be repeated}} \cdot \underbrace{Q_{im}}_{\text{Maximum capacity}}$$

Maximum lbs of product i that can be produced on unit m at time period t if product i is assigned repeatedly throughout the time period

$$FP_{imt} \le Bound_{imt} \cdot YP_{imt}$$

$$FP_{imt} \ge Q_{im} \cdot YP_{imt}$$

- Defines a bound on the production levels of product i on unit m at time period t.
- Sets the production level to zero if product i is not assigned on unit m at time period t.

$$NB_{i,m,t} = FP_{i,m,t} / Q_{i,m}$$

Number of batches of product i in unit m at time t

Mass balance on each state node:

$$\underbrace{P_{jt}}_{\text{purchases}} + \underbrace{\sum_{i \in PS_j} \rho_{ji} \sum_{m \in MT_i} FP_{imt}}_{\text{production}} = \underbrace{S_{jt}}_{\text{sales}} + \underbrace{\sum_{i \in CS_j} \bar{\rho}_{ji} \sum_{m \in MT_i} FP_{imt}}_{\text{consumption}} + \underbrace{INV_{jt} - INV_{jt-1}}_{\text{change in inventory}}$$

Proposed Planning Model

2. Sequence dependent changeovers:

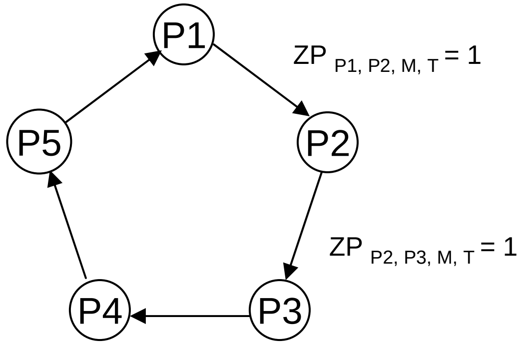
2a. Sequence dependent changeovers within each time period:

1. Generate a cyclic schedule where total transition time is minimized.

KEY VARIABLE:

$ZP_{ii'mt}$:becomes 1 if product i is after product i' on unit m at time period t, zero otherwise

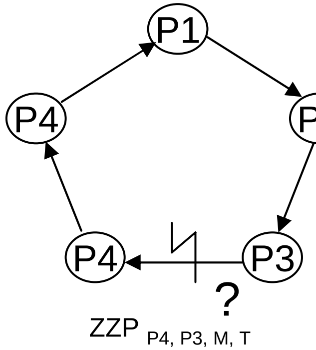
P1, P2, P3, P4, P5



2. Break the cycle at the pair with the maximum transition time to obtain the sequence.

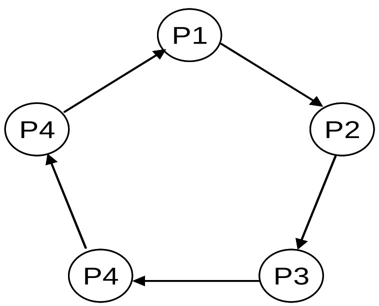
KEY VARIABLE:

$ZZP_{ii'mt}$:becomes 1 if the link between products i and i' is to be broken, zero otherwise



Proposed Planning Model

According to the location of the link to be broken:



P2, P3, P4, P5, P1 ⟶ $ZZP_{P1, P2, M, T} = 1$

P3, P4, P5, P1, P2 ⟶ $ZZP_{P2, P3, M, T} = 1$

P4, P5, P1, P2, P3 ⟶ $ZZP_{P3, P4, M, T} = 1$

P5, P1, P2, P3, P4 ⟶ $ZZP_{P4, P5, M, T} = 1$

P1, P2, P3, P4, P5 ⟶ $ZZP_{P5, P1, M, T} = 1$

The sequence with the minimum total transition time is the **optimal sequence** within time period t.

$$YP_{imt} = \sum_{i'} ZP_{ii'mt} \qquad \forall i, m, t$$

$$YP_{i'mt} = \sum_{i} ZP_{ii'mt} \qquad \forall i', m, t$$

$$\sum_{i} \sum_{i'} ZZP_{ii'mt} = 1 \qquad \forall m, t$$

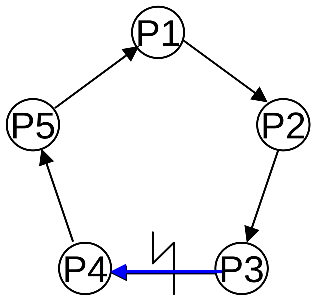
$$ZZP_{ii'mt} \le ZP_{ii'mt} \qquad \forall i, i', m, t$$

Generate the cycle and break the cycle to find the optimum sequence where transition times are minimized.

Having determining the sequence, we can determine **the total transition time** within each week.

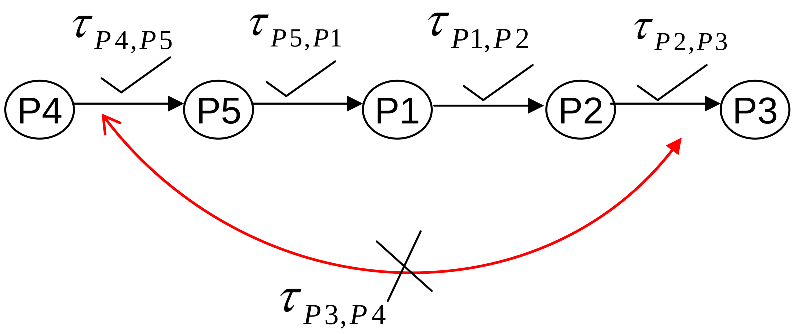
Proposed Planning Model

1) generate the cycle



$ZZP_{P4,\,P3,\,M,\,T} = 1$

2) break the cycle to obtain the sequence



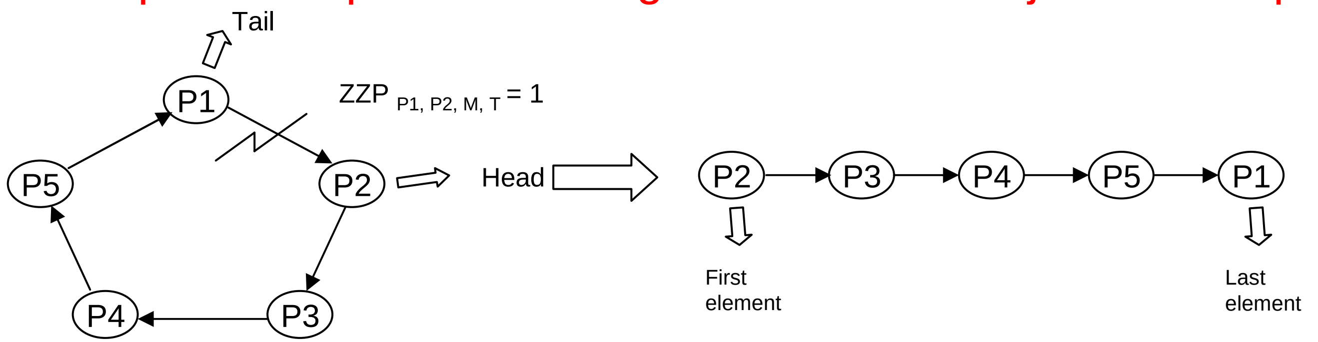
$$TRNP_{m,t} = \tau_{P4,P5} + \tau_{P5,P1} + \tau_{P1,P2} + \tau_{P2,P3} + \tau_{P3,P4} - \tau_{P3,P4}$$

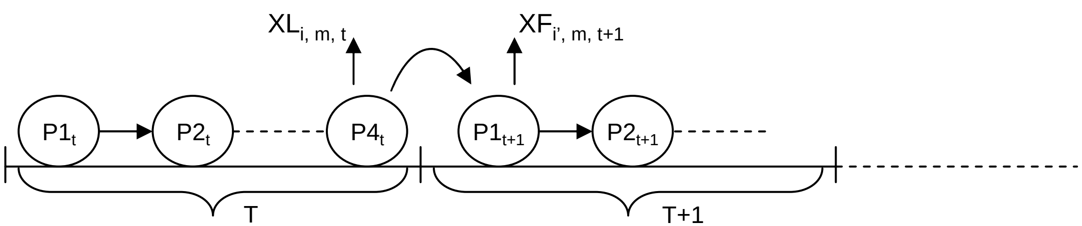
Transition time required to change the operation from P1 to P2

Total transition time within period t on unit m

$$TRNP_{m,t} = \sum_{i}\sum_{i'} \tau_{i,i'} \cdot ZP_{i,i',m,t} - \sum_{i}\sum_{i'} \tau_{i,i'} \cdot ZZP_{i,i',m,t} \qquad \forall m,t$$

2b. Sequence dependent changeovers across adjacent time periods:





$$ZZZ_{i,i',m,t} \geq XL_{i,m,t} + XF_{i',m,t+1} - 1 \quad \forall i, i', m, t$$

Transitions across adjacent weeks

Proposed Planning Model

3. The time balance constraint:

$$\sum_{i} NB_{i,m,t} \cdot BT_{i,m} + TRNP_{m,t} + \sum_{i}\sum_{i'} ZZZ_{i,i',m,t} \cdot \tau_{ii'} \leq H_t \qquad \forall m,t$$

- summation of batch times of assigned products to unit m at period t.
- total transition time within time period t
- transition time between period t and period t+1
- total available time for unit m

4. Objective Function:

Maximize PROFIT:

$$Z^p = \sum_j\sum_t cp_{jt} \cdot S_{jt} - \sum_j\sum_t c_{jt}^{inv} \cdot INV_{jt} - \sum_i\sum_m\sum_t c_{it}^{oper} \cdot FP_{imt} - \sum_i\sum_{i'}\sum_m\sum_t c_{i,i'}^{trans} \cdot ZP_{i,i',m,t} + \sum_i\sum_{i'}\sum_m\sum_t c_{i,i'}^{trans} \cdot ZZP_{i,i',m,t} - \sum_i\sum_{i'}\sum_m\sum_t c_{i,i'}^{trans} \cdot ZZZ_{i,i',m,t}$$

- Sales
- Inventory costs
- Variable operating costs
- Transition costs

MILP Aggregate Planning Model

$$Z^p = \sum_j \sum_t cp_{jt} \cdot S_{jt} - \sum_j \sum_t c_{jt}^{inv} \cdot INV_{jt} - \sum_i \sum_m \sum_t c_{it}^{oper} \cdot FP_{imt} - \sum_i \sum_{i'} \sum_m \sum_t c_{i,i'}^{trans} \cdot ZP_{i,i',m,t} + \sum_i \sum_{i'} \sum_m \sum_t c_{i,i'}^{trans} \cdot ZZP_{i,i',m,t} - \sum_i \sum_{i'} \sum_m \sum_t c_{i,i'}^{trans} \cdot ZZZ_{i,i',m,t}$$

$$Bound_{imt} = \left(H_t / BT_{im} \right) \cdot Q_{im} \quad \forall i,m,t$$

$$FP_{imt} \le Bound_{imt} \cdot YP_{imt} \quad \forall i,m,t$$

$$FP_{imt} \ge Q_{im} \cdot YP_{imt} \quad \forall i,m,t$$

$$NB_{i,m,t} = FP_{i,m,t} / Q_{i,m} \quad \forall i,m,t$$

$$S_{i,t} \le D_{i,t} \quad \forall i,t$$

$$P_{jt} + \sum_{i \in PS_j} \rho_{ji} \sum_{m \in MT_i} FP_{imt} = S_{jt} + \sum_{i \in CS_j} \bar{\rho}_{ji} \sum_{m \in MT_i} FP_{imt} + INV_{jt} - INV_{jt-1} \quad \forall j,t$$

Assignment of products to equipment and mass balances

$$YP_{imt} = \sum_{i'} ZP_{ii'mt} \quad \forall i,m,t$$

$$YP_{i'mt} = \sum_{i} ZP_{ii'mt} \quad \forall i',m,t$$

$$\sum_i \sum_{i'} ZZP_{ii'mt} = 1 \quad \forall m,t$$

$$ZZP_{ii'mt} \le ZP_{ii'mt} \quad \forall i,i',m,t$$

Generate the cycle and break the cycle to find the optimum sequence where transition times are minimized.

$$XF_{i',m,t} \ge ZZP_{i,i',m,t} \quad \forall i,i',m,t$$

$$XL_{i,m,t} \ge ZZP_{i,i',m,t} \quad \forall i,i',m,t$$

$$ZZP_{i,i',m,t} \ge XF_{i',m,t} + XL_{i,m,t} - 1 \quad \forall i,i',m,t$$

$$\sum_i XF_{imt} = 1 \quad \forall m,t$$

$$\sum_i XL_{imt} = 1 \quad \forall m,t$$

$$ZZZ_{i,i',m,t} \ge XL_{i,m,t} + XF_{i',m,t+1} - 1 \quad \forall i,i',m,t$$

Determine the first and last elements of each sequence in order to account for transitions across adjacent periods

$$TRNP_{m,t} = \sum_i \sum_{i'} \tau_{i,i'} \cdot ZP_{i,i',m,t} - \sum_i \sum_{i'} \tau_{i,i'} \cdot ZZP_{i,i',m,t} \quad \forall m,t$$

$$\sum NB_{i,m,t} \cdot BT_{i,m} + TRNP_{m,t} + \sum_i \sum_{i'} ZZZ_{i,i',m,t} \cdot \tau_{ii'} \le H_t \quad \forall m,t$$

Sequence dependent changeover times and the time balances

Integer Cuts

Integer Cuts

$$\sum_{(i,m,t)\in Z_1} YP_{i,m,t} - \sum_{(i,m,t)\in Z_0} YP_{i,m,t} \le |Z_1| - 1 \quad Z_1 = \{ i,m,t | YP_{i,m,t} = 1 \} \quad Z_0 = \{ i,m,t | YP_{i,m,t} = 0 \}$$

*Ref. Jeroslow and Balas (1972)

Integer cuts exclude previously obtained feasible configurations.

Solution of the aggregated scheduling model (**upper** level):

	Time Periods			
Products	1	2	3	4
A	**1**	**1**	**0**	**1**
B	**1**	**0**	**1**	**1**
C	**1**	**1**	**1**	**1**

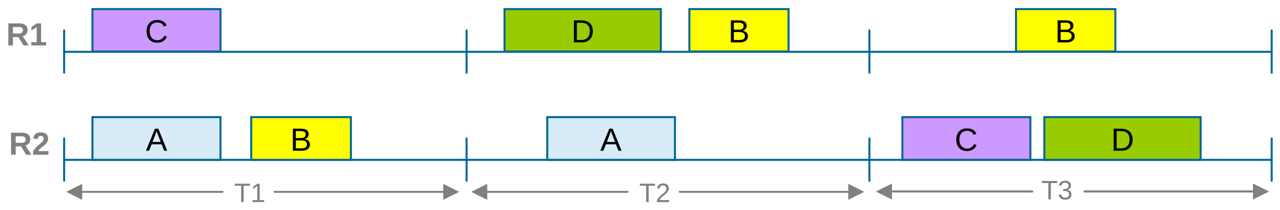
Product A is produced in the 4th period.

Product B is NOT produced in the 2nd period.

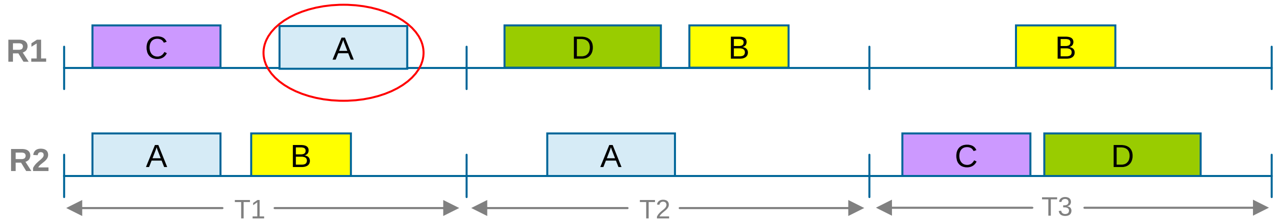
Cover Cuts

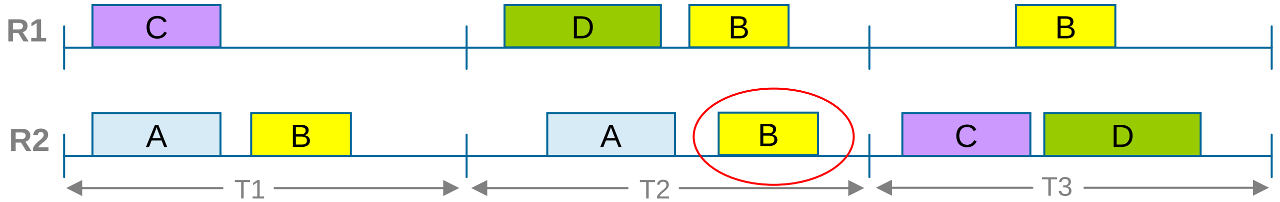
- ✓**Stronger than the integer cuts**
- ✓**Exclude the supersets of the previously obtained configurations**

$$\sum_{(i,m,t)\in Z^r} YP_{i,m,t} \le \left|Z^r\right| - 1 \qquad Z^r = \left\{ i,m,t \mid YP^r_{i,m,t} = 1 \right\}$$

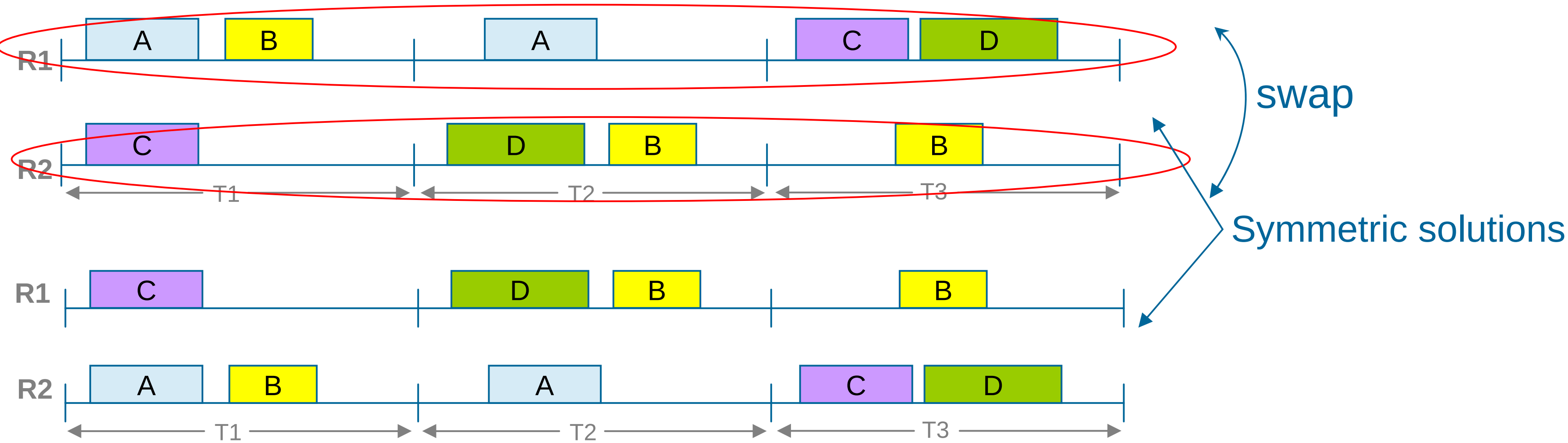


Supersets:





Symmetry Breaking Cuts



✓ *Result in increase in the number of iterations !*

Symmetric configurations are excluded:

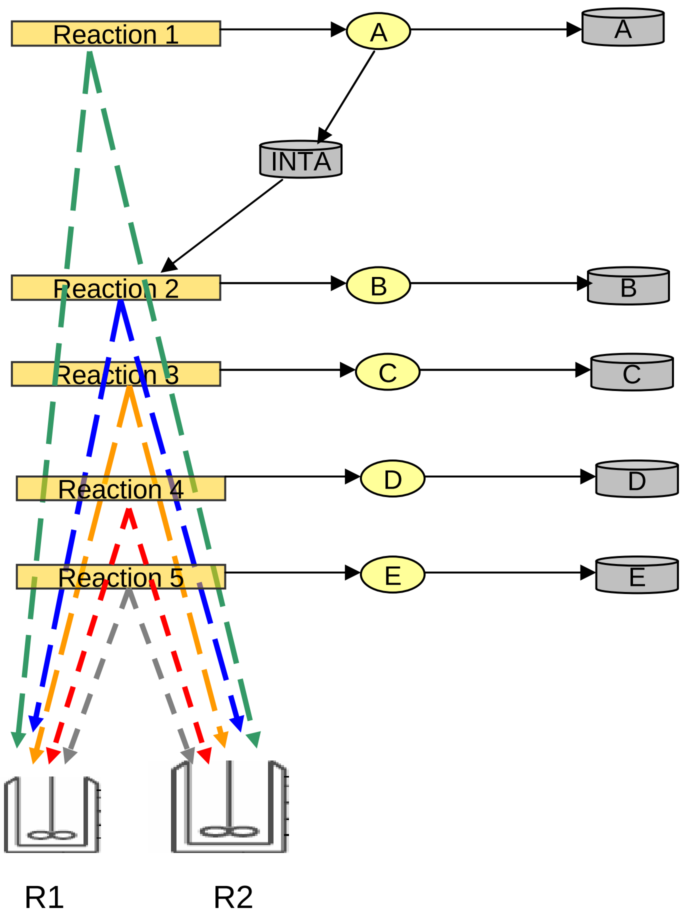
$$\sum_{(i,t)\in Z_r^2} YP_{i,m,t} + \sum_{(i,t)\in Z_r^1} YP_{i,m',t} \le \left|Z_r^1\right| + \left|Z_r^2\right| - 1 \quad \forall m, m', m \ge m'$$

$$Z_r^1 = \left\{ i, m, t \mid YP^r_{i,m,t} = 1 \right\}$$

$$Z_r^2 = \left\{ i, m', t \mid YP^r_{i,m',t} = 1 \right\}$$

EXAMPLE 1- 5 Products, 2 Reactors, 1 Week

- Determine the **schedule** for **5** products, **2** reactors plant for **1 week** so as to maximize **profit.**



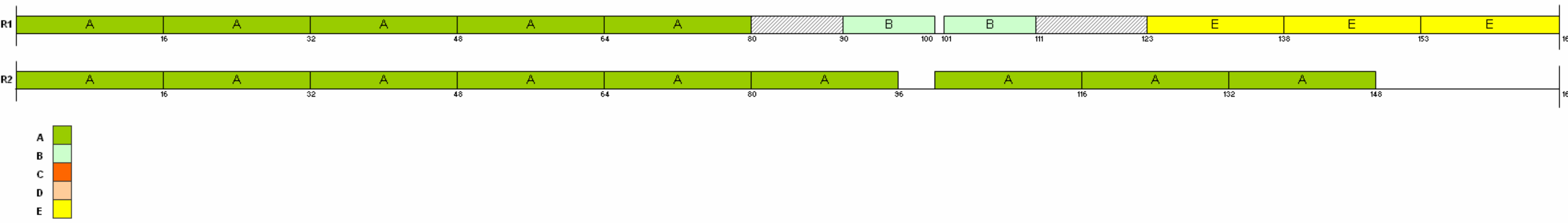
- 5 Products, A,B,C,D,E
- To produce 1 lb of “B”, 0.2lb of “A” is required.
- 2 Reactors, R1,R2
- End time of the week is defined as due dates
- Demands are upper bounds

EXAMPLE 1- 5 Products, 2 Reactors, 1 Week

Method	Number of binary variables	Number of continuous variables	Number of Equations	Time (CPUs)	Solution ($)
Full Space	500	2,615	2,185	**60.0**	**1,055,127.0**
Proposed algorithm				**1.2**	**1,055,127.0**
Problem UB	140	207	335	0.6	1,055,127.0
Problem LB	500	2,615	2,185	0.6	1,055,127.0

- ✓ *% **0.00** Gap between **Upper** Level and **Lower** Level*
- ✓ *Convergence obtained in a **single** iteration*

Gantt Chart:



EXAMPLE 1A- Higher Transition Times and Demands

✓ Same as Example 1, transition times and demand data has been changed.

Method	Number of binary variables	Number of continuous variables	Number of Equations	Time (CPUs)	Solution ($)
Full Space	500	2,615	2,185	**2,778.2**	**920,205.6**
Proposed algorithm				**143.5**	**920,205.6**
Problem UB	140	207	350	5.9	**926,090.0**
Problem LB	500	2,615	2,185	137.6	**920,205.6**

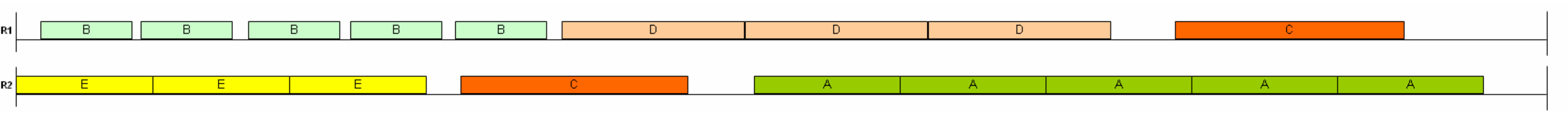
Solutions at each iterations

Iteration	Upper Bound (UB)	Lower Bound (LB)
1	974,290.0	919,205.2
2	946,220.0	920,170.0
3	946,200.0	920,205.6
4	944,340.0	918,320.0
5	935,210.0	909,190.0
6	926,090.0	915,100.0

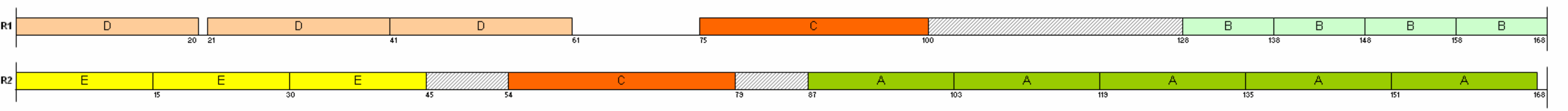
Transition times (hrs)

	A	B	C	D	E
A	0	28	16	20	12
B	30	0	15	18	16
C	8	28	0	20	10
D	13	36	14	0	25
E	22	25	9	5	0

Gantt Chart for solution obtained from the upper level (aggregate scheduling model) at the last iteration :

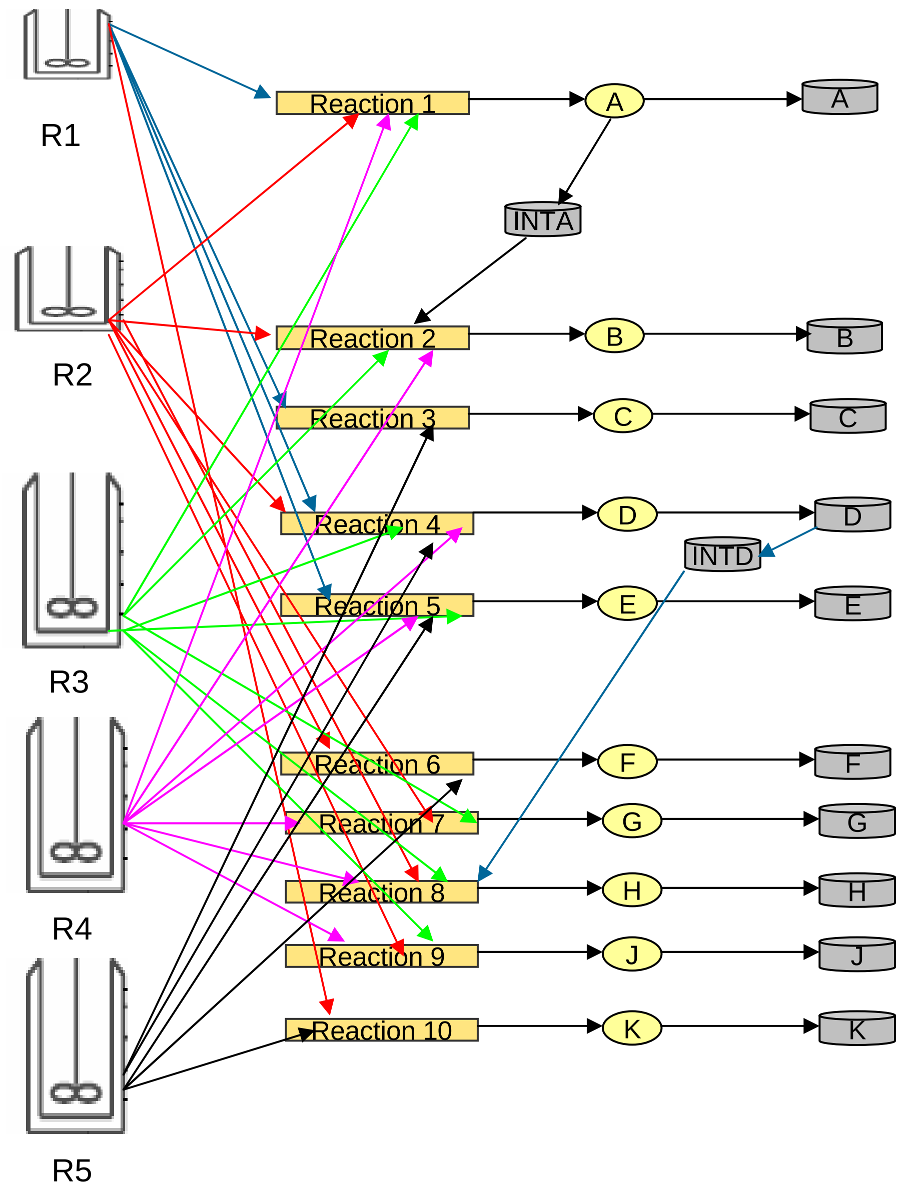


Gantt Chart for solution obtained from the lower level at the last iteration :



EXAMPLE 2 – 10 Products, 5 Reactors, 1 Week

- Determine the **schedule** for **10** products, **5** reactors plant for **1 week** so as to maximize **profit.**

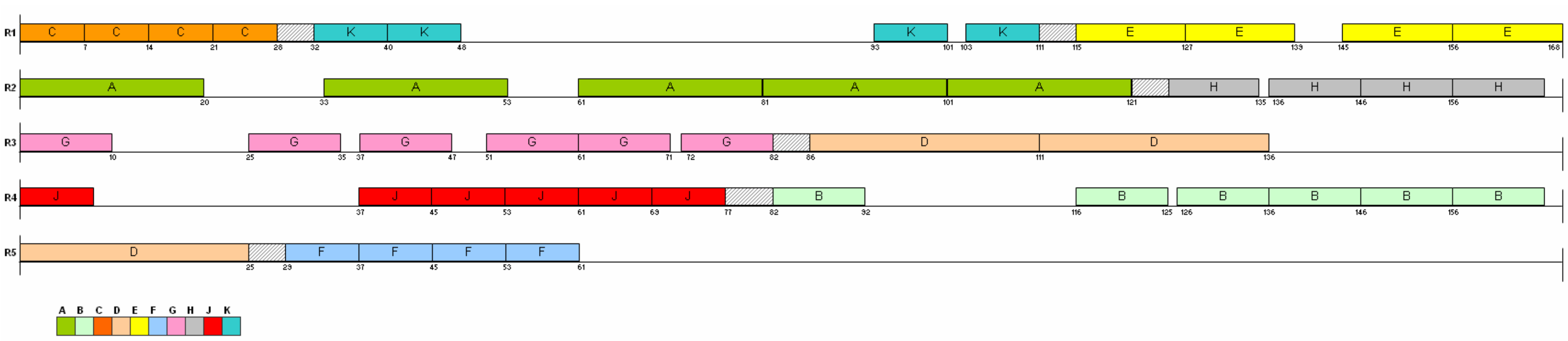


EXAMPLE 2 – 10 Products, 5 Reactors, 1 Week

Method	Number of binary variables	Number of continuous variables	Number of Equations	Time (CPUs)	Solution ($)
Full Space	3,984	18,716	22,903	**85,000.0**	**2,300,824.0**
Proposed algorithm				**428.3**	**2,300,824.0**
Problem UB	480	542	648	217.8	**2,300,824.0**
Problem LB	3,984	18,716	22,903	210.5	**2,300,824.0**

- ✓ *% **0.00** Gap between **Upper** Level and **Lower** Level*
- ✓ *Convergence obtained in a **single** iteration*

Gantt Chart:



EXAMPLE 3- 6 Products, 4 Reactors, 12 Weeks

Method	Number of binary variables	Number of continuous variables	Number of Equations	Time (CPUs)	Solution ($)
Planning	2352	3517	5853	945	17,582,989.51
Scheduling	13,152	54,011	72,570	6136.57	17,543,092.82

	Planning	Scheduling
Sales	26511800	26408630
Operating Costs	8918720	8838270
Inventory Costs	6948.335	18247
Transition Costs	7950	9020

Note: Planning model has been integrated with scheduling model by providing information on number of slots

EXAMPLE 4 – 5 Products, 2 Reactors, 24 Weeks

Method	Number of binary variables	Number of continuous variables	Number of Equations	Time (CPUs)	Solution ($)
Planning	3360	5083	8385	636	13,559,343.04
Scheduling	5,026	33,371	30,053	521	13,444,109.84

	Planning	Scheduling
Sales	21,871,700	21,713,620
Operating Costs	8,290,760	8,234,760
Inventory Costs	7286	16,140.16
Transition Costs	14,310	18,610

Note: Planning model has been integrated with scheduling model providing information on number of slots