Global Optimization for Scheduling Refinery Crude Oil Operations

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Enterprise-wide Optimization Project

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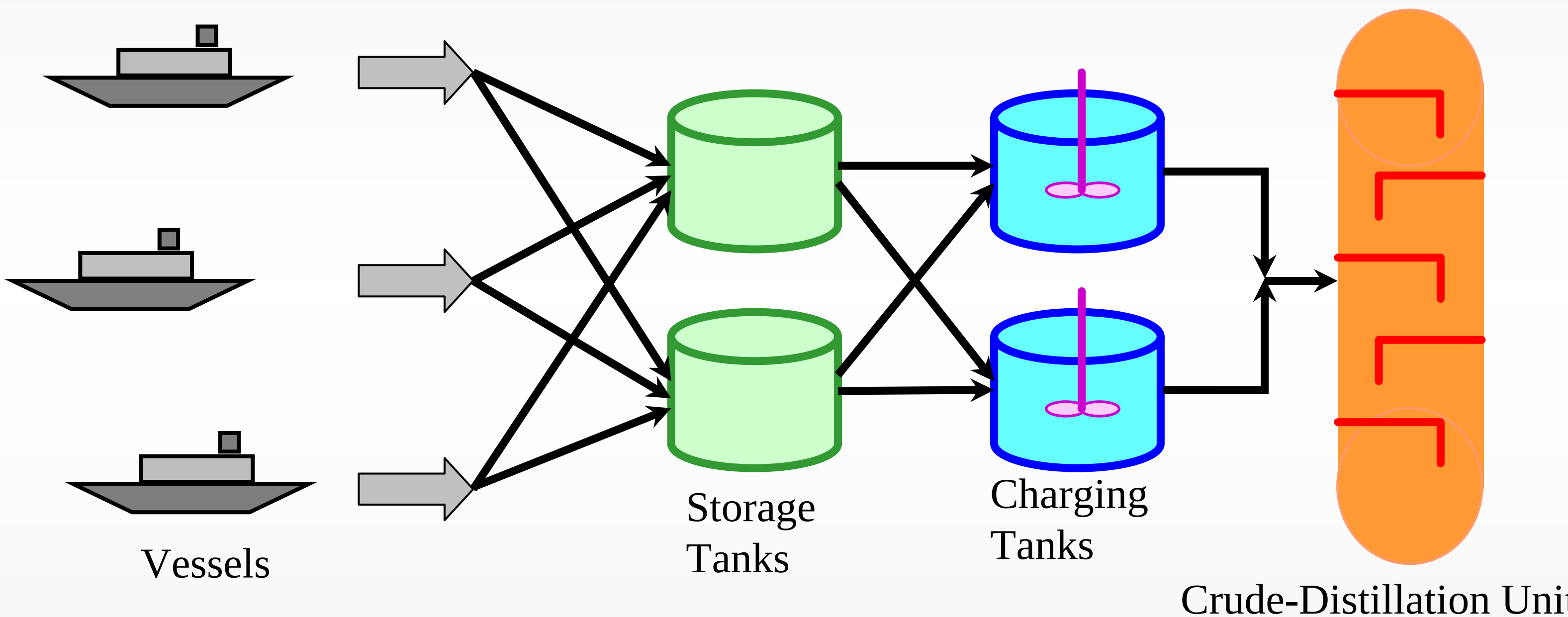
Motivation

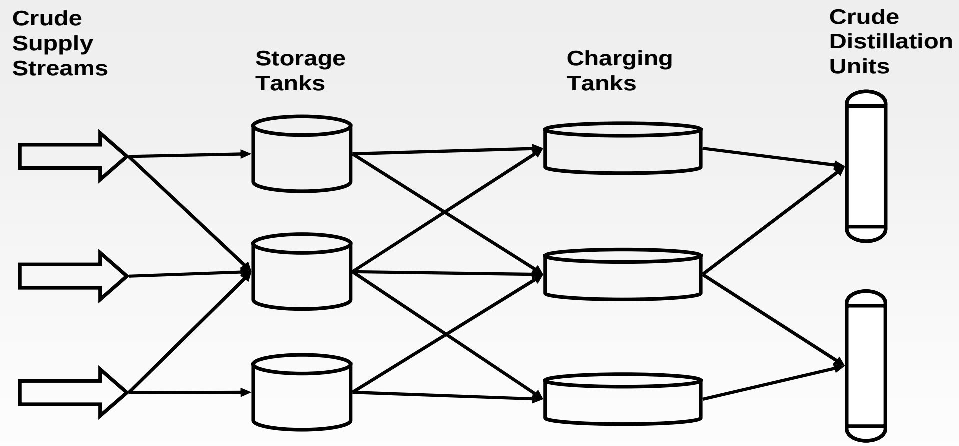
- Scheduling and Planning of crude oil is key problem in petrochemical refineries
- Large cost savings can be realized with an optimum schedule for the movement of crude oil



How to coordinate discharge of vessels with loading to storage?
How to synchronize charging tanks with crude-oil distillation?

⇒ **Economic refinery operation**

Problem Statement



Given:

(a) Maximum and minimum inventory levels for a tank
(b) Initial total and component inventories in a tank
(c) Upper and lower bounds on the fraction of key components in the crude inside a tank
(d) Times of arrival of crude oil in the supply streams
(e) Amount of crude arriving in the supply streams
(f) Fractions of various components in the supply streams
(g) Bounds on the flowrates of the streams in the network
(h) Time horizon for scheduling

Determine:

(i) **Inventory levels** in the tanks at various points of time
(ii) **Flow volumes** from one unit to another in a certain time interval
(iii) **Start and end times** of the flows in the network

Objective: Minimize Cost

Previous Work

Lee, H.; Pinto, J. M.; Grossmann, I. E.; Park, S. (1996). Mixed Integer Linear Programming Model for Refinery Short-Term Scheduling of Crude Oil Unloading with Inventory Management. *I&EC Res.*, 35, 1630 -1641.

Jia, Z.; Ierapetritou, M. G. (2003). Refinery Short-Term Scheduling Using Continuous Time Formulation: Crude Oil Operations. *I&EC Res.*, 42, 3085 -3097.

Furman, K. C.; Jia, Z.; Ierapetritou, M. G. (2005). A Robust Event-Based Continuous Time Formulation for Tank Transfer Scheduling. *Work in Progress.*

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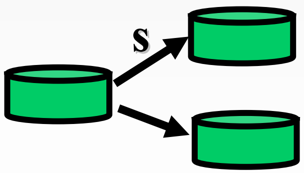
Scheduling Model

- Continuous time formulation by Furman et al. (2006)
- Based on time events where inputs and outputs for a unit can take place in the same time event
  - Assumption: No simultaneous input and output for a tank
  - Transfers from one tank to another are denoted by **streams**



- Formulation reduces number of binary variables required in the scheduling model

Optimization model

Minimize *cost objective*

s.t. Tank constraints

Distillation unit (CDU) constraints

Supply stream constraints

Variable bounds (P)

Binary Variables ⇒

Variables in the model

$I_{b,t}^{tot}$ – Total inventory in tank *b* at end of time event *t*

$I_{b,t}^{j}$ – Inventory of component *j* in tank *b* at end of time event *t*

$V_{s,t}^{tot}$ – Total flow in stream *s* in time event *t*

$V_{s,t}^{j}$ – Flow of component *j* in stream *s* in time event *t*

$T_{s,t}^{1}$ – Start time of flow in stream *s* in time event *t*

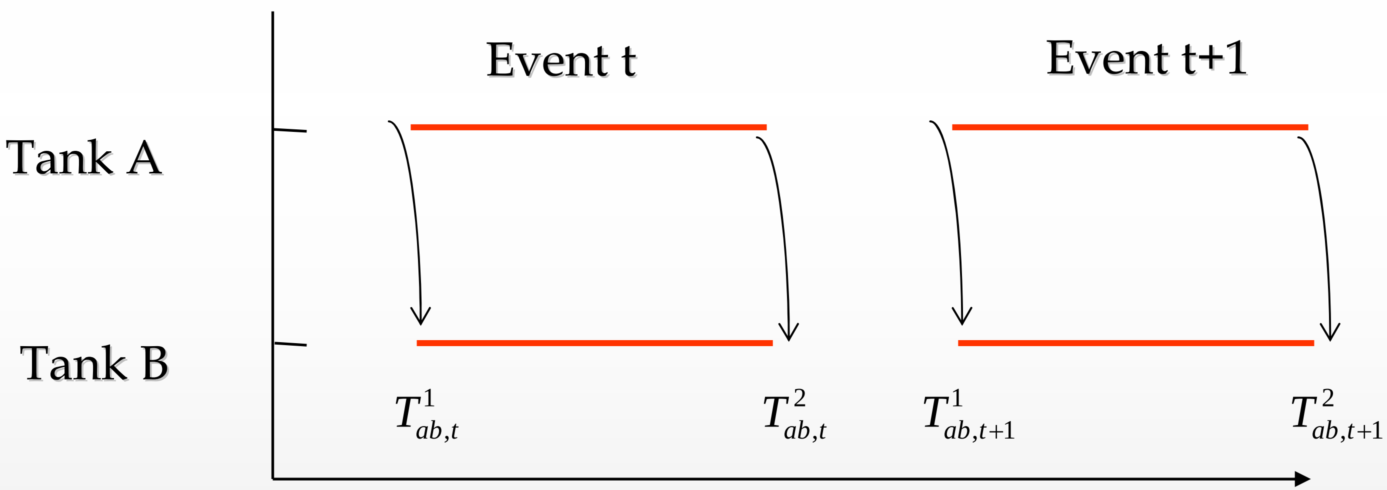
$T_{s,t}^{2}$ – End time of flow in stream *s* in time event *t*

$w_{s,t}$ – Existence of flow in stream *s* in time event *t*

Time representation

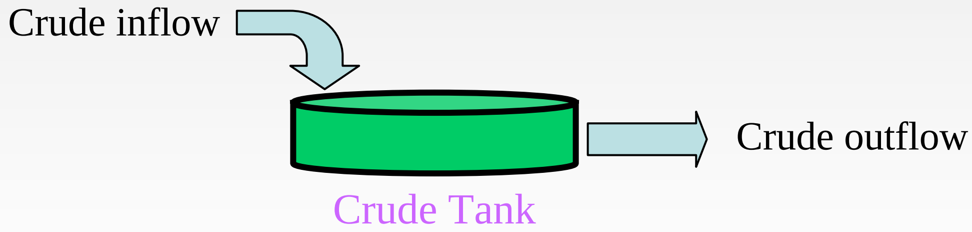
- Instead of using global times t, events t are used

- Instead of timing of individual operations, timing of transfers is used



Model Constraints

Tank constraints



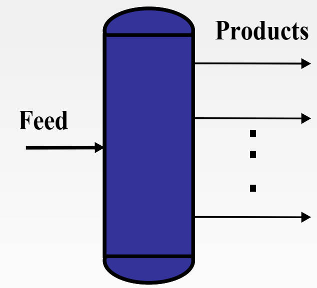
- **Total Inventory balances**
- **Individual component balances**

Non-linear equations containing Bilinearities

- **Duration constraints**
  - To bound the flow of a stream into/from a tank in a particular time event
- **Simple sequencing constraints**
- **Bounds on component fractions inside a tank**

Model Constraints (Contd …)

Distillation unit constraints



- **Continuous operation constraint**
- **Allocation constraints**
  - At most one CDU can be charged by a charging tank at a time
  - At most one charging tank can charge a CDU at any point of time
- **Crude-mix demand constraints**

Crude supply stream constraints

- **Overall mass balances**
- **Component mass balances**
- **Start and end timing constraints**

Non-convex MINLP

Objective function:

Minimize a cost objective similar to the one by Jia and Ierapetritou (2003)

min *total cost* = *waiting cost for supply streams*
+ *unloading cost of supply streams*
+ *inventory cost for each tank over scheduling horizon*
+ *setup cost for charging CDUs with different charging tanks*

- Scheduling problem modeled as a Mixed Integer Nonlinear Program (MINLP)
  - Discrete variables used to determine which flows should exist and when
  - Model is non-linear and non-convex

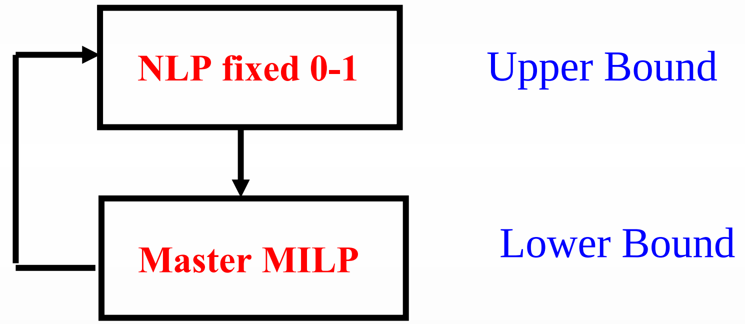
Overall model ≡ (P) ⇒ Non-convex MINLP

Convex relaxation of (P) (obtained by linearizing non-linear equations in *Tank constraints* and introducing McCormick estimators (1976) for bilinear terms) ≡ (R) ⇒ MILP

Global Optimization of MINLP

- Large-scale non-convex MINLPs such as (P) are very difficult to solve
  - Commercial global optimization solvers *fail to converge to solution* in tractable computational times

- Special Outer-Approximation algorithm proposed to solve problem to global optimality

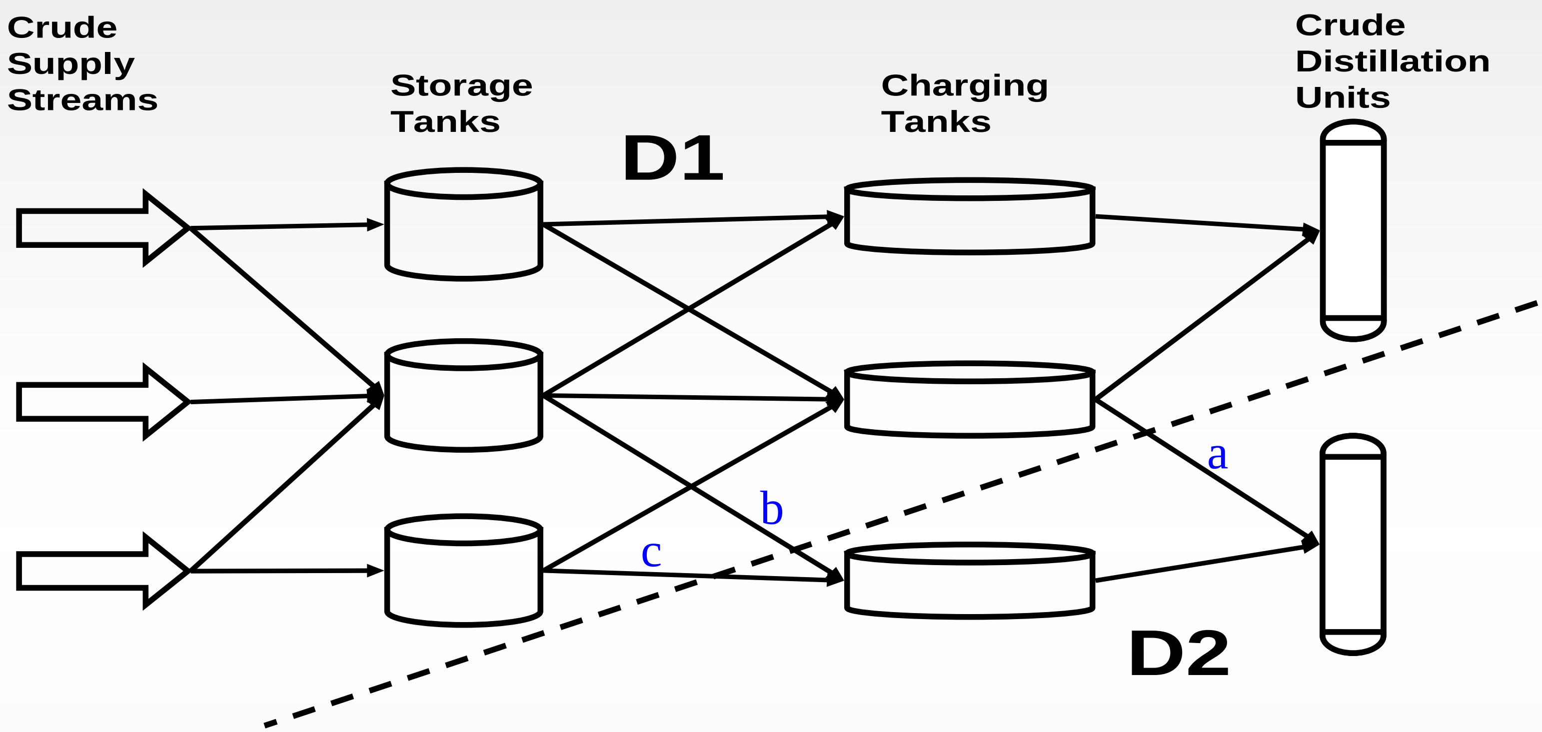


  - ***Guaranteed to converge*** to global optimum given certain tolerance between lower and upper bounds

Upper Bound : Feasible solution of (P)

Lower Bound : Obtained by solving a MILP relaxation (R) of the non-convex MINLP model with Lagrangean Decomposition based cuts added to it

Spatial Decomposition of the Network



- Network is split into two decoupled sub-structures D1 and D2
  - Physically interpreted as cutting some pipelines (Here ***a***, ***b*** **and** ***c***)
  - Set of split streams denoted by $p \in \{a, b, c\}$

Decomposition of the model

- Create two copies of the variables pertaining to the split streams $\{V^{tot}_{p,t}, V^{j}_{p,t}, T^{1}_{p,t}, T^{2}_{p,t}, w_{p,t}\}$ and get two sets of *duplicate variables* :

$$\{V^{tot,1}_{p,t}, V^{j,1}_{p,t}, T^{1,1}_{p,t}, T^{2,1}_{p,t}, w^{1}_{p,t}\}$$

and

$$\{V^{tot,2}_{p,t}, V^{j,2}_{p,t}, T^{1,2}_{p,t}, T^{2,2}_{p,t}, w^{2}_{p,t}\}$$

  - The equations involving the split streams are re-written in terms of the newly created variables
  - These ***duplicate variables*** are related by equality constraints which are added to (R) to get model (RP):

$$V^{tot,1}_{p,t} - V^{tot,2}_{p,t} = 0 \qquad \forall p,t$$

$$V^{j,1}_{p,t} - V^{j,2}_{p,t} = 0 \qquad \forall j,p,t$$

$$T^{1,1}_{p,t} - T^{1,2}_{p,t} = 0 \qquad \forall p,t$$

$$T^{2,1}_{p,t} - T^{2,2}_{p,t} = 0 \qquad \forall p,t$$

$$w^{1}_{p,t} - w^{2}_{p,t} = 0 \qquad \forall p,t$$

Non-anticipativity constraints

- Non-anticipativity constraints in (RP) are multiplied by Lagrange multipliers and transferred to objective function to bring model to a decomposable form which is decomposed into sub-models (LD1) and (LD2)

Decomposed Sub-models

Sub-problem involves duplicate variables

$\{V^{tot,1}_{p,t}, V^{j,1}_{p,t}, T^{1,1}_{p,t}, T^{2,1}_{p,t}, w^1_{p,t}\}$

min z_1 = *waiting cost for supply streams + unloading cost of supply streams + inventory cost for tanks in D1 over scheduling horizon + setup costs for charging CDUs in D1 with different charging tanks +*

$$\sum_p\sum_t \lambda^{Vtot}_{p,t} V^{tot,1}_{p,t} + \sum_j\sum_p\sum_t \lambda^{V}_{j,p,t} V^{j,1}_{p,t} + \sum_p\sum_t \lambda^{T1}_{p,t} T^{1,1}_{p,t} + \sum_p\sum_t \lambda^{T2}_{p,t} T^{2,1}_{p,t} + \sum_p\sum_t \lambda^{w}_{p,t} w^{1}_{p,t}$$

s.t. Tank constraints
Distillation unit constraints
Supply stream constraints
Variable bounds

(LD1)

Optimize to get solution z_1^*

Sub-problem involves duplicate variables

$\{V^{tot,2}_{p,t}, V^{j,2}_{p,t}, T^{1,2}_{p,t}, T^{2,2}_{p,t}, w^2_{p,t}\}$

min z_2 = *inventory cost for tanks in D2 over scheduling horizon + setup costs for charging CDUs in D2 with different charging tanks +*

$$-\sum_p\sum_t \lambda^{Vtot}_{p,t} V^{tot,2}_{p,t} - \sum_j\sum_p\sum_t \lambda^{V}_{j,p,t} V^{j,2}_{p,t} - \sum_p\sum_t \lambda^{T1}_{p,t} T^{1,2}_{p,t} - \sum_p\sum_t \lambda^{T2}_{p,t} T^{2,2}_{p,t} - \sum_p\sum_t \lambda^{w}_{p,t} w^{2}_{p,t}$$

s.t. Tank constraints
Distillation unit constraints
Variable bounds

(LD2)

Optimize to get solution z_2^*

Cut Generation

- Using solutions z_1^* and z_2^* we develop the following cuts :

$z_1^* \leq$ *waiting cost for supply streams + unloading cost of supply streams + inventory cost for tanks in D1 over scheduling horizon + setup costs for charging CDUs in D1 with different charging tanks +*

$$\sum_p \sum_t \lambda_{p,t}^{Vtot} V_{p,t}^{tot} + \sum_j \sum_p \sum_t \lambda_{j,p,t}^{V} V_{p,t}^{j} + \sum_p \sum_t \lambda_{p,t}^{T1} T_{p,t}^{1} + \sum_p \sum_t \lambda_{p,t}^{T2} T_{p,t}^{2} + \sum_p \sum_t \lambda_{p,t}^{w} w_{p,t}$$

Lagrange Multipliers

$z_2^* \leq$ *inventory cost for tanks in D2 over scheduling horizon + setup costs for charging CDUs in D2 with different charging tanks +*

$$-\sum_p \sum_t \lambda_{p,t}^{Vtot} V_{p,t}^{tot} - \sum_j \sum_p \sum_t \lambda_{j,p,t}^{V} V_{p,t}^{j} - \sum_p \sum_t \lambda_{p,t}^{T1} T_{p,t}^{1} - \sum_p \sum_t \lambda_{p,t}^{T2} T_{p,t}^{2} - \sum_p \sum_t \lambda_{p,t}^{w} w_{p,t}$$

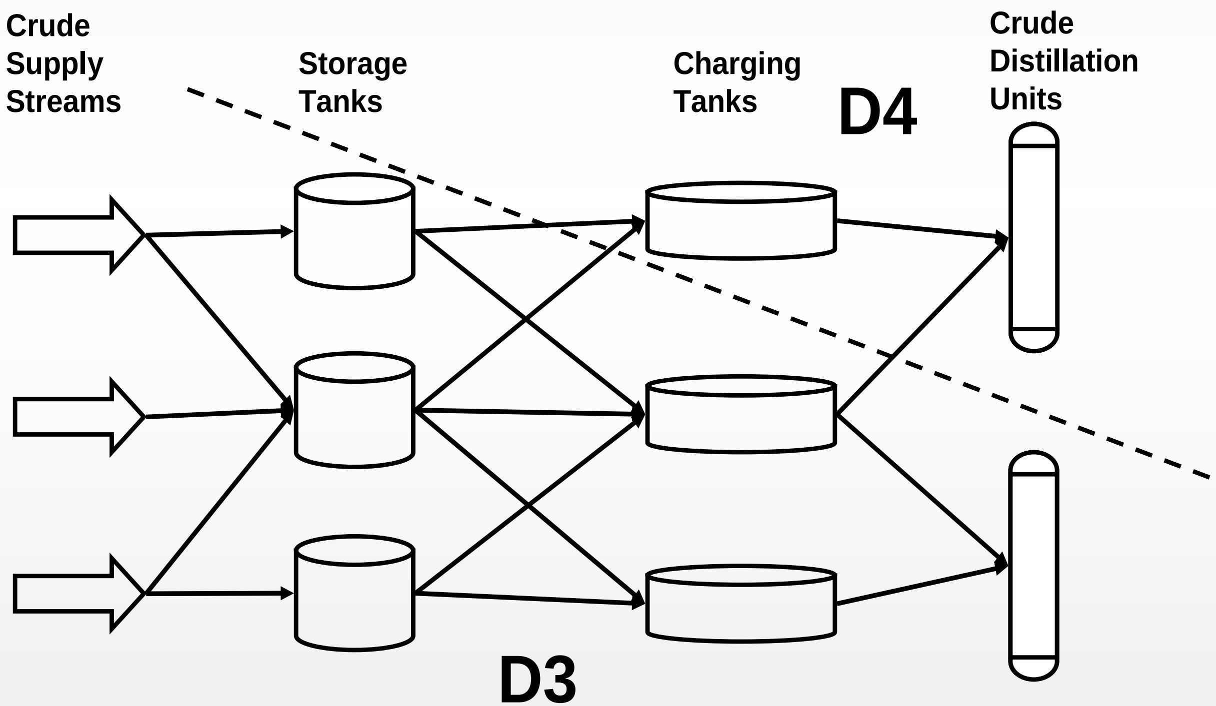
- Add above cuts to (R) to get (R′) which is solved to obtain a valid lower bound on global optimum of (P)

Remark: Update Lagrange multipliers and generate more cuts to add to (R)

Advantages of Cut Generation

- Lower bound obtained is at least as strong as one from conventional Lagrangean decomposition or LP relaxation of (R)

- Alternative decomposition schemes can be used to generate more cuts to add to relaxation (R)



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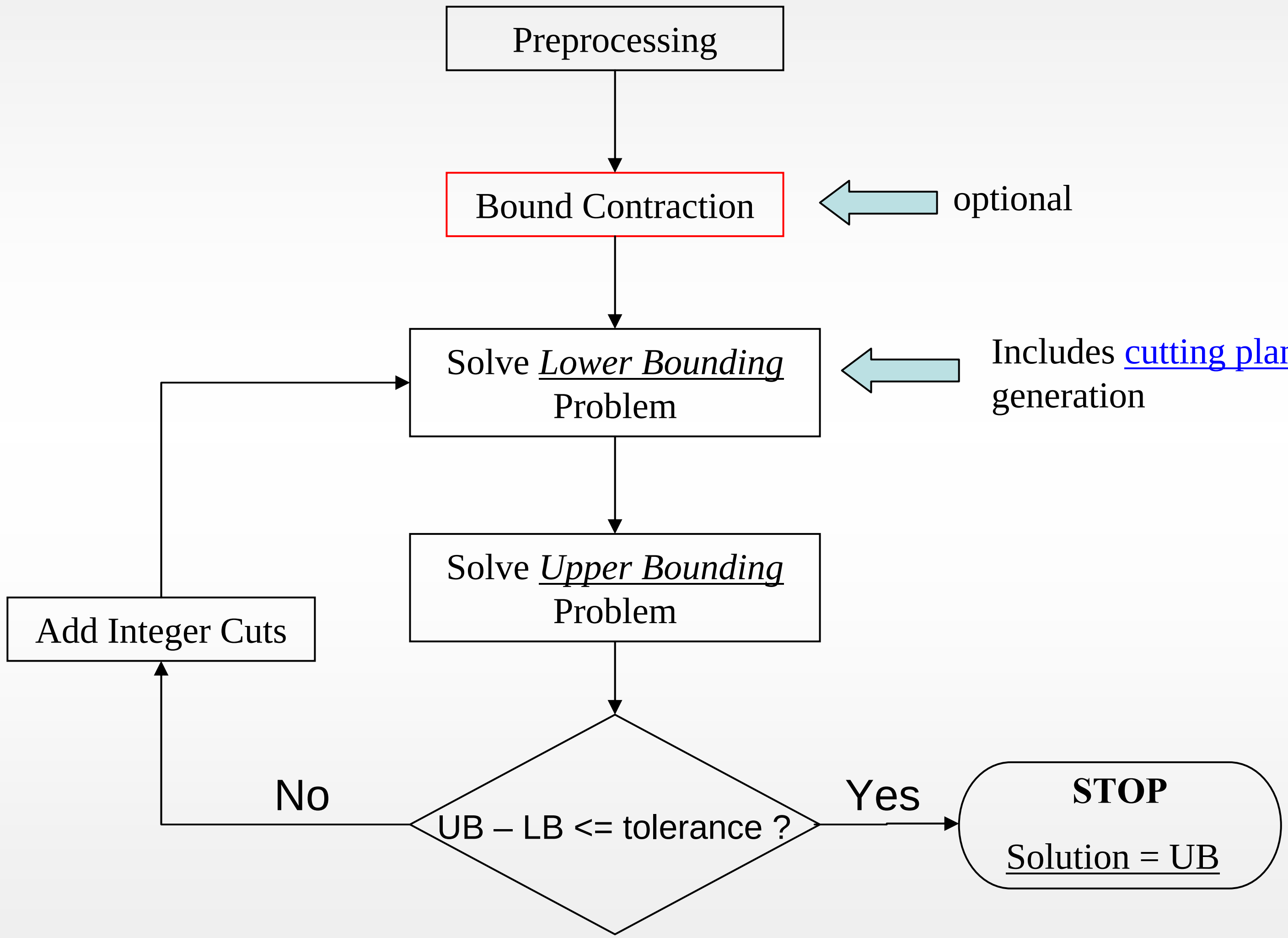
Proposed Algorithm

- Variant of **Outer-Approximation** (Duran and Grossmann, 1986)



Illustrative Example

3 Supply streams – 6 Storage Tanks – 4 Charging Tanks – 3 Distillation units

Scheduling Horizon	15 hours		
Number of supply streams	3		
	Arrival Time	Incoming Volume of crude	Fraction of key component
IN1	1	60	0.03
IN2	6	60	0.05
IN3	11	60	0.065

Number of Charging Tanks			3
	Capacity	Initial Inventory	Initial Fraction of key component (min – max)
Tank1	80	5	0.0317 (0.03 – 0.035)
Tank2	80	30	0.0483 (0.043 – 0.05)
Tank3	80	30	0.0633 (0.06 – 0.065)
Tank4	80	30	0.075 (0.071 – 0.08)

Number of Storage Tanks		6	
	Capacity	Initial Inventory	Initial fraction of key component (min – max)
Tank1	10 – 90	60	0.031 (0.025 – 0.038)
Tank2	10 – 110	10	0.03 (0.02 – 0.04)
Tank3	10 – 110	50	0.05 (0.04 – 0.06)
Tank4	10 – 110	40	0.065 (0.06 – 0.07)
Tank5	10 – 90	30	0.075 (0.07 – 0.08)
Tank6	10 – 90	60	0.075 (0.07 – 0.08)

Number of CDUs : 3
Waiting cost for supply streams (*Csea*): 5
Unloading cost for supply streams (*Cunload*): 7
Tank inventory costs (*Cinv(b)*): storage tanks – 0.05
charging tanks – 0.06
Changeover cost for charged oil switch (*Cset*): 30

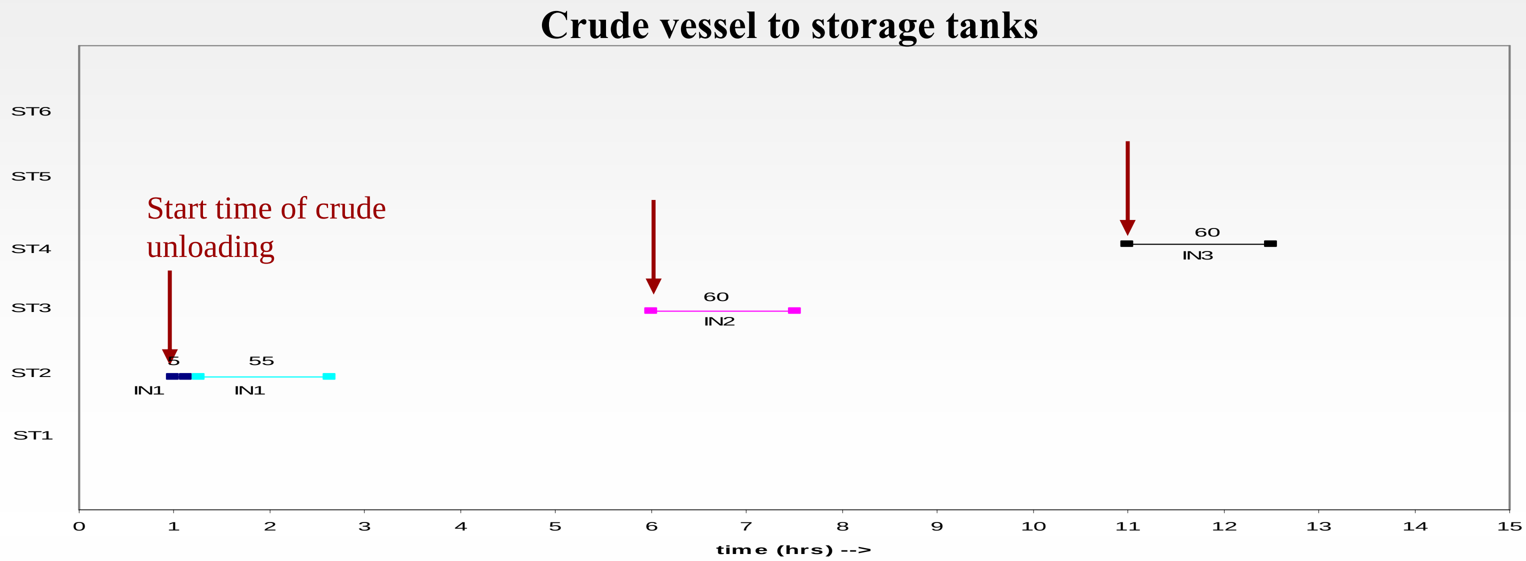
Demand of mixed oils by CDUs :

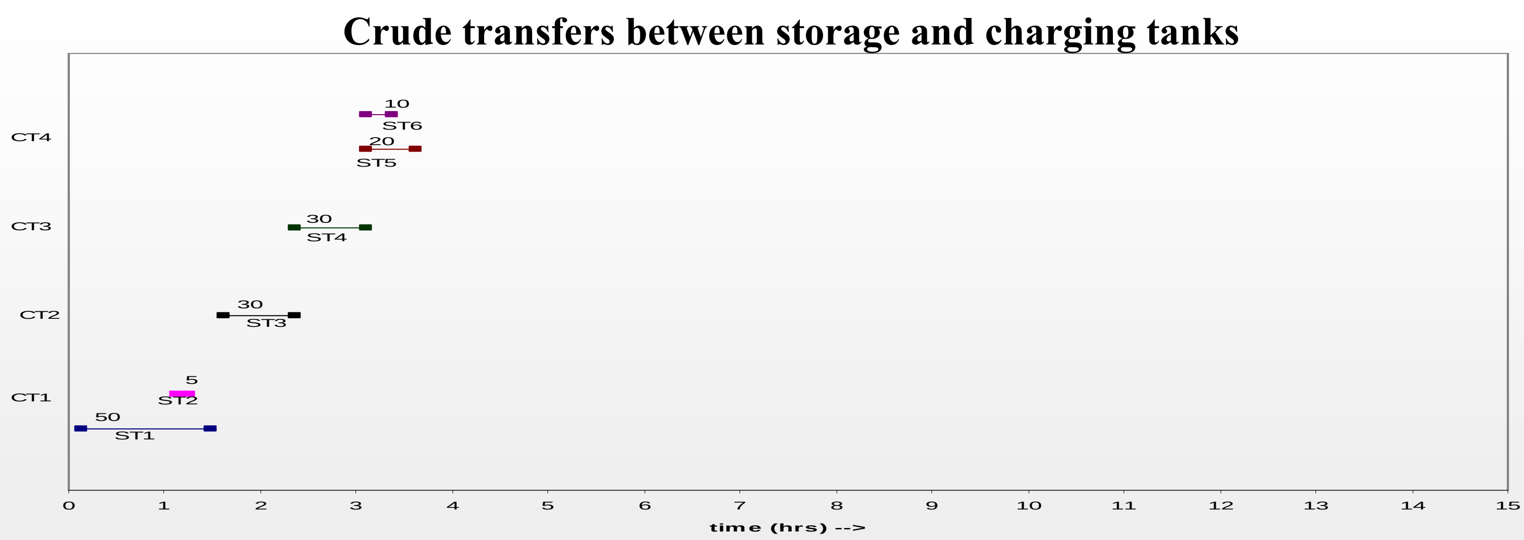
oil mix 1	60
oil mix 2	60
oil mix 3	60
oil mix 4	60

Bounds on flowrates in the streams: *Lower Bound : 1, Upper Bound : 40*

Optimal Crude Flow Schedule

Gantt chart of optimal schedule





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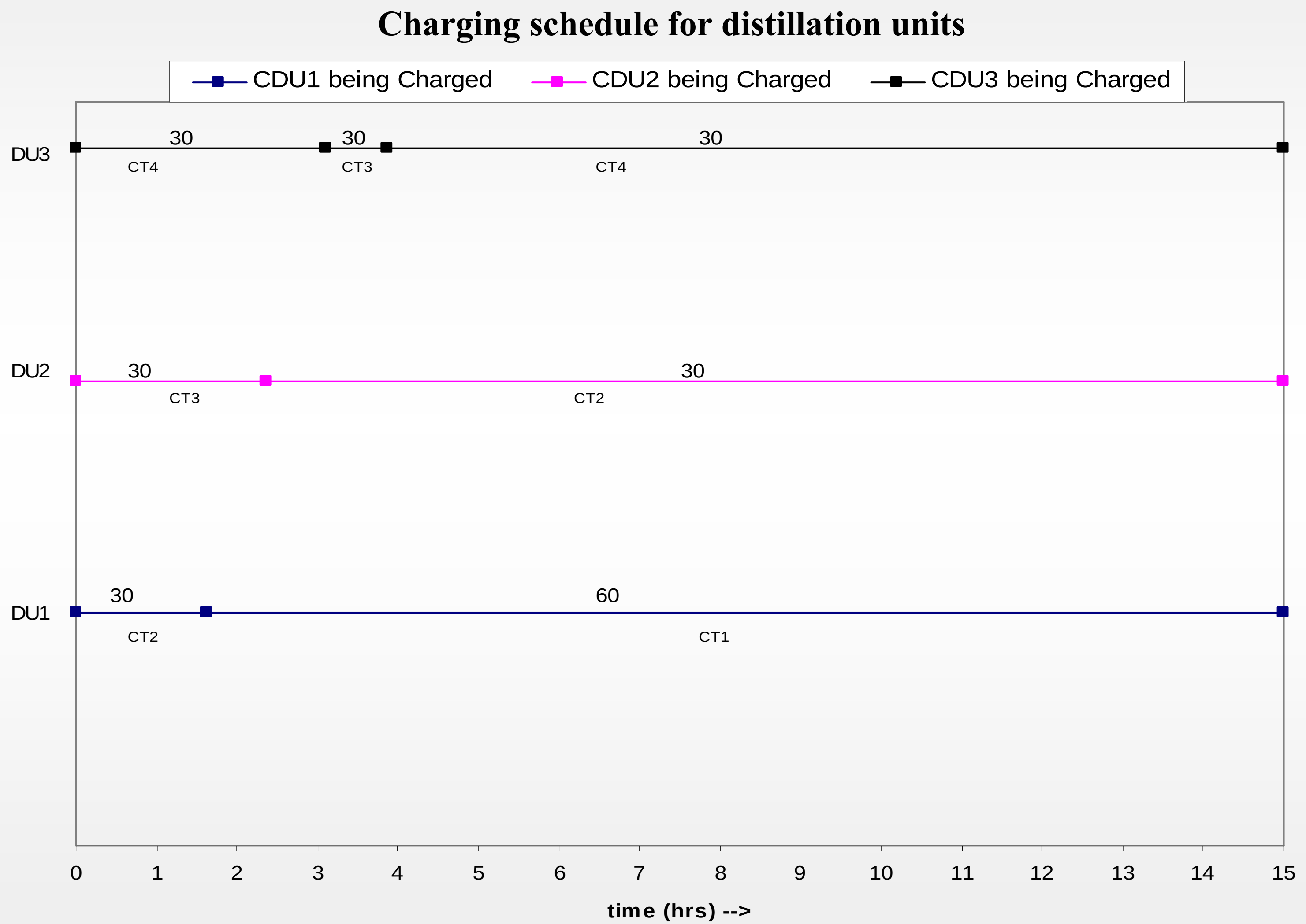
Optimal Crude Flow Schedule

Gantt chart of optimal schedule



Computational Results

Example	Original MINLP model (P)		
	Number of Binary Variables	Number of Continuous Variables	Number of Constraints
1	**48**	**300**	**946**
2	**42**	**330**	**994**
3	**57**	**381**	**1167**

3 Supply streams – 3 Storage tanks – 3 Charging tanks – 2 Distillation units

3 Supply streams – 3 Storage tanks – 3 Charging tanks – 2 Distillation units

3 Supply streams – 6 Storage tanks – 4 Charging tanks – 3 Distillation units

Solvers : **MILP** ⟹ CPLEX 9.0, **NLP** ⟹ BARON 7.2.5 (Sahinidis, 1996)

Example	Lower bound [obtained by solving relaxation (RP)] (z^{RP})	Upper bound [on solving (P-NLP) using BARON] ($z^{P\text{-}NLP}$)	Relaxation gap (%)	Total time taken for one iteration of algorithm* (CPUsecs)	Local optimum (using DICOPT)
1	281.14	282.19	0.37	827.7	291.93
2	351.32	359.48	2.27	6913.9	361.63
3	383.69	383.69	0	8928.6	383.69

Example	Solving MILP model (R)				Solving MILP model (RP) (including proposed cuts)			
	Solution (z^{R})	LP relaxation at root node	No. of nodes	Time taken to solve (R)* (CPUsecs)	Solution (z^{RP})	LP relaxation at root node	No. of nodes	Time taken to solve (RP)* (CPUsecs)
1	281.14	-55.24	940800	1953.3	281.14	68.45	334300	758.8
2	351.32	113.35	931700	14481.7	351.32	133.80	310600	5873.2
3	383.69	147.24	3029600	15874.8	383.69	189.19	1258100	8025.9

BARON could not guarantee global optimality in more than 10 hours*

* Pentium IV, 2.8 GHz , 512 MB RAM

Summary

- New continuous time formulation used to represent the scheduling of crude oil at the front-end of a refinery
  - Scheduling model is a non-convex MINLP

- Special Outer-Approximation algorithm proposed to solve problem to global optimality
  - Main idea : Generation of cutting planes for speeding up solution of MILP relaxation