Models for Mitigating Supply Chain Disruptions

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Supply Chain Disruptions

- All supply chains are subject to disruptions
- Common sources
  - Natural disasters, weather
  - Strikes
  - Terrorism, war
  - Product defects
  - Equipment breakdowns
  - Transit/customs delays
  - Supplier bankruptcy
  - etc.
- Only recently have academics and practitioners studied supply disruptions in earnest
- Why the recent interest?

Supply Chain Disruptions Are as Old as Supply Chains

East India Company

Supply Chain Disruptions Are as Old as Supply Chains

Wells Fargo

Why the Recent Interest? (cont'd)

1. Recent high-profile disruptions
   - September 11 (2001)
   - West-coast port lockout (2002)
   - Flu vaccine shortage (2004)
   - Hurricanes Katrina and Rita (2005)
   - Bird-flu pandemic (???)
2. Focus on lean supply chain management
   - aka just-in-time (JIT), etc.
   - Systems contain very little slack
   - Very efficient—as long as there is little uncertainty
   - Very fragile—easily disrupted
   - **There is value to having slack in a system**

Why the Recent Interest? (cont'd)

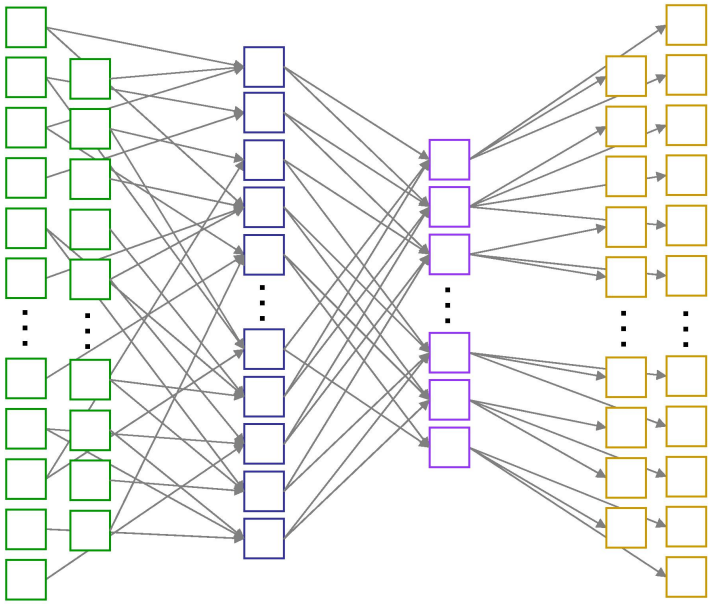
3. Increasingly global supply chains
   - A single supply chain may span the globe
   - Firms are less vertically integrated
     - "Manufacturing" firms may actually manufacture very little
     - Instead, they assemble components that are made by suppliers
     - Thomas Friedman, *The World is Flat*
   - Firms depend critically on parts from unstable regions
     - Unstable politically, economically, militarily, climatologically, seismically, ...
     - Barry Lynn, *End of the Line*

Cascading Disruptions

- A **supply chain** is the system of locations and activities that move products from where they are produced to where they are consumed
- Supply chains consist of many locations (**stages**)
- Stages are grouped into tiers (**echelons**)
- Disruptions are never purely local
- They **cascade** through the system
- Upstream disruptions cause downstream stockouts

Cascading Disruptions: GM Example

- In 1998, strikes at two General Motors parts plants



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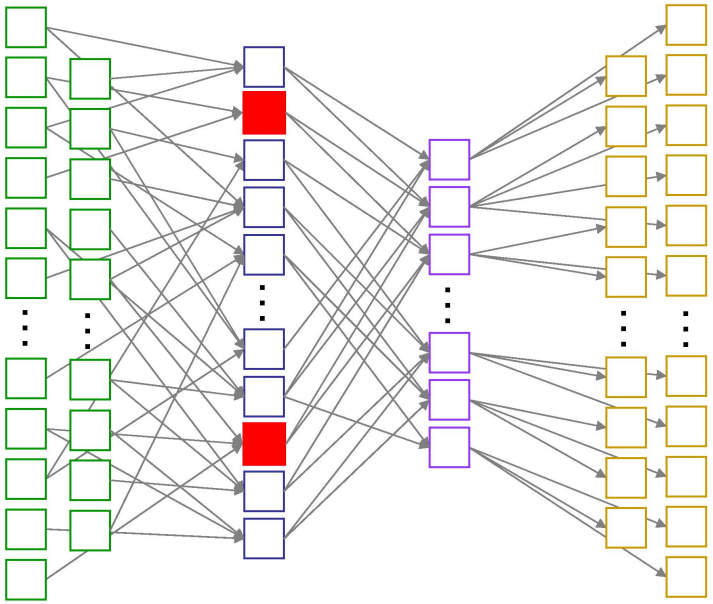

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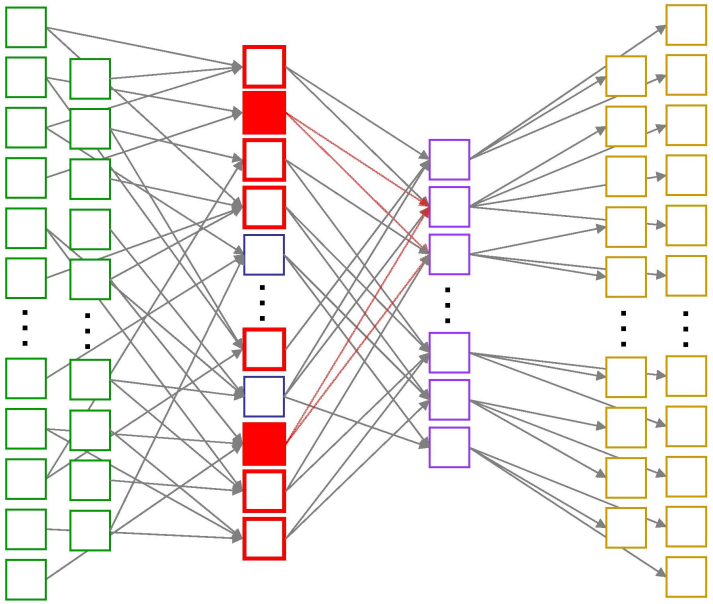
- Led to shutdown of 100+ other parts plants...



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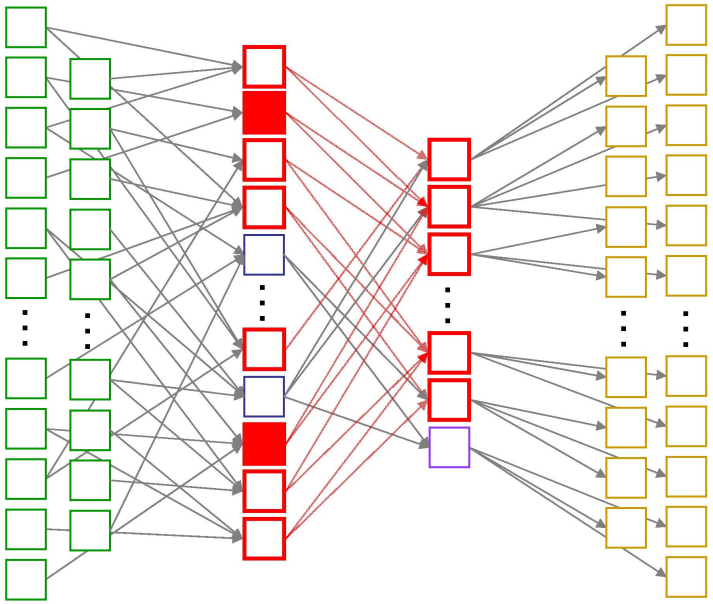
- ...and then to closures of 26 assembly plants...



Cascading Disruptions: GM Example

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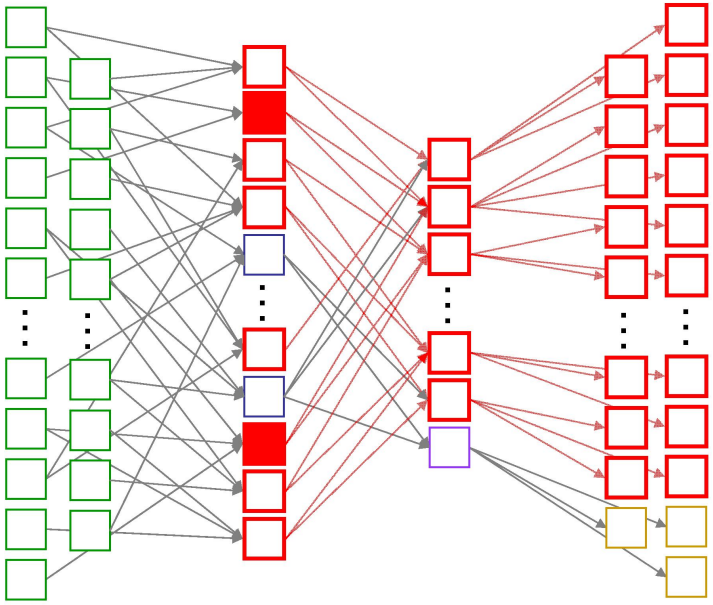
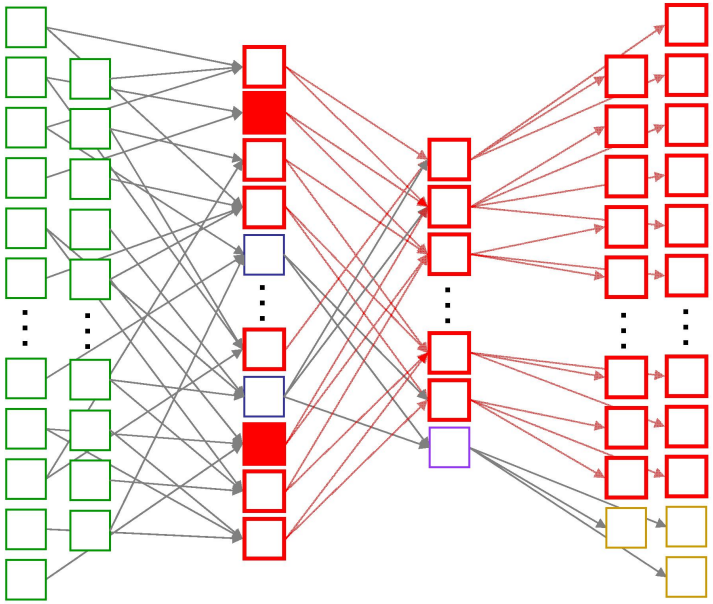
- ...and finally to vacant dealer lots for months



Cascading Disruptions: GM Example

- In 1998, strikes at two General Motors parts plants
- Led to shutdown of 100+ other parts plants...
- ...and then to closures of 26 assembly plants...
- ...and finally to vacant dealer lots for months
- 500K cars, 37% ↓ sales, 33% ↓ market share, $809M qrtly loss



A Scarier Example

- A terrorist attack on New York Harbor in winter would halt shipments of heating fuel
- New England and upstate New York would run out of heating fuel within 10 days
  - (according to national security analysis)
- Even a temporary halt would have significant cascading effects

Source: Finnegan (2006)

Most Research is on Single-Stage Systems

- Despite the importance of studying disruptions in a multi-stage context, most research focuses on a single stage
  - e.g., how should a firm plan for disruptions to its suppliers or itself?
  - Examines purely local effects
- (There are a few exceptions)
- I will focus primarily on disruptions in multi-stage systems

Roadmap

- Motivation
- Literature overview
- Methodology and assumptions
- Supply vs. demand uncertainty
- Reverse bullwhip effect
- Conclusions and future research directions

Literature Overview

Classical Inventory Models with Supply Uncertainty

- Classical models + disruptions:
  - ZIO: Parlar and Berkin (1991), Berk and Arreola-Risa (1994), Snyder (2006)
  - Non-ZIO: Parlar and Perry (1995, 1996)
  - Stochastic demand: Gupta (1996), Parlar (1997), Mohebbi (2003, 2004)
- Classical models + yield uncertainty
  - Gerchak et al. (1998), Bassok and Akella (1991), Yano and Lee (1995), Wang and Gerchak (1996)
- All are single-stage models
- Most must be solved numerically
  - Even if non-disruption models can be solved analytically
- General insight:
  - Hold more inventory

Strategic Questions

- More recently, papers addressing strategic questions
- What strategy is optimal?
- How does this change as disruption characteristics or other parameters change?
- For example:
  - Optimal strategy: Tomlin (2006)
  - Advanced warning: Tomlin and Snyder (2006)
  - Supplier flexibility: Tomlin and Wang (2004)
  - Effect of border closures: Lewis, Erera, and White (2005)
  - Error from "bundling" disruptions and yield uncertainty: Chopra et al. (2006), Schmitt and Snyder (2006)

Multi-Echelon Models

- Kim et al. (2005)
  - Yield uncertainty in 3-echelon SC, risk-averse objective
- Hopp and Yin (2006)
  - Optimal placement and size of inventory and capacity buffers in assembly network
  - More severe upstream disruptions $\implies$ buffers further upstream
- Today's talk:
  - Supply vs. demand uncertainty:
    - Snyder and Shen (2006)
    - Schmitt, Snyder, and Shen (2007)
  - Reverse bullwhip effect:
    - Rong, Snyder, and Shen (2007a,b)

The Newsboy/Base-Stock Problem

Theorem

In a single-stage base-stock system with stochastic demand distributed as $N(\mu, \sigma^2)$ and deterministic supply, the optimal base-stock level is given by

$$S^* = \mu + \sigma \Phi^{-1}\left(\frac{p}{p+h}\right),$$

where Φ is the standard normal cdf.

- Now suppose supplier may experience disruptions
  - i.e., supply is stochastic
  - But demand is deterministic
- Maybe buy extra items today in case supplier is down tomorrow

The Newsboy/Base-Stock Problem with Disruptions

Theorem (Tomlin 2006)

In a single-stage base-stock system with deterministic demand and stochastic supply disruptions, the optimal base-stock level is given by

$$S^* = d + dF^{-1}\left(\frac{p}{p+h}\right),$$

where d is the demand per period and F is the cdf of supply.

- $F(x) = P(\text{we are in a disruption lasting } x \text{ periods or fewer})$
- Cycle/safety stock interpretation
- Similar (but less sharp) result given by Güllü et al. (1997)

Are DU and SU the same?

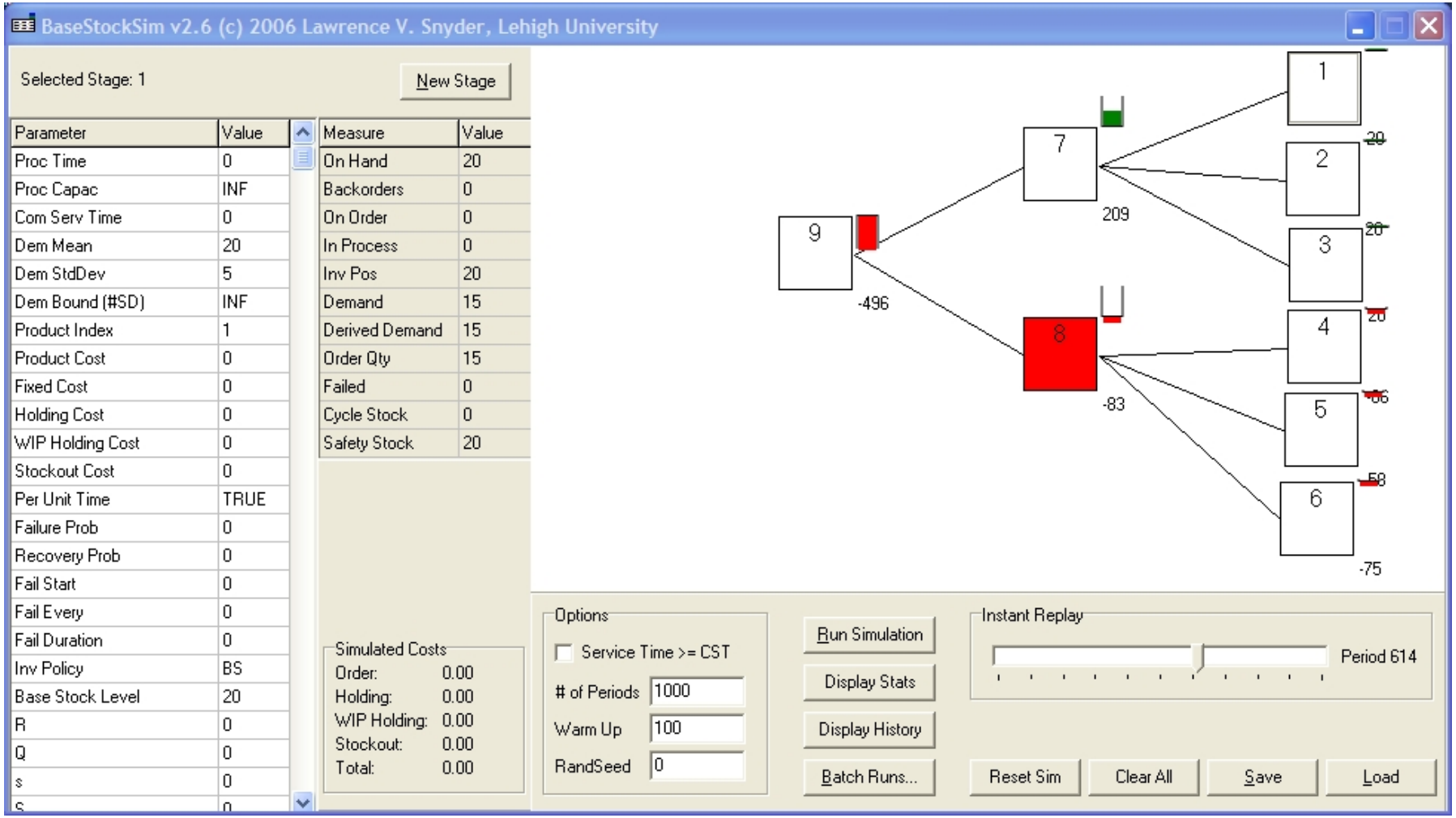
- This theorem suggests a symmetry between demand uncertainty (DU) and supply uncertainty (SU)
- Maybe they are two sides of the same coin?
- Under both DU and SU, the main issue is the same:
  - Not enough supply to meet demand
  - May be irrelevant whether mismatch came from DU or SU
- Moreover, mitigation strategies are similar for DU and SU
  - Inventory, excess capacity, supplier redundancy, etc.
- The good news:
  - We know a lot about supply chains under DU
- The bad news:
  - The "conventional wisdom" from DU is often **wrong** under SU!

Methodology and Assumptions

Methodology

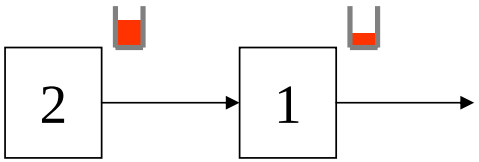
- Some of our results are proved analytically
  - Key theoretical results:
    - Tomlin (2006): optimal base-stock level in single-stage system with disruptions
    - Others that I'll present later
- Others we demonstrate using simulation
  - BaseStockSim software
  - Rough optimization of base-stock levels

BaseStockSim



Supply Chain Assumptions

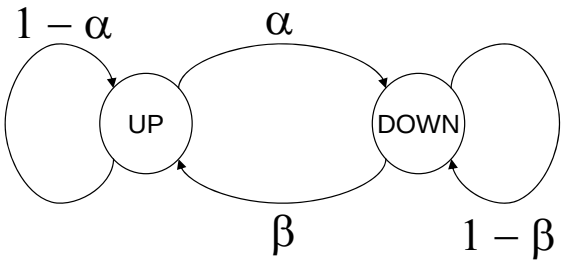
- Multi-echelon SC
  - Each stage has processing function and output buffer:



  - May represent physical location, processing activity, or SKU
- Excess demands are backordered
- Costs h, p
- Deterministic processing (lead) time T

Demand and Supply Processes

- Under DU, demands are $N(\mu, \sigma^2)$
- Under SU, disruption process follows 2-state Markov process
  - Disruption probability α
  - Recovery probability β
  - Simplest case:
    - Capacity $= \infty$ when UP
    - Capacity $= 0$ when DOWN
    - (But can make it more general)



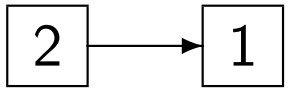
Supply vs. Demand Uncertainty

SU vs. DU: Roadmap

1. Order frequency
2. Centralization vs. decentralization
   - The risk-diversification effect
   - DU and SU
   - Implication for facility location
3. Supplier redundancy
4. The cost of reliability

Order Frequency

- Two-stage supply chain
- $\mu = 20$, $p = 100$ at retailer
- $T = 1$ at supplier
- Under DU, $\sigma = 5$
- Two possible cost structures:
  1. $h = 2.85$ and $K = 0$
  2. $h = 0.1$ and $K = 250$



Key Question:

Does firm prefer #1 (one-for-one ordering) or #2 (batch ordering)?

Order Frequency: DU

- Option 1: $h = 2.85$, $K = 0$
  - Base-stock policy is optimal, with

$$S^* = \mu + \sigma\Phi^{-1}\left(\frac{p}{p+h}\right) \approx 30$$

  - E[cost] ≈ 32.8
- Option 2: $h = 0.1$, $K = 250$
  - (s, S) policy is optimal with

$$s^* \approx 31, \quad S^* \approx 349$$

  - E[cost] ≈ 32.8
- So the firm is indifferent between the two options under DU

Order Frequency: SU

- Option 1: $h = 2.85$, $K = 0$
  - Base-stock policy is optimal (Tomlin 2006), with

$$S^* = \mu + \mu F^{-1}\left(\frac{p}{p+h}\right) \approx 60$$

  - E[cost] $\approx$ 497.7

Order Frequency: SU

- Optimal policy not known:
  - Deterministic demand
  - Stochastic disruptions
  - (Possibly) non-zero fixed cost

Conjecture

Under deterministic demand and stochastic disruptions, an (s, S) policy is optimal.

- Proof will rely on establishing **K-convexity** of objective function

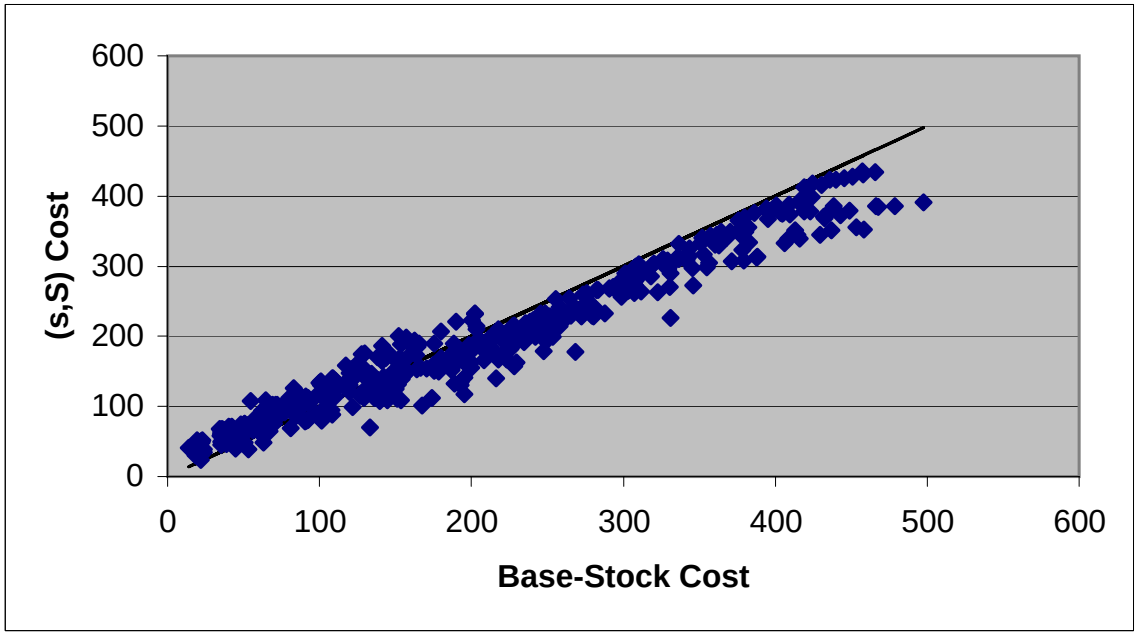
Lemma (Schmitt, Snyder, and Shen 2007)

s^* and S^* are integer multiples of μ.

Order Frequency: SU

- Option 2: $h = 0.1$, $K = 250$
  - $s^* \approx 40$, $S^* \approx 340$, E[cost] $\approx$ 391.1
  - (Recall: under base-stock policy, E[cost] $\approx$ 497.7)
- So the batch ordering policy is preferred
- Why?
  - If an order is disrupted, the impact is the same under either policy
  - But the likelihood of a disruption affecting an order is smaller under batch policy

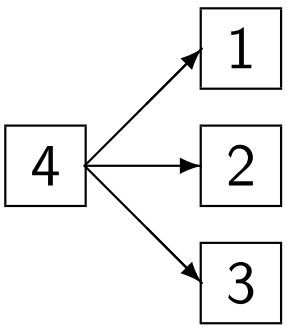
Simulation Experiment



- Batch policy is usually—though not always—preferred
  - s and S may not be optimal
- Instances are generated so that batch and base-stock policies are equivalent under DU

Centralization vs. Decentralization

- One warehouse, multi-retailer (OWMR) system
- Cost of holding inventory is equal at the two echelons
- Lead times are negligible



Key Question:

Should we hold inventory at the warehouse or at the retailers?

- (Equivalently: Should we centralize many stocking points into one?)

OWMR under DU

- Let C_D, C_C be cost under decentralized and centralized systems, resp.

Theorem (Eppen 1979)

Under DU,

- $E[C_D] \propto N$
- $E[C_C] \propto \sqrt{N}$

- Therefore, centralization is optimal
  - The **risk-pooling effect**

OWMR under SU

- Under SU:
  - Disruptions affect inventory sites
  - In decentralized system, a disruption affects one retailer
  - In centralized system, a disruption affects the whole supply chain

Theorem (Schmitt, Snyder, and Shen 2007)

Under SU,

(a) $E[C_D] = E[C_C]$

(b) $V[C_D] \propto N$
$V[C_C] \propto N^2$

- Therefore decentralization is preferable
- We call this the **risk-diversification effect**

OWMR under DU and SU

- What if both DU and SU are present?
- Numerical results suggest risk-diversification usually trumps risk-pooling
  - Even if disruptions are relatively rare
  - Even if demand uncertainty is relatively high
  - Even if decision maker is relatively risk averse
- Single-stage problem with DU and SU is significantly harder

OWMR under DU and SU

Theorem (Schmitt, Snyder, and Shen 2007)

The expected cost of a single-stage system under DU and SU under base-stock level S is

$$c(S) = \sum_{i=1}^{\infty} \pi_{i-1} \left[h(S - i\mu) + \sigma\sqrt{i}(p+h)G\left(\frac{S - i\mu}{\sigma\sqrt{i}}\right)\right],$$

where

- π_i = steady-state probability of being in the ith period of a disruption
- $G(\cdot)$ = standard normal loss function

Theorem (Schmitt, Snyder, and Shen 2007)

$c(S)$ is convex.

What Makes This Hard?

- Easy to optimize numerically
- Hard (impossible) to find closed-form expression for S^*
- $c(S)$ has the form

$$\gamma + \kappa S + \eta_1 G\left(\frac{S - 1\mu}{\sqrt{1}\sigma}\right) + \eta_2 G\left(\frac{S - 2\mu}{\sqrt{2}\sigma}\right) + \eta_3 G\left(\frac{S - 3\mu}{\sqrt{3}\sigma}\right) + \ldots$$
$$= \gamma + \kappa S + \eta_1 G(f_1(S)) + \eta_2 G(f_2(S)) + \eta_3 G(f_3(S)) + \ldots$$

where γ, κ, η_i are constants.

An Approximation

$$c(S) = \gamma + \kappa S + \underbrace{\eta_1 G(f_1(S)) + \eta_2 G(f_2(S))}_{G(x)\approx 0} + \eta_3 G(f_3(S)) + \underbrace{\eta_4 G(f_4(S)) + \eta_5 G(f_5(S)) + \ldots}_{G(x)\approx -x}$$

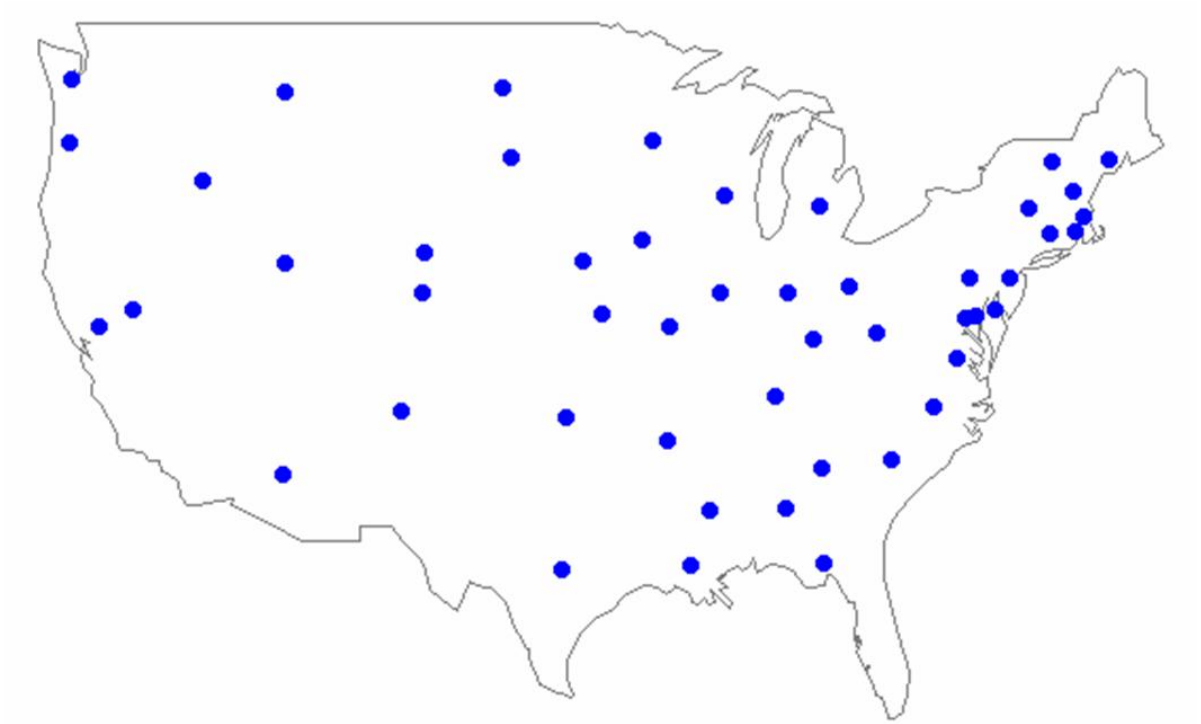
Approximation (Schmitt, Snyder, and Shen 2007)

$$c(S) \approx \gamma' + \kappa' S + \eta_{\hat{\imath}} G\left(\frac{S - \hat{\imath}\mu}{\sqrt{\hat{\imath}}\sigma}\right)$$

- We can find a closed-form expression for S^* using approximate cost function.
- This approximation is generally very accurate
  - How to identify $\hat{\imath}$?

Facility Location Problems

- Nodes represent demand locations
- Where to open facilities?
- Formulate as integer programming problems
- Solve using variety of optimization techniques
  - Branch & bound, Lagrangian relaxation, decomposition methods, etc.



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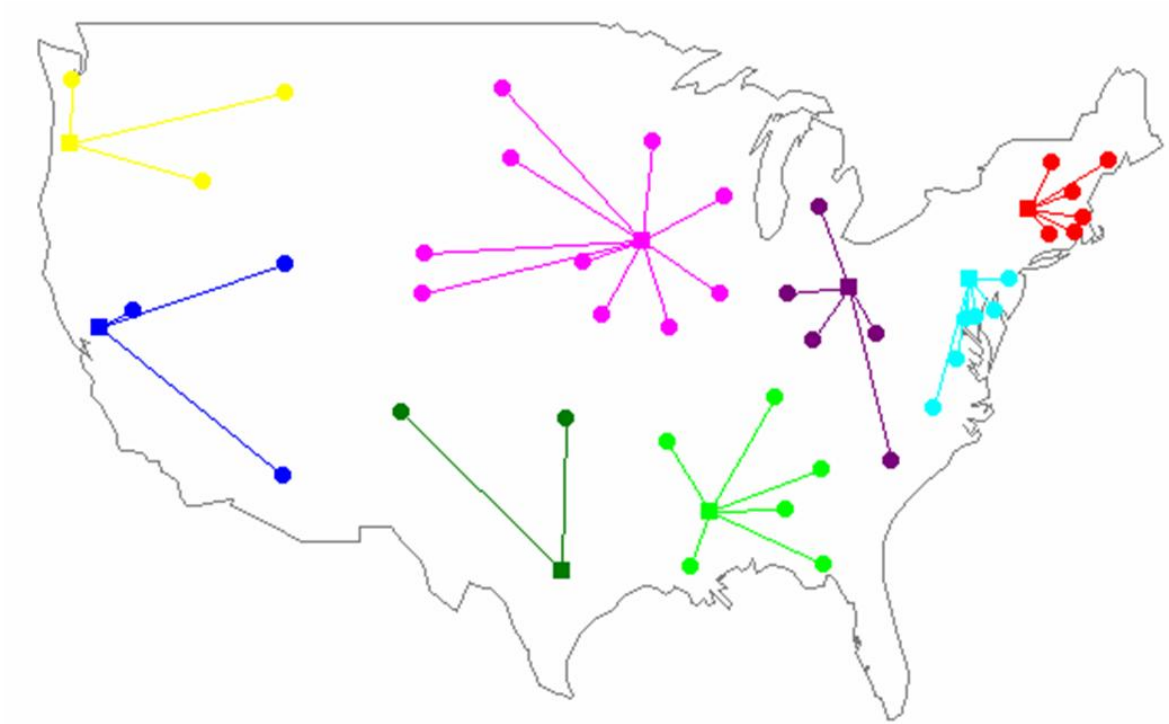
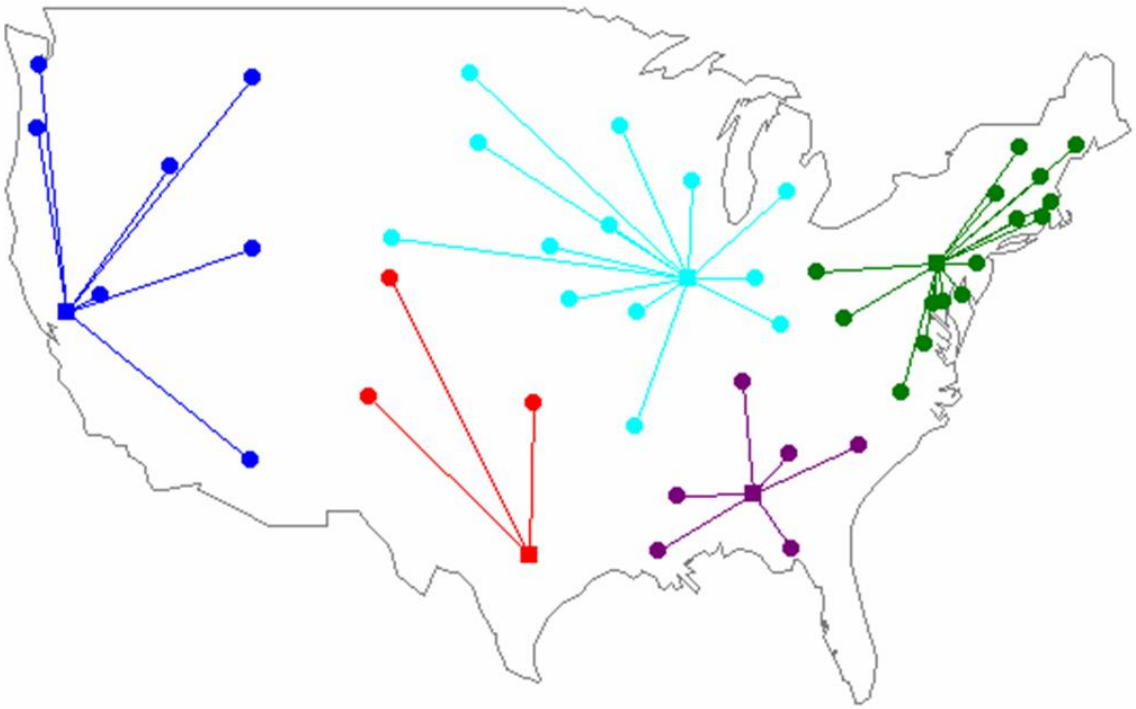


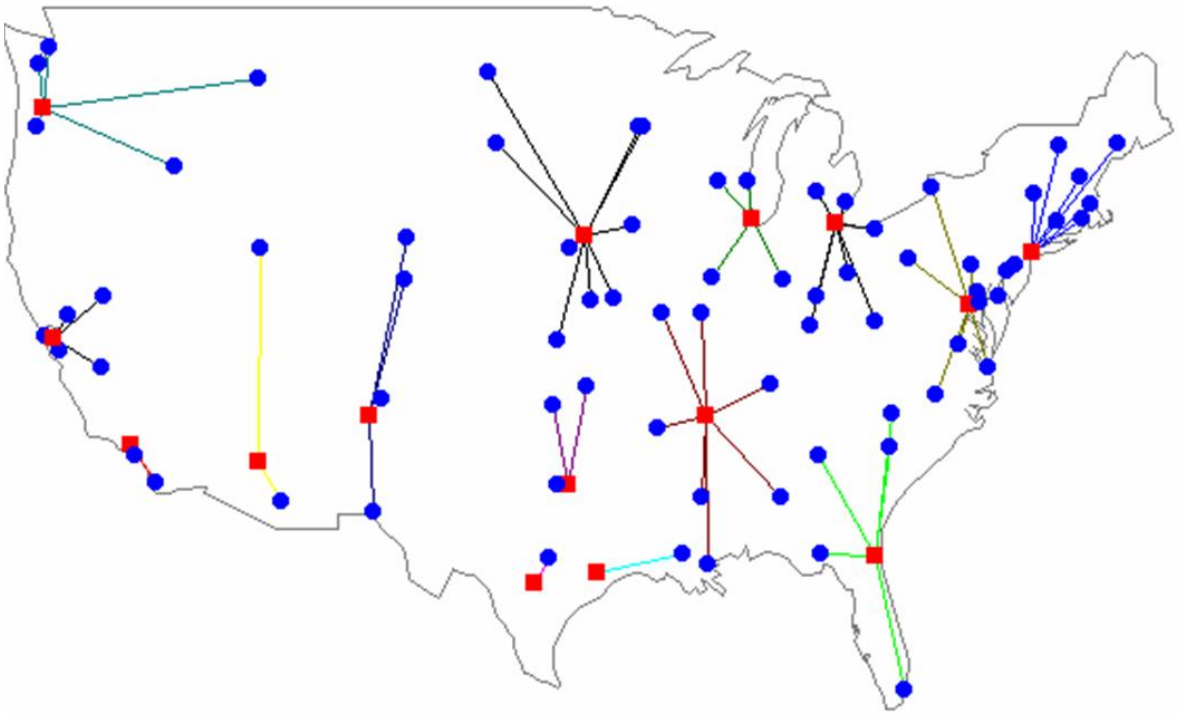
Facility Location and the Risk-Pooling Effect

- Joint location–inventory model by Shen, Coullard, and Daskin (2003)
  - Considers DU via concave inventory costs in location model
  - Optimal # of facilities decreases because of risk-pooling effect (and inventory economies of scale)



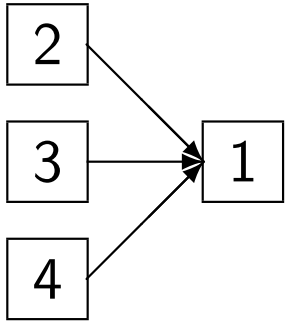
Facility Location and the Risk-Diversification Effect

- Reliability model by Snyder and Daskin (2005)
  - Considers SU in the form of facility failures
  - Optimal # of facilities increases—related to risk-diversification effect
  - Jeon et al. (2006), Qi et al. (2006) balance these competing tendencies



Supplier Redundancy

- Single retailer with one or more suppliers
- Suppliers are identical in terms of cost, capacity, reliability



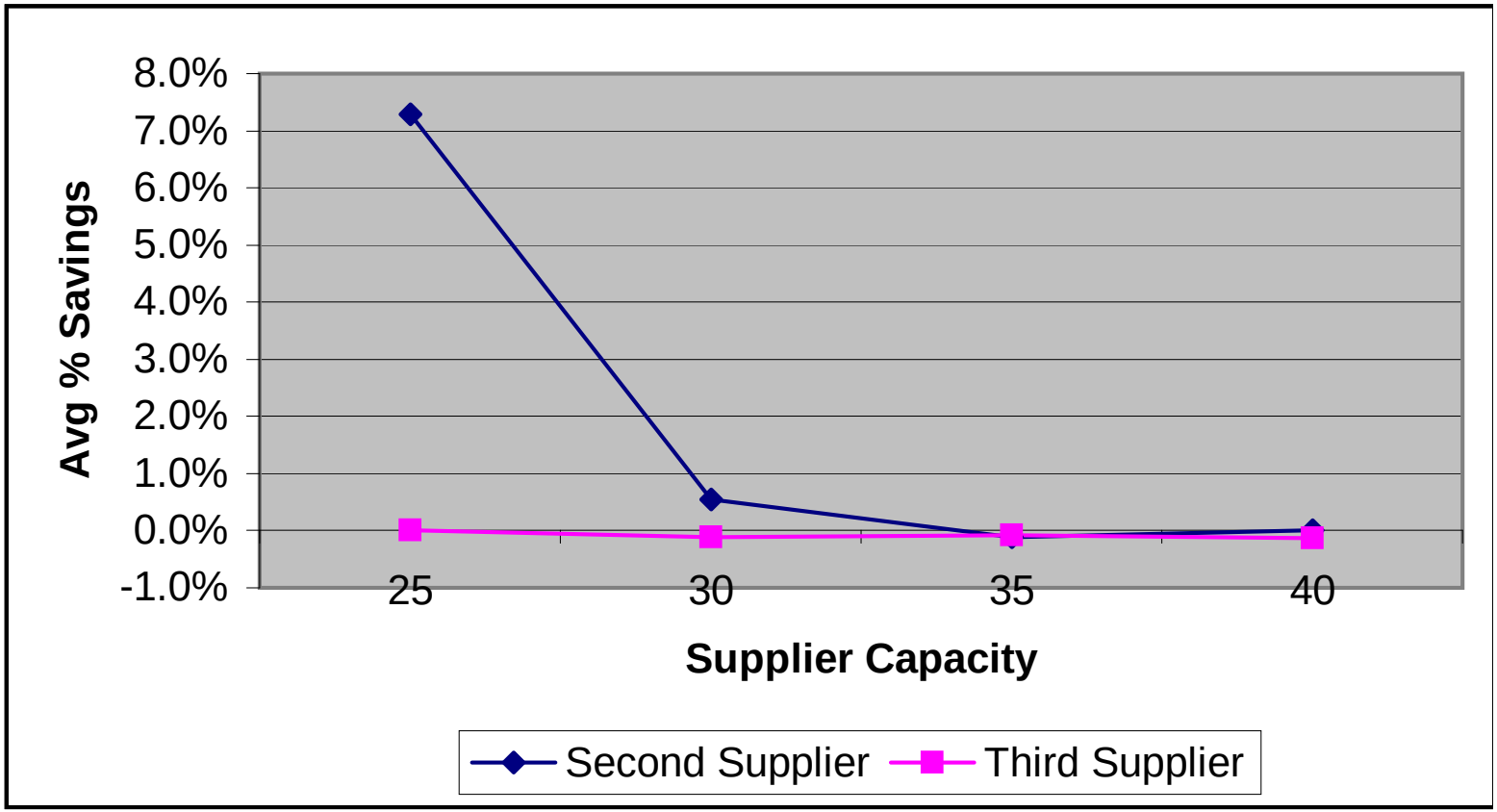
Key Question:

What is the value of having backup suppliers?

Supplier Redundancy under DU

- Under DU, second supplier provides value if capacities are tight
  - e.g., if capacity $= \mu + \sigma$
  - But value decreases quickly as capacity increases
  - Third, etc. suppliers provide little value

Value of Backup Suppliers: DU

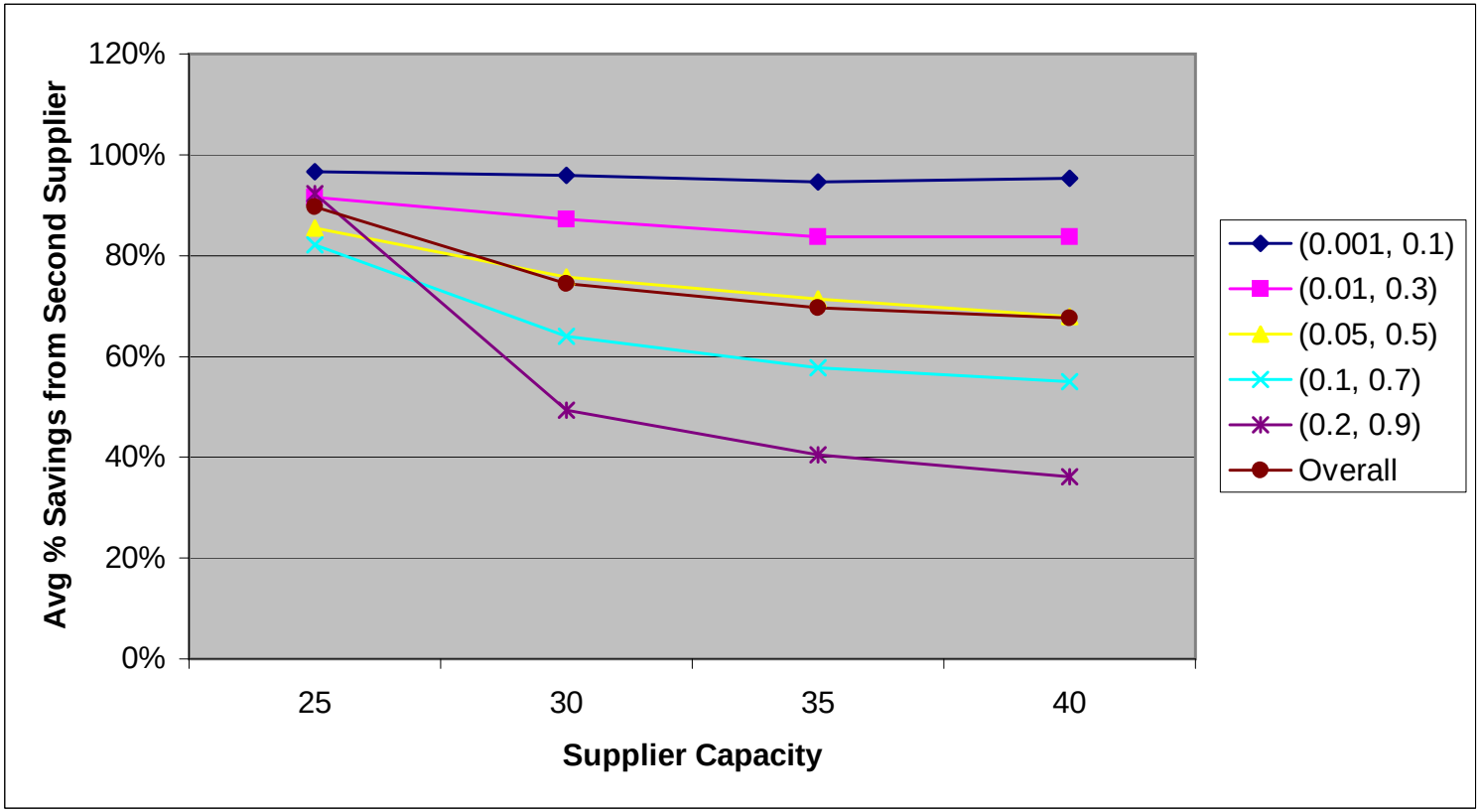


- $\mu = 20$, $\sigma = 5$

Supplier Redundancy under SU

- Under SU, second supplier provides great benefit
  - Fills in when primary supplier is disrupted
  - Also helps ramp back up after disruption
  - Even third+ supplier provides some benefit

Value of Backup Suppliers: SU



The Cost of Reliability

- Firms are accustomed to planning for DU
- Often reluctant to plan for SU if it requires large investment

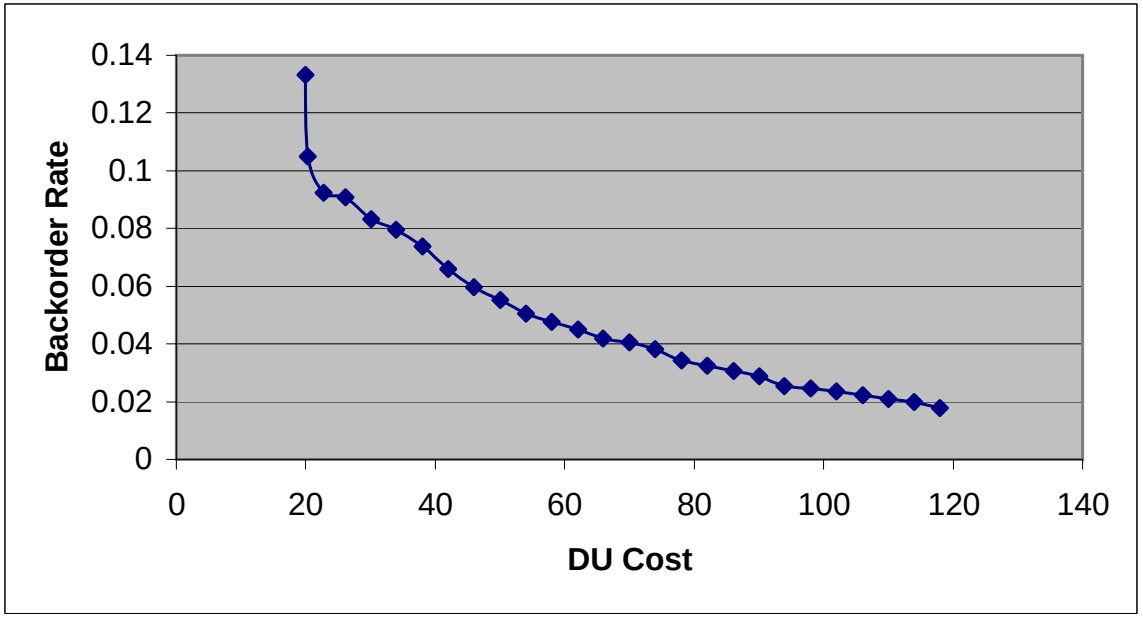
Key Question:

How much DU cost must be sacrificed to achieve a given level of reliability?

- The short answer: Not much

Tradeoff Curve

- Each point represents a solution (set of base-stock levels) for serial system
  - Left-most point is "optimal" solution considering DU only
  - Second point: 21% fewer stockouts, 2% more expensive
- "Steep" left-hand side of tradeoff curve is fairly typical
  - Especially for combinatorial problems



The Reverse Bullwhip Effect

Motivation: Hurricane Katrina

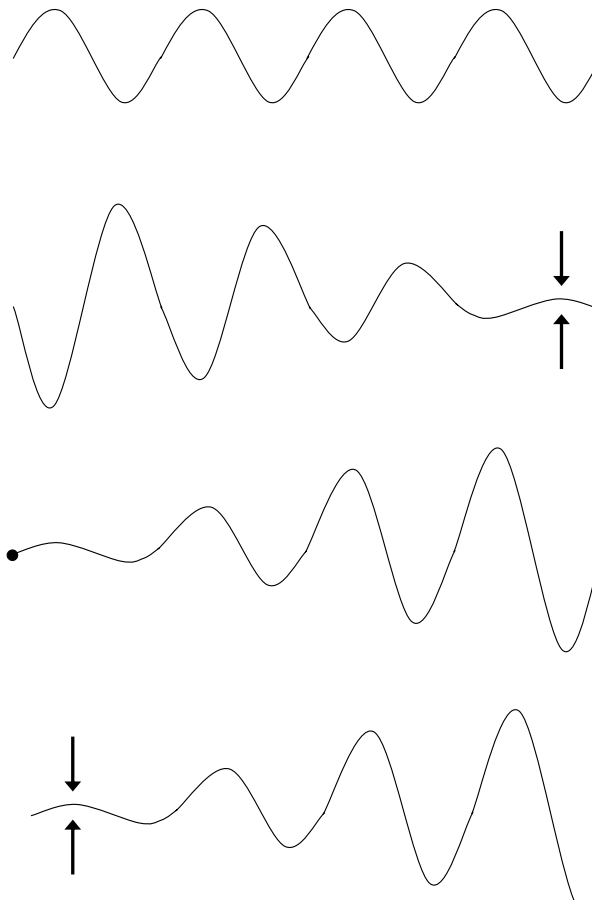
- Hurricanes Katrina and Rita crippled U.S. oil drilling and refining capacity
- As a result, demand for gasoline became very volatile
- The classical **bullwhip effect** (BWE):
  - Demand volatility increases as we move upstream
  - Sterman (1989), Lee, Padmanabhan, and Whang (1997a,b)
- We conjecture that the **reverse bullwhip effect** (RBWE) occurred after Katrina and Rita:
  - Demand volatility increases as we move downstream

Empirical Evidence

- Recent empirical study
  - Only 50% of industries exhibited BWE
  - Only 18% after data are deseasonalized
  - Cachon, et al. (2006)
- Behavioral studies using **beer game**
  - Many find a significant portion of players not exhibiting BWE
  - Croson et al. (2003, 2004, 2006), Kaminski and Simchi-Levi (2000), Wu and Katok (2005)

String-Vibration Analogy

- With no amplification
- With a demand shock
- With a fixed point upstream
- **With a supply shock**



Beer Game Experiment

- Like classical beer game, but
  - No demand uncertainty
  - Supply disruptions

- (**Heavy blue** curve is aggregate.)

Capacity/Price/Demand Model

- Model describes relationship between random capacity and resulting price and demand
- Use it to demonstrate that capacity shocks create RBWE

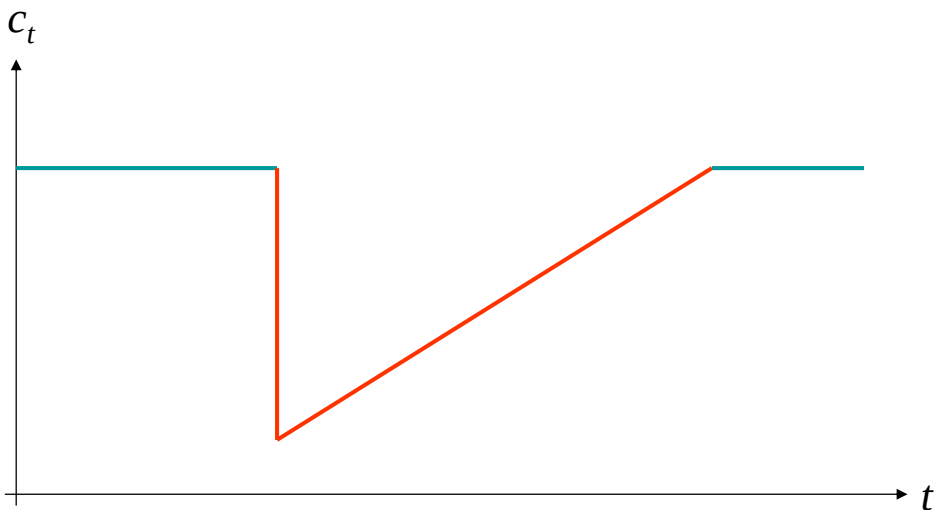
- 2 stages, supplier and buyer
- Supplier's capacity follows process $\{c_t\}_{t=1}^{\infty}$
- Capacity changes produce price changes
- Buyer anticipates future price changes and sets demand accordingly
  - Linear, downward-sloping demand curve
- We assume capacity is always tight
  - i.e., supplier's production quantity always equals capacity

Notation

- c = supplier's production capacity = production quantity
- p = equilibrium price
- Q = quantity demanded by buyer
- All state variables are indexed by t (time)

Capacity Process

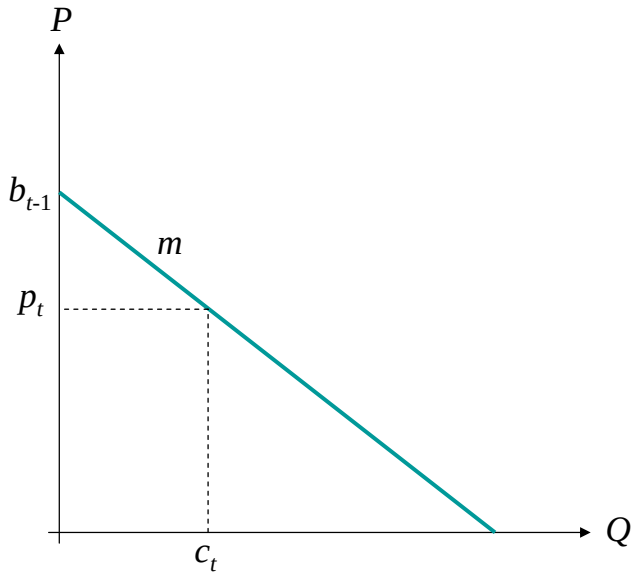
- For now, we assume a deterministic process for c_t



- Many of our results can also be proven for iid random c_t
- We intend to extend to more general capacity processes

Capacity $\Longrightarrow$ Price

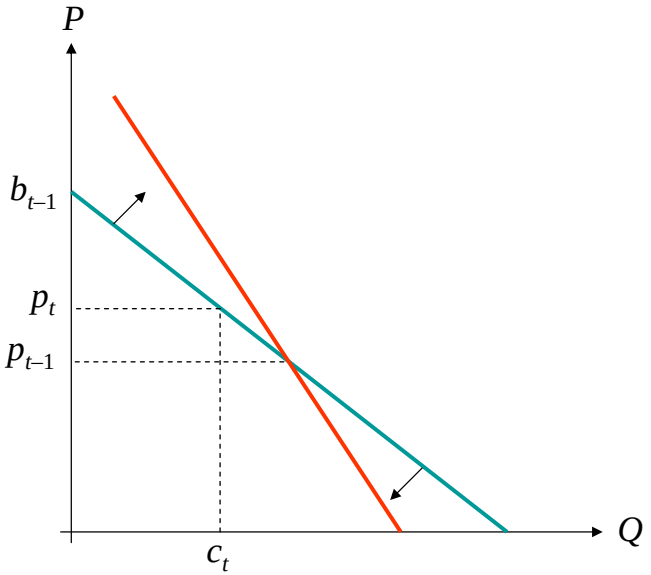
- For each capacity c_t we determine market-clearing price p_t



$$p_t = mc_t + b_{t-1}$$

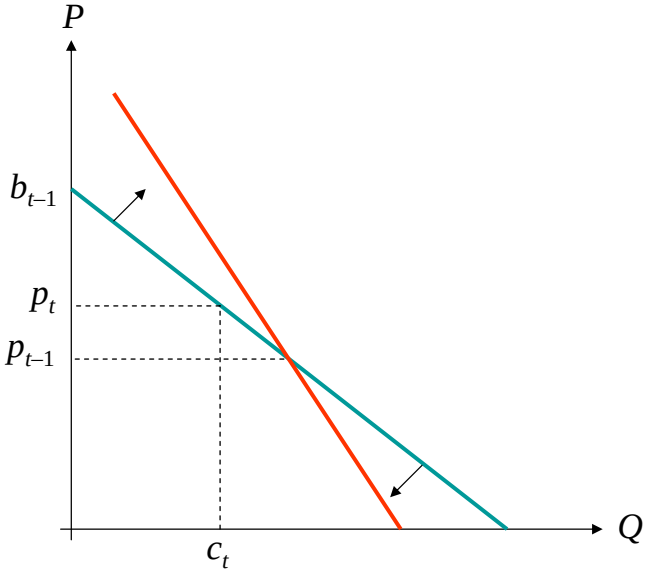
Price $\Longrightarrow$ Demand Curve Shift

- Buyer observes price p_t *and* change in price from last period
- Adjusts demand curve based on change in price
  - Assumes price trend will continue
  - Replaces p_t with $p_t - r(p_t - p_{t-1})$
  - $r \in [0, 1)$ is a "reaction factor"

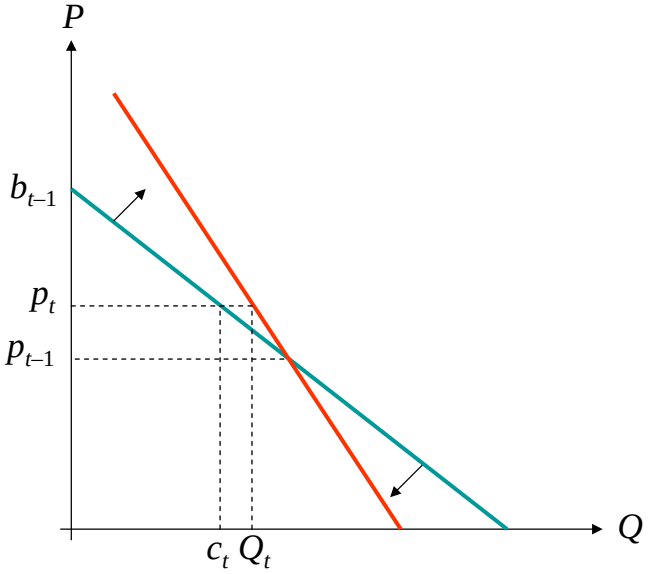


$$p_t = \frac{1}{1-r}\left[mc_t + b_{t-1} - rp_{t-1}\right]$$

Demand Curve $\Longrightarrow$ Order Quantity



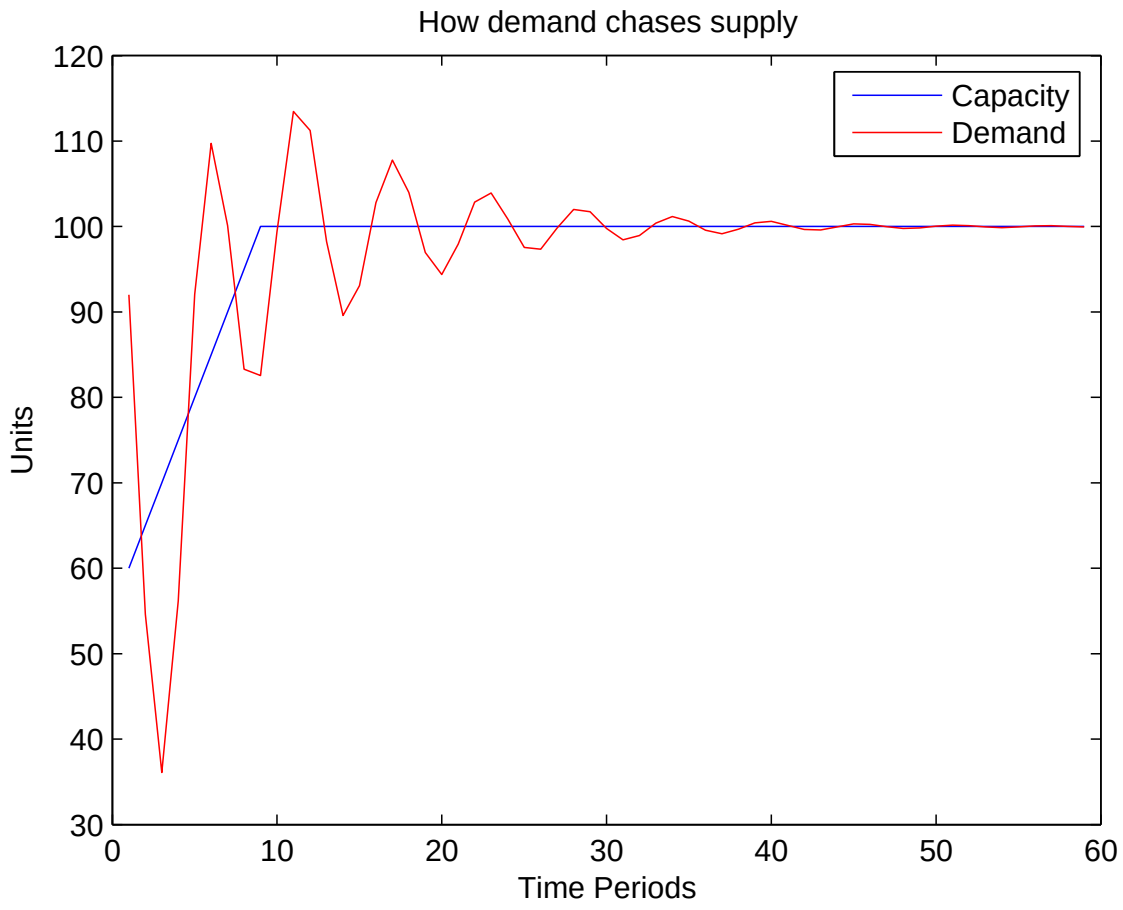
Demand Curve $\Longrightarrow$ Order Quantity



- New curve (and current price) $\Longrightarrow$ demand:

$$Q_t = \frac{(1-r)p_t + rp_{t-1} - b}{m}$$

Plot of Demand vs. Supply (Capacity)



- Demand is more variable than supply $\Longrightarrow$ RBWE

Approximation of Demand Variance

- p_t is polynomial function of r
  - (Recall: r = shift in demand curve)
- Let $\hat{p}_t$ = first-order approximation of p_t with respect to r
- And $\hat{Q}_t$ the resulting demand
- Then we calculate the variance of $\hat{Q}_t$
- **Key Question:** Is
  - $V(\hat{Q}_t) > V(c_t)$ [RBWE] or
  - $V(\hat{Q}_t) < V(c_t)$ [BWE]?

BWE or RBWE?

Theorem (Rong, Snyder, and Shen 2007b)

There exists a unique $r^* > 0$ such that:

(a) If $r = 0$ or $r = r^*$, then $V(\hat{Q}_t) = V(c_t)$ [no BWE or RBWE].

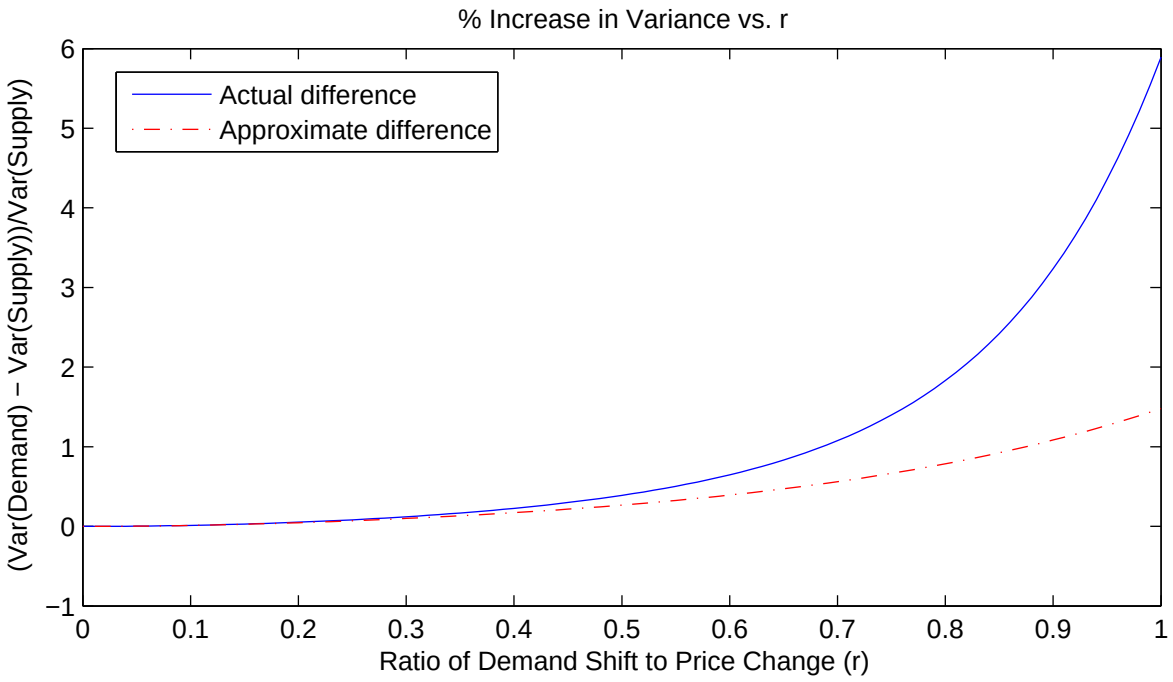
(b) If $r \in (0, r^*)$, then $V(\hat{Q}_t) < V(c_t)$ [BWE].

(c) If $r \in (r^*, \infty)$, then $V(\hat{Q}_t) > V(c_t)$ [RBWE].

- We know that $r^* \in (0, 0.2547)$ and are working on narrowing this range further.

Numerical Study: Difference in Variance vs. r

- We conjecture that $V(\hat{Q}_t)$ always underestimates $V(Q_t)$
  - Then RBWE is more frequent and more exaggerated than suggested by the Theorem.



Severity of RBWE

Proposition (Rong, Snyder, and Shen 2007b)

The magnitude of RBWE ($V(\hat{Q}_t) - V(c_t)$):

(a) increases with Δc (drop in capacity)

(b) increases with T (time to recovery)

Conclusions

Conclusions

- Planning for disruptions is critical
- "Mirror image" between supply and demand uncertainty
  - Risk pooling vs. risk diversification
  - BWE vs. RBWE
  - etc.
- Research uses a variety of OR tools
  - Optimization (LP, IP, SP)
  - Stochastic processes
  - Simulation
  - Inventory theory
  - Behavioral studies

My Research Wish List

- Optimal inventory policies and settings for multi-echelon systems with disruptions
- Strategies for modeling and mitigating cascading of disruptions
  - Including RBWE and other phenomena
- Methods for identify bottlenecks/vulnerability points
- Mitigation strategies
  - Methods for identifying buffer points
  - Strategies for counteracting RBWE
- Good models (or approximations) that include both DU and SU
- Robust models: Insensitive to errors in disruption parameters
- Application to other complex systems

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- Working papers available at `www.lehigh.edu/~lvs2/research.html`
- BaseStockSim software available at `www.lehigh.edu/~lvs2/software.html`

Questions?

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