Continuous Approximation Model for Vehicle Routing -Tank Sizing Optimization

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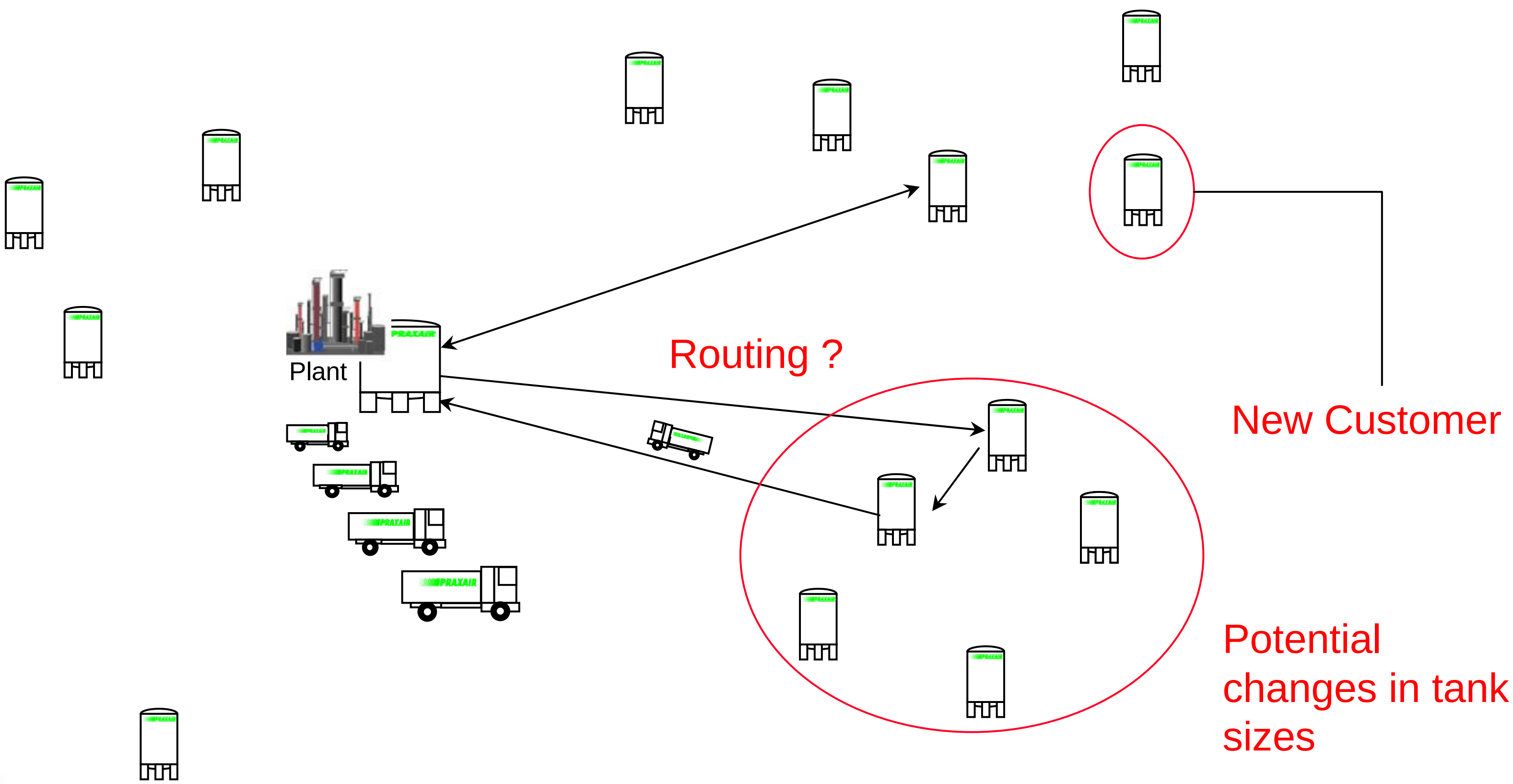
Tank Sizing Problem

- New customers need new tanks
- All or some of existing customers subject to tank upgrades or downgrades
- Different trailer sizes and available tank sizes



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Tank Sizing Issues

- Major Features
  - Key **tradeoff**: distribution (routing) vs. capital (tank sizing) cost
  - Capture the effects of customer synergies and tanker availability
- Nontrivial Problem
  - Non obvious ways of grouping customers for lower total costs
    - Tank size at a particular customer may influence distribution (routing) of **all** other customers
  - Many **thousands of** customer combinations are possible
    - Analyzing a small set of clusters at one time is useful
  - Lots of possible routes for each fixed tank sizing decision
    - Ex. a large customer may be delivered along with a small customer to empty tanker

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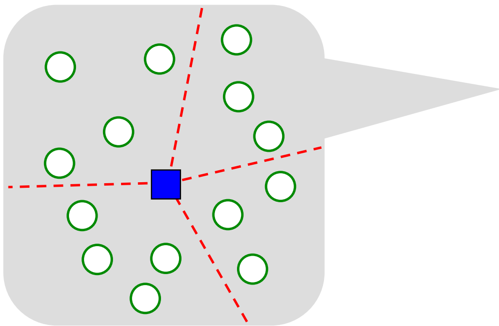
Algorithm Flowchart for Detailed Model



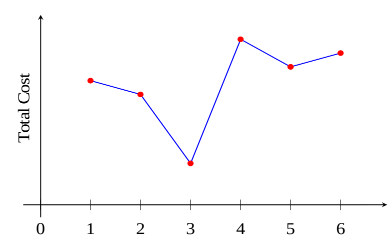
Customer Clustering

Select the first clustering solution

Detailed Integrated Model
(obtain routing and tank sizing decisions simultaneously)

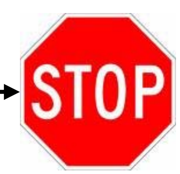
Pros: accurate

Cons: large scale MILP, long CPU time

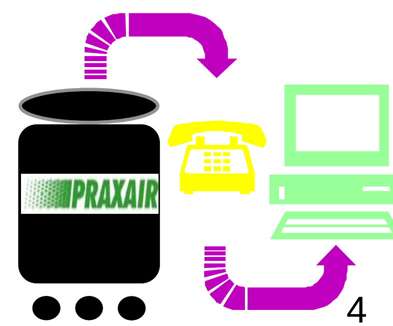


Next clustering solution

Termination?



Key Tradeoff: Routing Cost vs. Tank Cost



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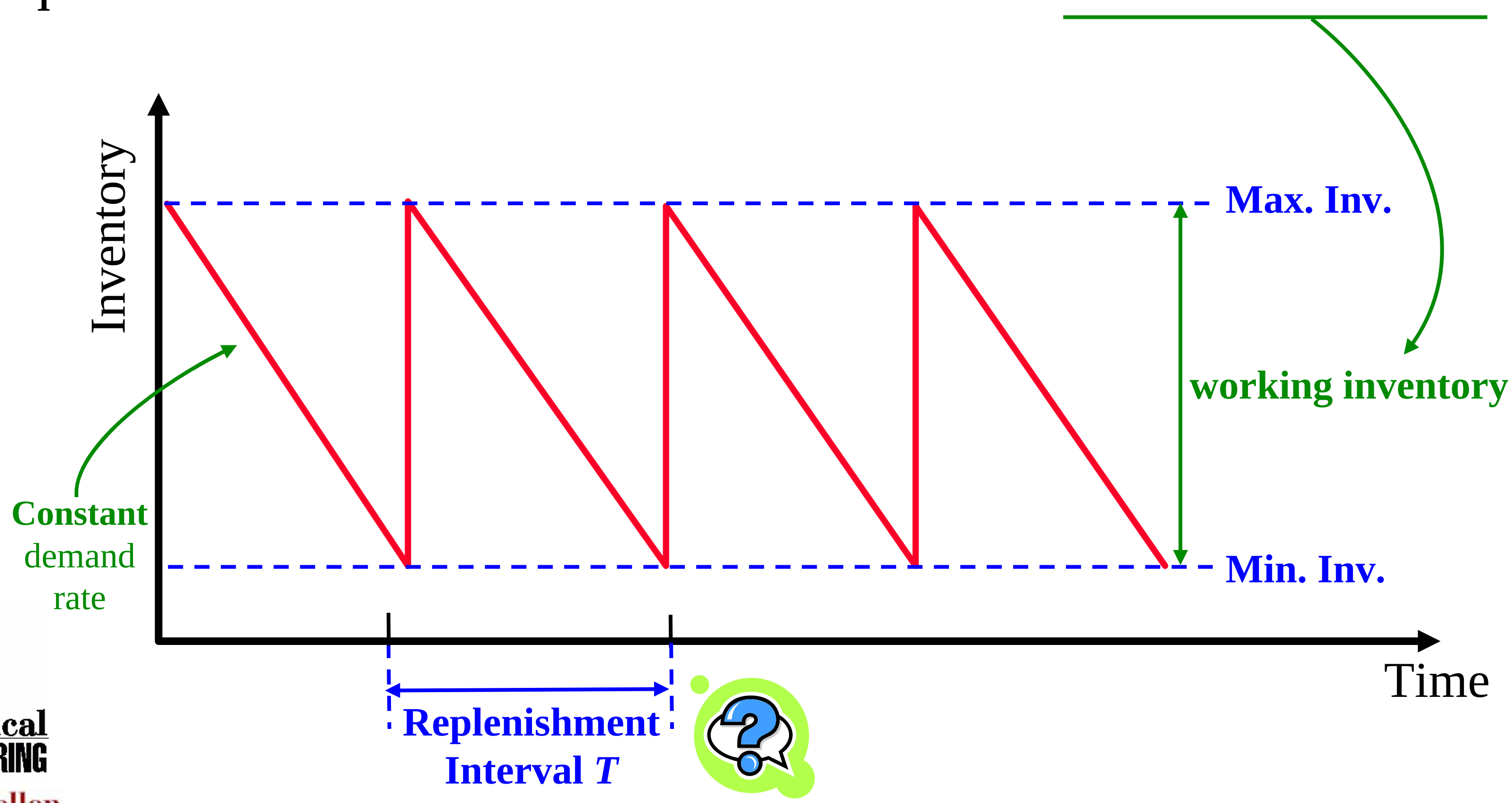
Why we need continuous approximation?

- Detailed Integrated Model (DIM)
  - Solve vehicle routing and tank sizing simultaneously
  - Tradeoff routing and tank sizing cost directly
  - Pros: more accurate result
  - Cons: very large scale MILP, long CPU time

- Continuous Approximation Model (CAM)
  - Approximate the routing cost in the long run (e.g. annually)
  - Tradeoff the tank sizing cost with approximated routing cost
  - Pros: smaller model, fast computation
  - Cons: total cost is approximated

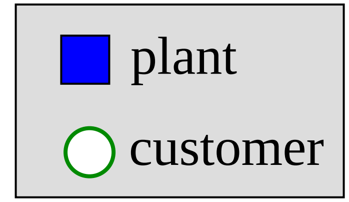
"Cyclic" Inventory-Routing in CAM

- Key Assumption: each customer is replenished in a "cyclic" way with fixed interval T
- Required tank size ≥ max. inv. = min. inv. + demand rate × T

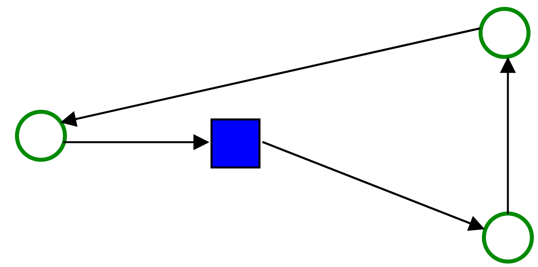


Routing & Replenishment in CAM

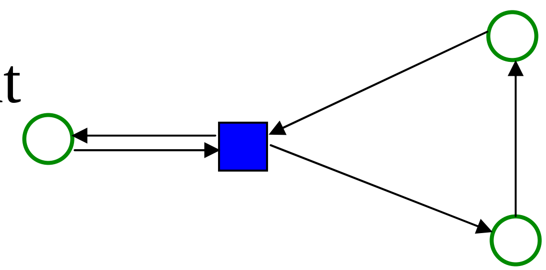
- $T = R$ / (ave. speed)
  - T - replenishment interval
  - R - minimum distance to replenish all the customer in a cluster once
  - Average travelling speed is known



- If only one trip for each replenishment
  - R = **TSP** distance of the cluster & plant



- If allowing multiple trips for replenishment
  - R = ?



CAM for Capacitated Routing Problems*

- Bounds for minimum routing distance ***R***

$$\max\{2\frac{n}{q}\cdot r,\ \mathrm{TSP}\} \le R \le 2\lceil \frac{n}{q} \rceil r + (1-\frac{1}{q})\cdot \mathrm{TSP}$$

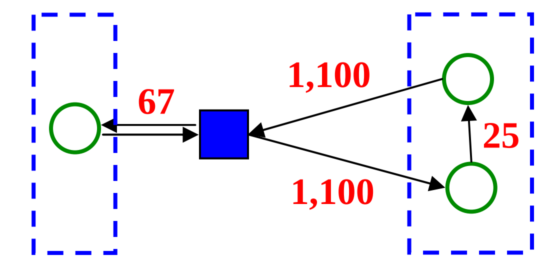
  - *n* – # of customers in the cluster
  - *q* – capacity, max. # of customers that can be visited in one trip or volume in terms of # of customers with unit demand
  - *r* – average distance from customers to the plant
  - TSP – traveling salesman distance to visit all customers once

- Examples
  - Cluster 1: *q*=1, TSP=0, *r* = 67

$$2\lceil \frac{n}{q} \rceil r + (1-\frac{1}{q})\cdot \mathrm{TSP} = 2r = 2\times 67 = \boxed{134\mathrm{km}}$$

  - Cluster 2: *q*=1, same as Cluster 1, $R = \boxed{4,400\mathrm{km}}$
  - Cluster 2: *q*=2, TSP=50, *r* = 1,100

$$2\lceil \frac{n}{q} \rceil r + (1-\frac{1}{q})\cdot \mathrm{TSP} = 2r + \frac{\mathrm{TSP}}{2} = \boxed{2,225\mathrm{km}}$$



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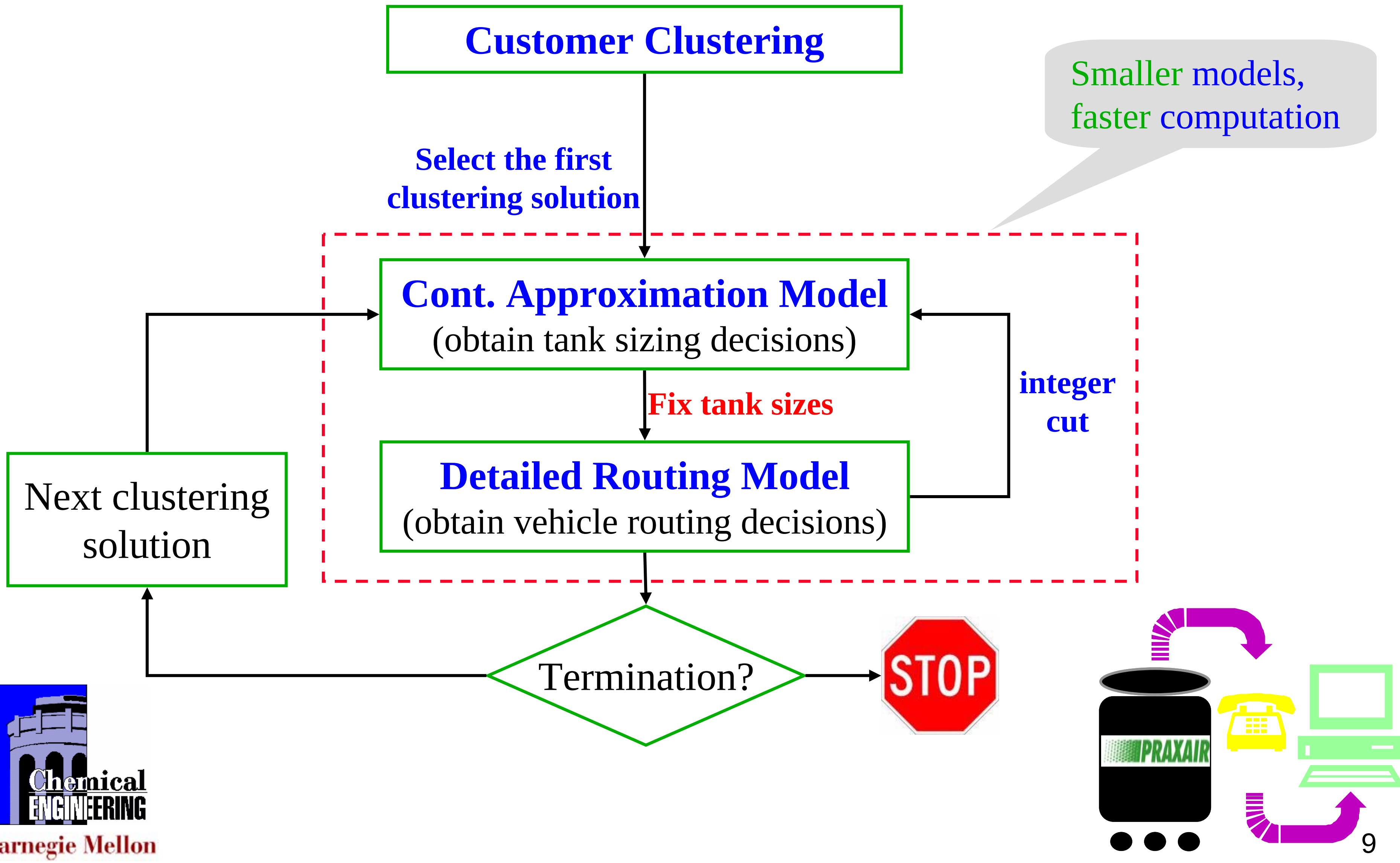
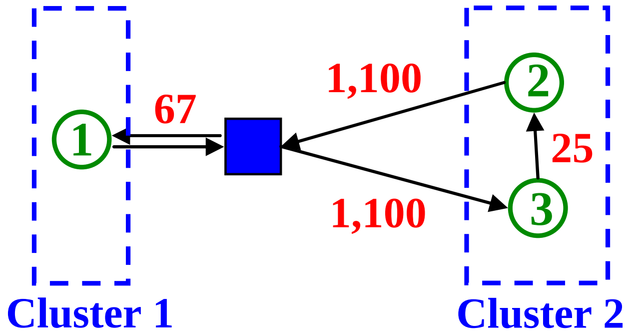
** M Haimovich, AHG Rinnooy Kan, "Bounds and heuristics for CRP", Math. of Oper. Res., 1985, 10(4), 527-541*

Algorithm Flowchart for CAM



Example: Comparison of DIM and CAM



- Problem Size
  - 3 customers (② is new) in 2 clusters
  - 6 available tank size, 4 types of trucks
- Detailed Integrated Model (DIM)
  - CPU time: ~ 8 min. (5% gap)
    - 747 disc. Var., 1,606 cont. var., 2,121 constraints
  - Total cost: $23,087
    - Upgrade ① by 6,000 L, add a new one for ② with 10,000 L, no change for ③
- Continuous Approximation Model (CAM)
  - CPU time: ~ 1 sec. for CAM, ~ 5 sec. for routing problem (5% gap)
    - 47 disc. Var., 26 cont. var., 35 constraints (CAM)
  - Total cost: $23,405, **Same** tank sizing decisions

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Conclusion / Future Work

- Conclusion
  - Continuous approximation model (CAM) for vehicle routing – tank sizing problem
  - Novel algorithm framework to reduce the computational effort w/o too much sacrifice in solution quality
- Future Work
  - Simplify MINLP model to MILP model
  - Bi-level decomposition algorithm
  - Consider uncertainties such as demand variation and adding or losing customers