Integrated Vehicle Routing -Tank Sizing Problem with Safety Stock Optimization

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PRAXAIR

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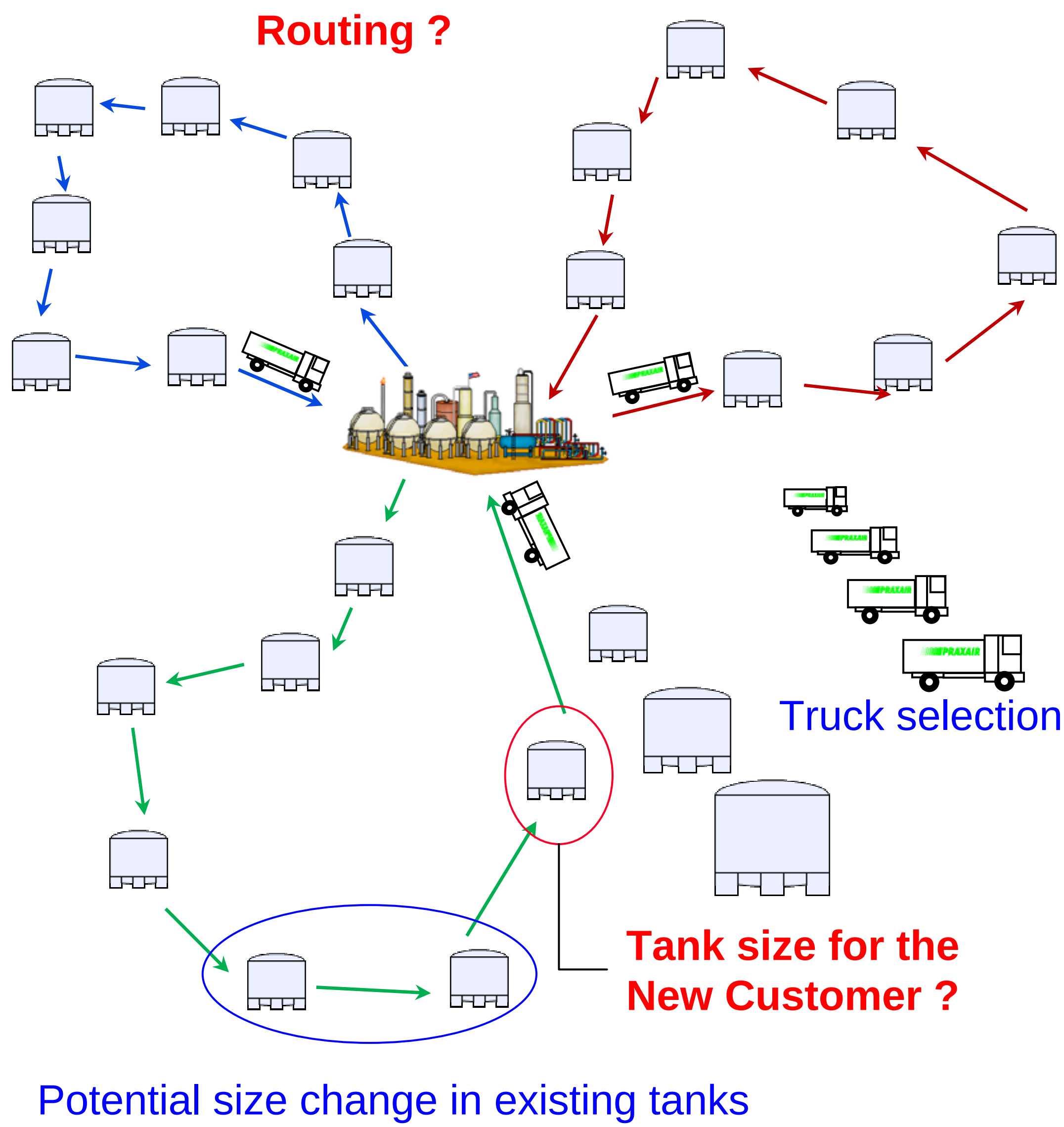
Vehicle Routing – Tank Sizing Problem

- **Tank Sizing:**
  - New customers need new tanks to be sized
  - All or some of existing customers subject to tank upgrades or downgrades
  - Selection among several available tank sizes
- **Vehicle Routing**
  - Selection among several truck sizes
  - Determine the routing and timing decisions



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Tank Sizing Issues

- **Major Features**
  - **Key tradeoff**: capital (tank sizing) vs. distribution (routing) cost
  - Capture the effects of customer synergies and truck availability
- **Challenges**
  - Routing decisions and tank sizing decisions should be integrated
    - A small change of tank size at a particular customer may influence distribution (routing) of **all** other customers
    - Many possible routes for each fixed tank sizing decision
  - Requires fast computational strategies for good solutions
    - Direct MILP model leads to large problem (millions of binary variables)
    - Difficult to obtain "good" solution in reasonable times
  - Account for the fluctuation of customer demand

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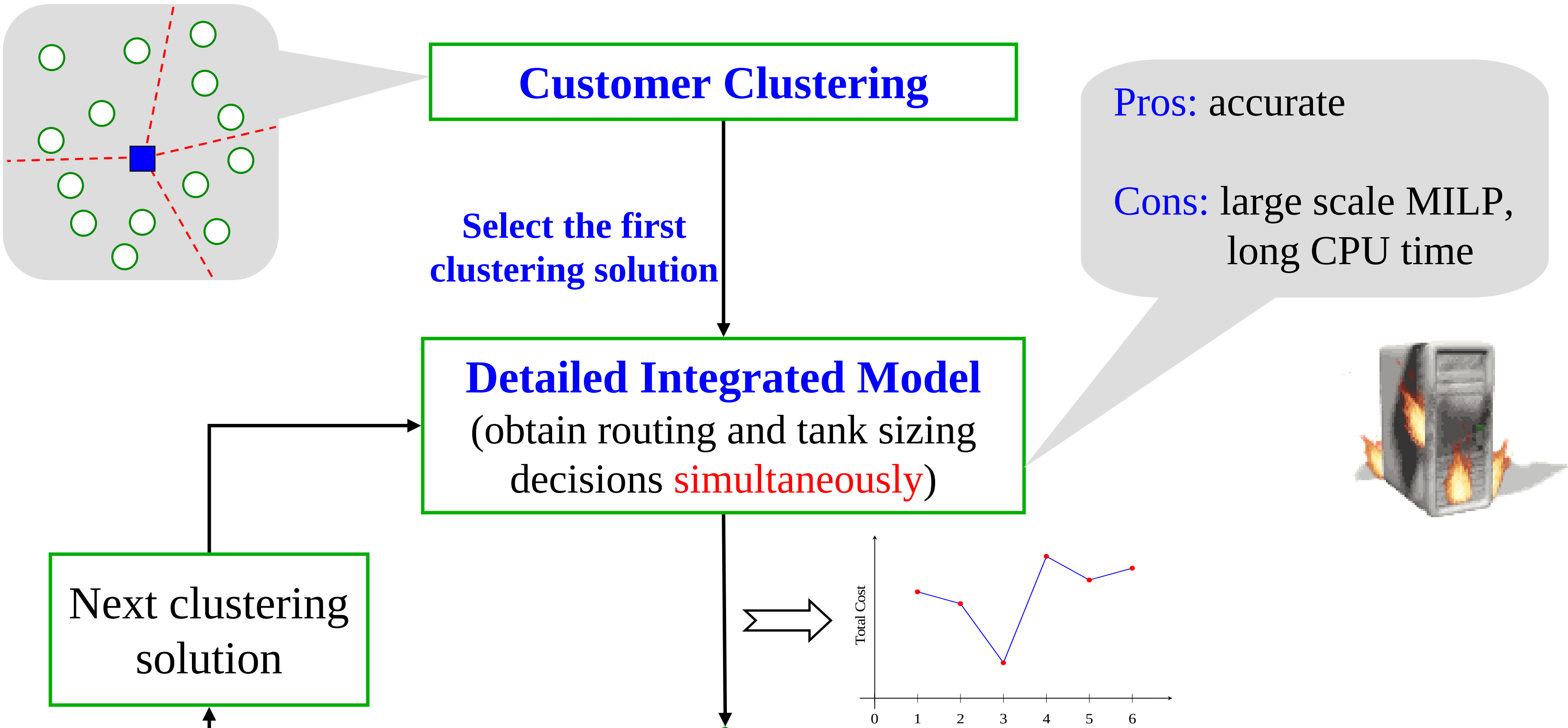
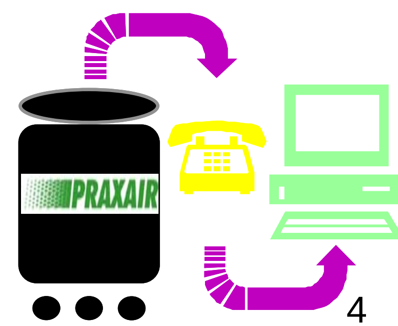
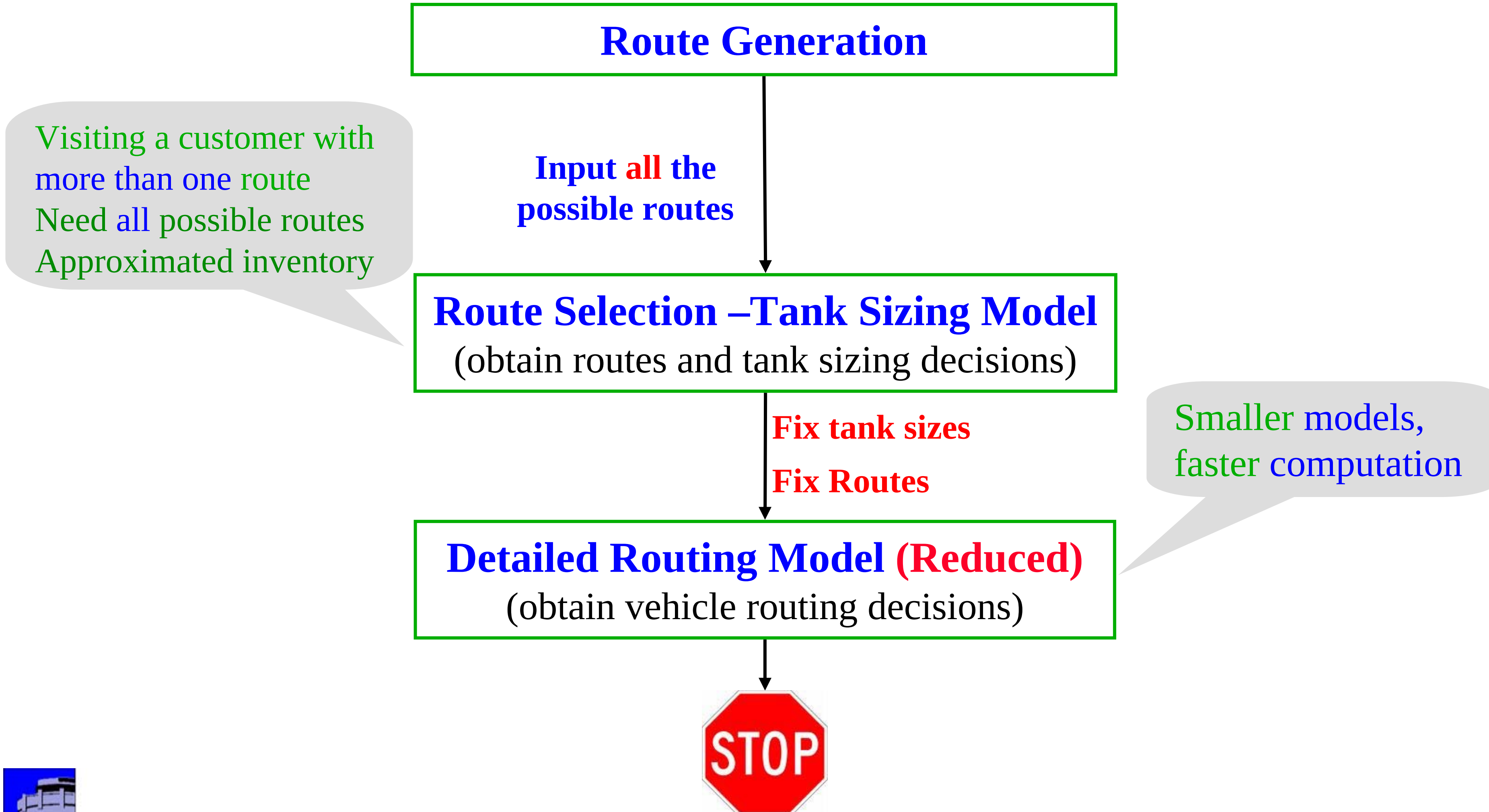
Clustering - Integrated Approach



Key Tradeoff: Routing Cost vs. Tank Cost



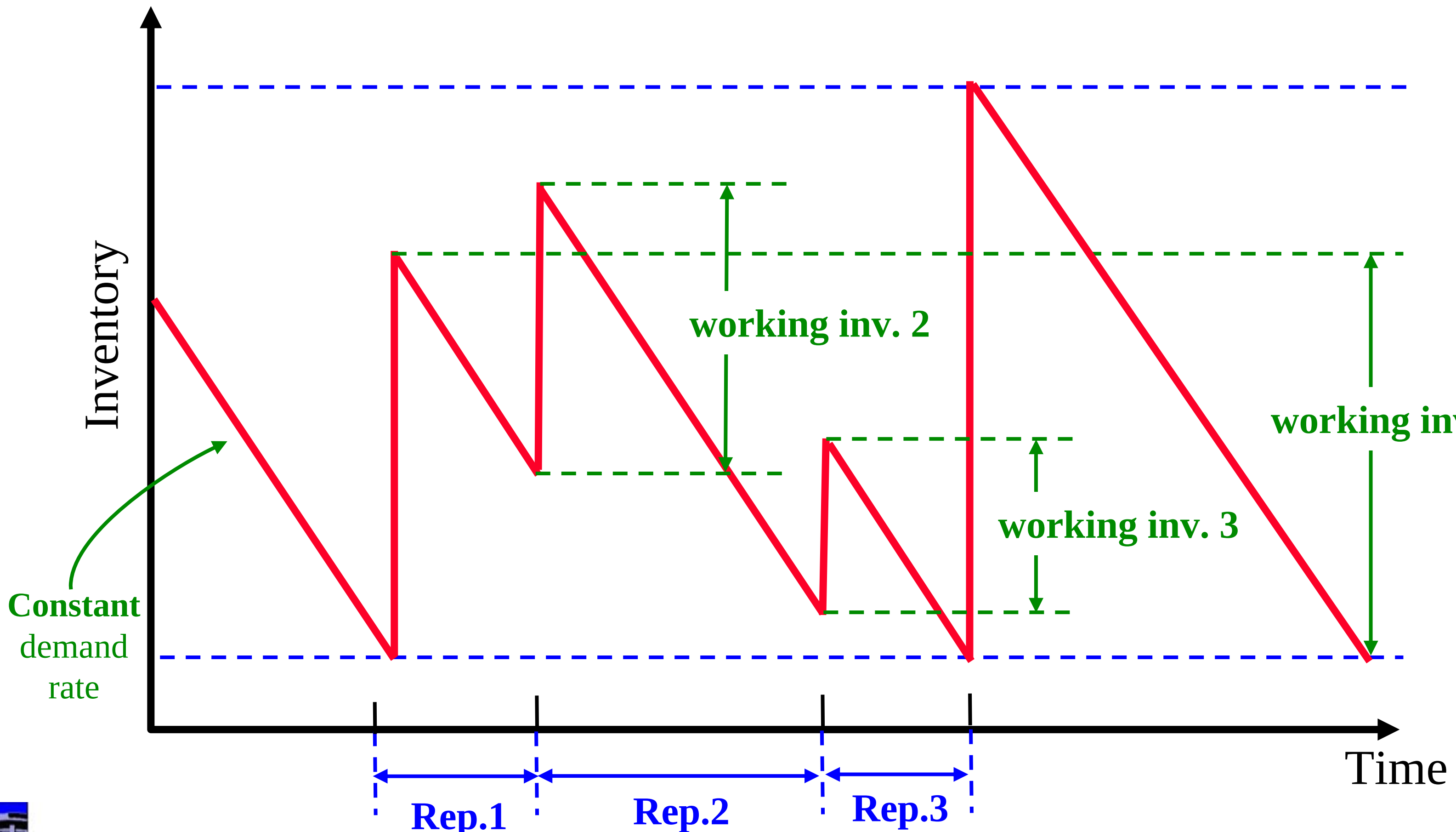
Route Selection – Tank Sizing Approach



Pros: fast for routing **Cons:** hard to estimate “global” gap

Inventory for Multiple Replenishments

Tank size = max. Inv. ≈ Min. Inv. + max{working Inv.}



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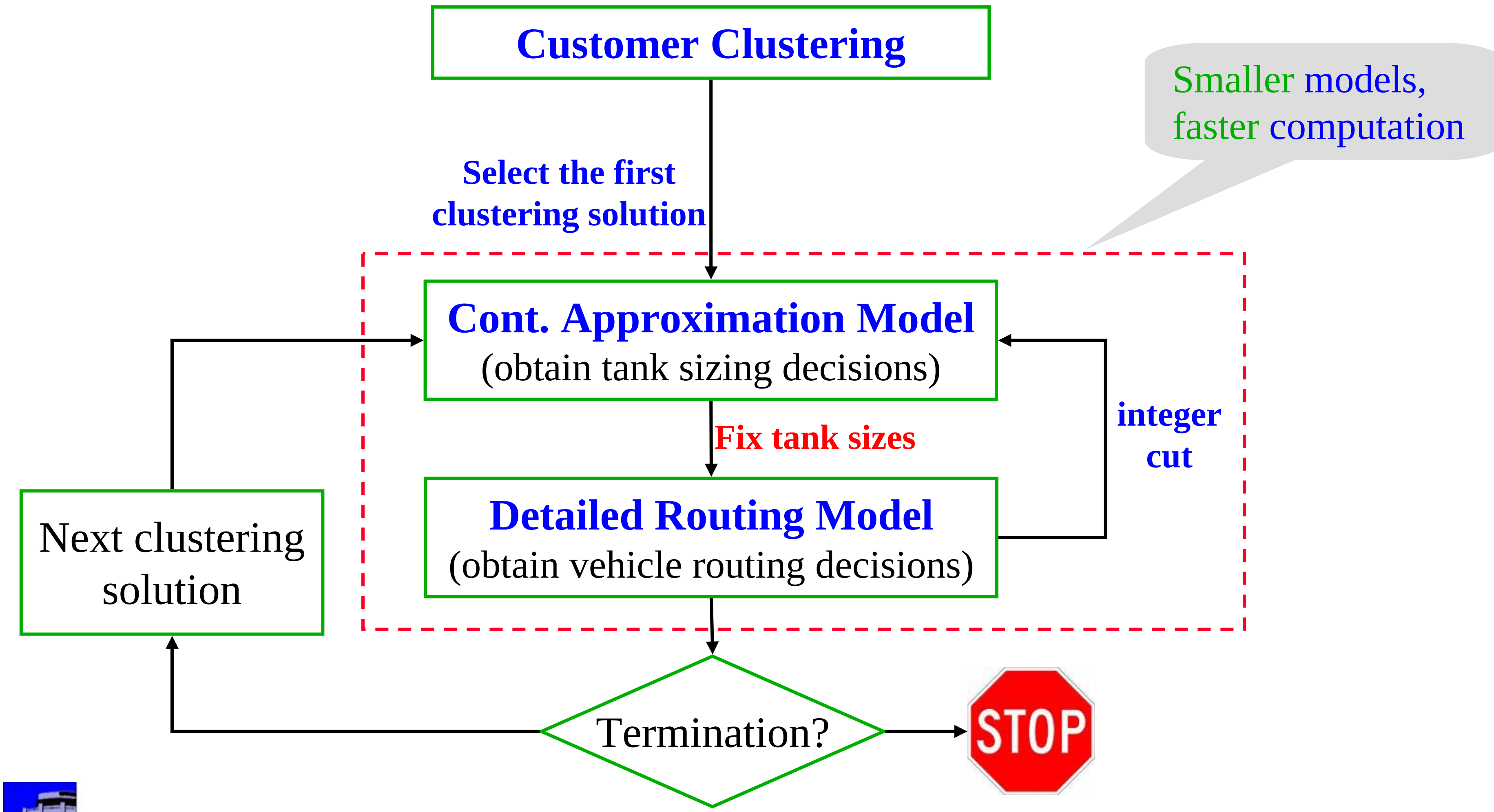
Continuous Approximation Approach



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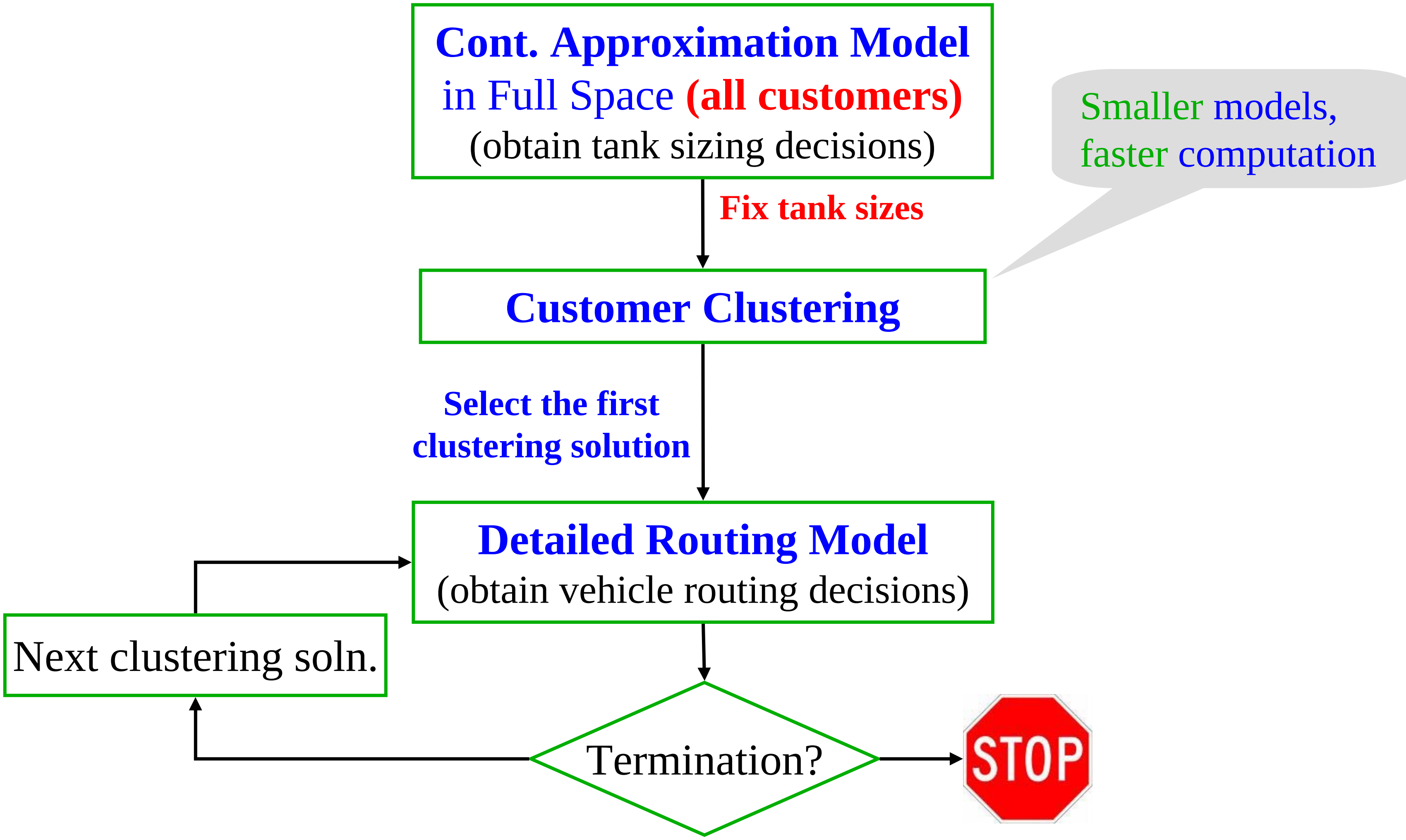
Pros: fast computation **Cons:** hard to estimate "global" gap

Improved CAM Approach



Note: more accurate, still fast, need to improve routing model

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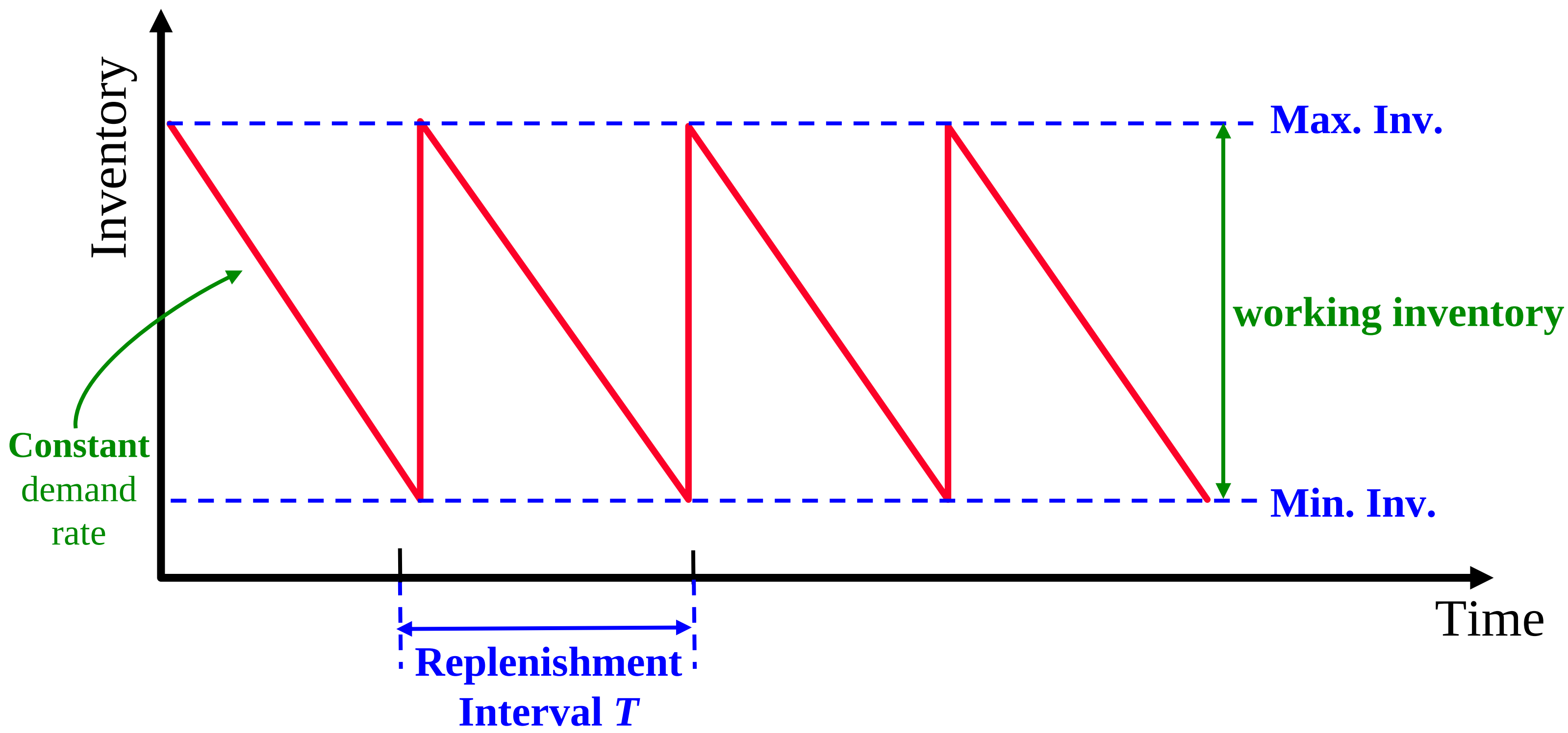
"Cyclic" Inventory-Routing in CAM

- Key Assumption: each customer is replenished in a "cyclic" way with fixed interval *T*
- Required tank size ≥ max. inv. = min. inv. + working inventory



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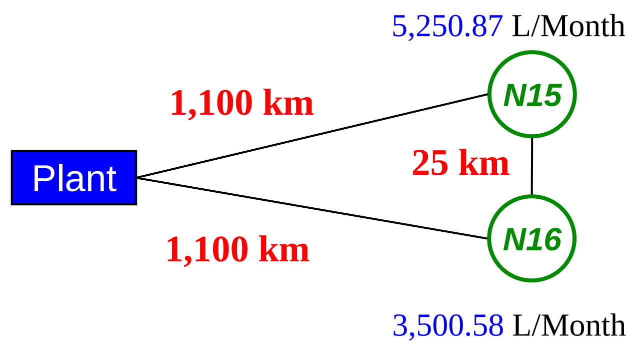
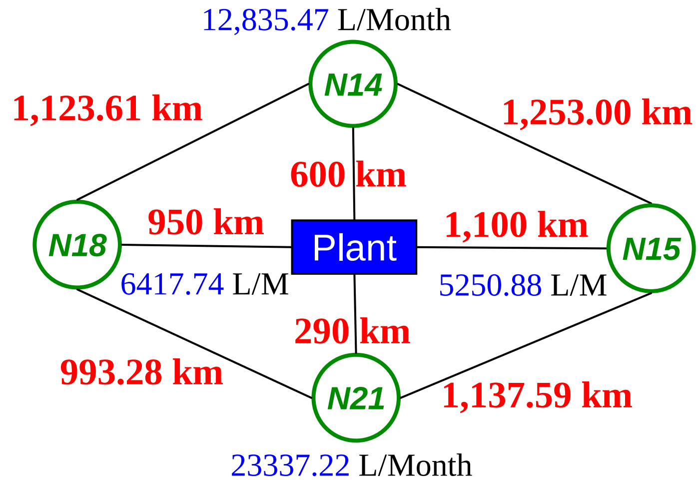
Deterministic Case: Example 1

- Problem Size:
  - A cluster with 2 customers, *N15* is new
  - *N16* has an existing tank of 13,000 L
  - 6 available tank size, 4 types of trucks



- Integrated Model
  - CPU time: ~ 8 min. (0% gap)
  - Total cost: $18,112, Tank size: 13,000 L for *N15*, no change for *N16*
- Routing Selection – Tank Sizing (RSTS)
  - CPU time: 43 sec. for RSTS, 53 sec. for routing problem (0% gap)
  - Total cost: $19,085, Tank size: 6,000 L for *N15*, no change for *N16*
- Continuous Approximation Model (CAM)
  - CPU time: < 1 sec. for CAM, 74 sec. for routing problem (0% gap)
  - Total cost: $18,112, Tank size: 13,000 L for *N15*, no change for *N16*

Deterministic Case: Example 2



- Problem Size:
  - A cluster with 4 customers, *N14* is new
  - All customers have a tank of 10,000 L
  - 6 available tank size, 4 types of trucks
- Integrated Model
  - CPU time: ~15 hours (4.39% gap, out of memory)
  - Total cost: $30,620; Tank size: 16,000 L for *N14*, no change for others
- Routing Selection – Tank Sizing (RSTS)
  - CPU time: ~10 hours. for RSTS (out of memory)
- Continuous Approximation Model (CAM)
  - CPU time: < 1 sec. for CAM, 2360 sec. for routing problem (0% gap)
  - Total cost: $30,299; Tank size: 16,000 L for *N14*, no change for others

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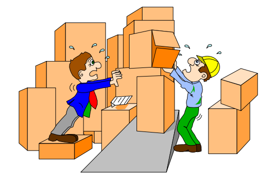
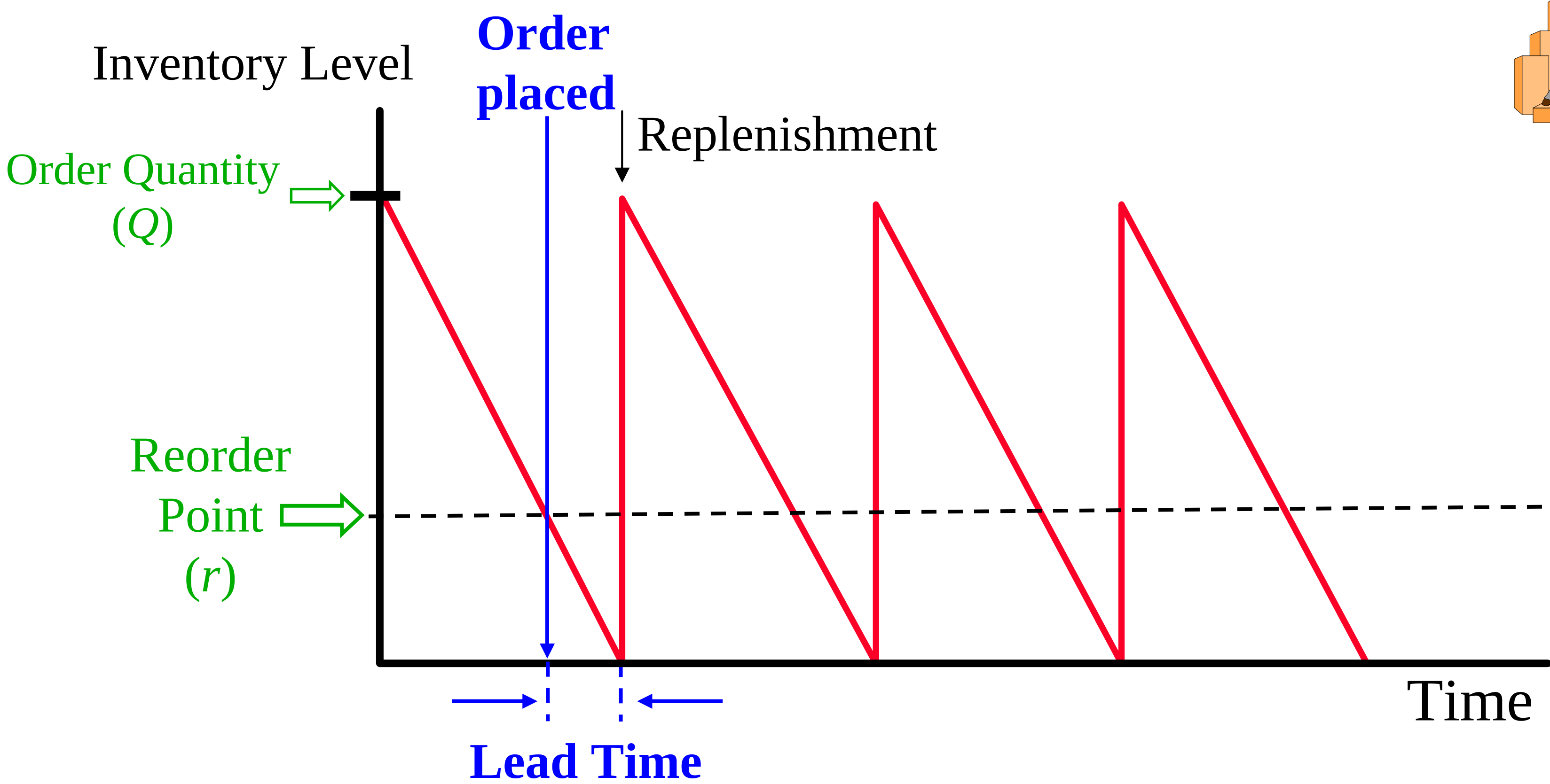
Inventory Policy

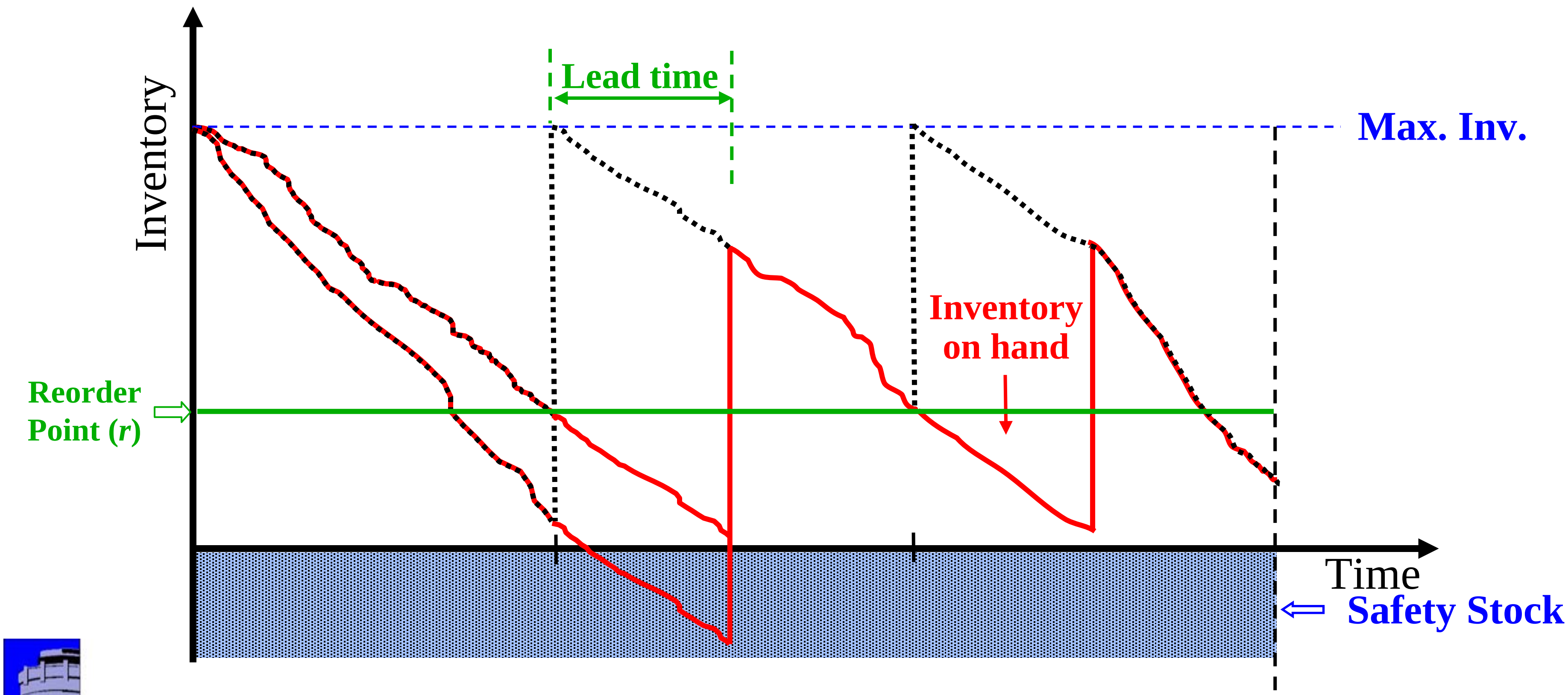




- **(*r*, *Q*) Inventory Policy**
  - When inventory level falls to *r*, order a quantity of *Q*
  - Reorder Point (*r*) = Demand over Lead Time

Stochastic Base-Stock Inventory Model

Maximum Inv. = Working Inv. + Safety Stock



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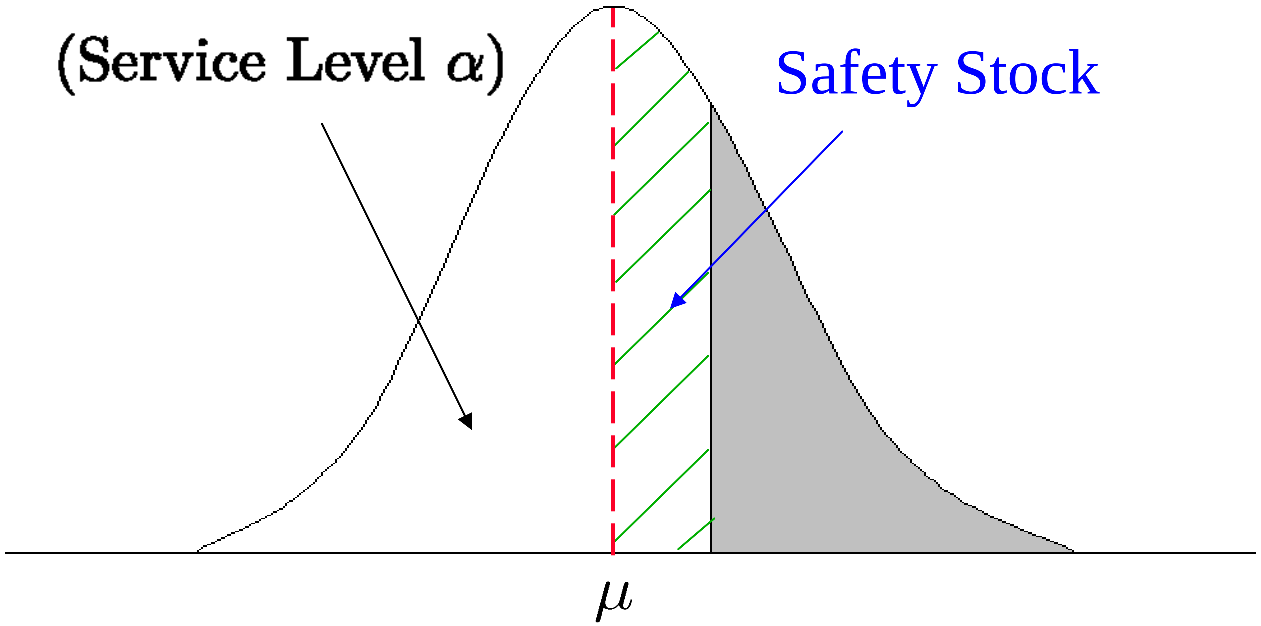
Safety Stock Level

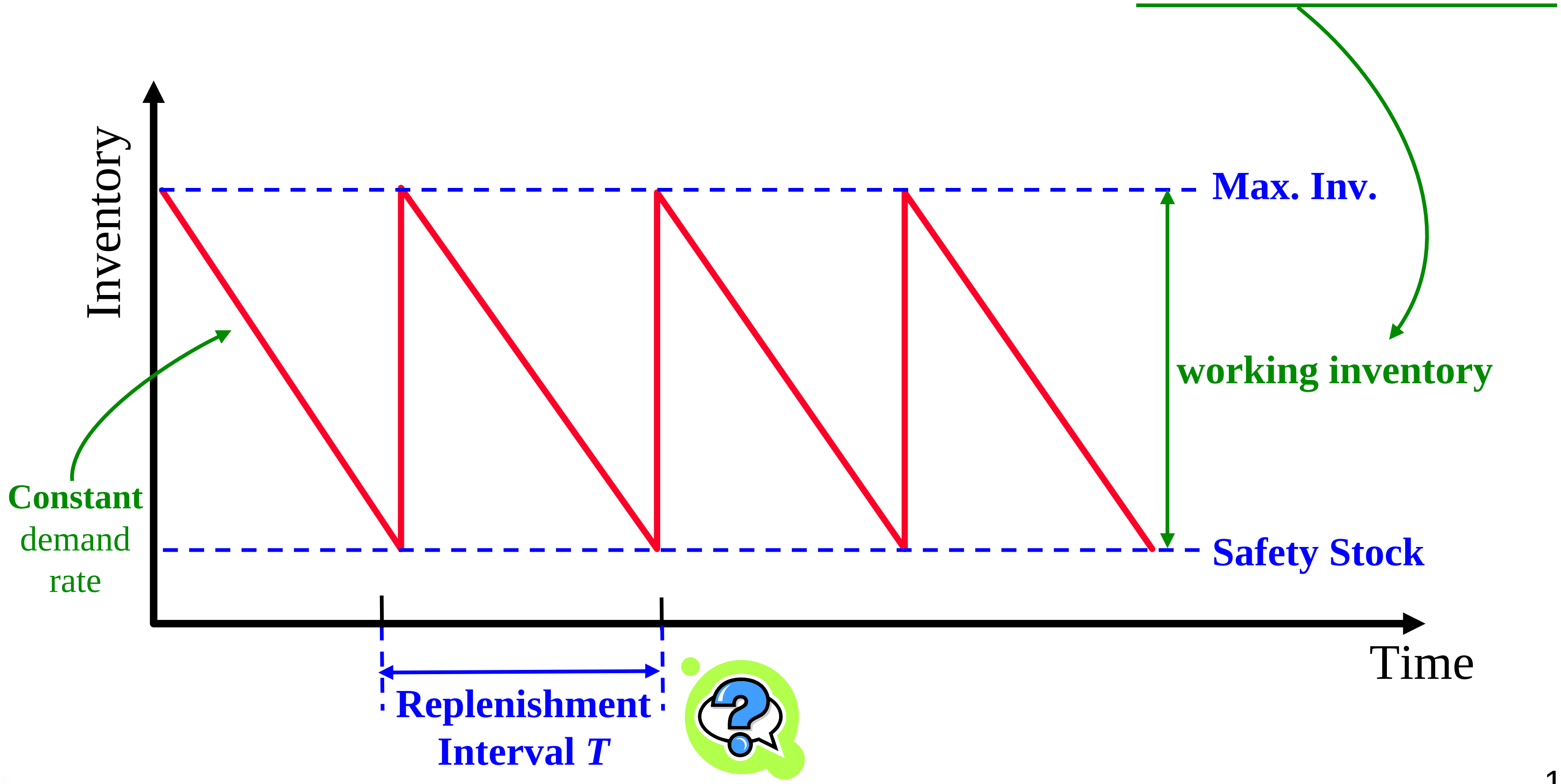
$D \sim N(\mu, \sigma^2) \Longrightarrow$ Safety Stock $= \boxed{z_\alpha \sigma}$, $P(z \leq z_\alpha) = \alpha$ **(Service Level)**

Lead time = *T* $\Rightarrow D \sim N(T \cdot \mu, T \cdot \sigma^2) \Rightarrow$ $\boxed{\text{Safety Stock} = z_\alpha \sigma \sqrt{T}}$



"Cyclic" Inventory-Routing in CAM

- Key Assumption: each customer is replenished in a "cyclic" way with fixed interval T
- Required tank size ≥ max. inv. = Safety Stock + demand rate × T



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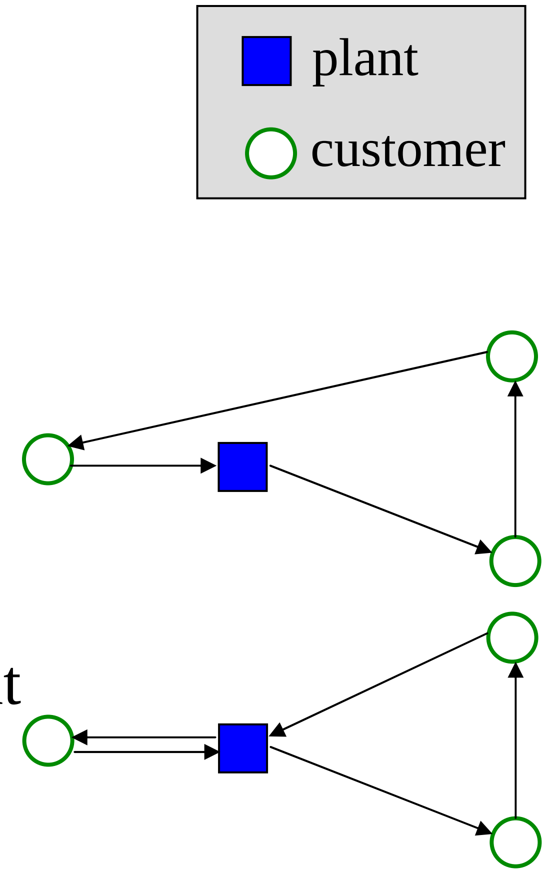
Routing & Replenishment in CAM

- $T = R$ / (ave. speed)
  - T - replenishment interval
  - R - minimum distance to replenish all the customer in a cluster once
  - Average travelling speed is known



- If only one trip for each replenishment
  - R = **TSP** distance of the cluster & plant
- If allowing multiple trips for replenishment
  - R = ?

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CAM for Capacitated Routing Problems*

- Bounds for minimum routing distance ***R***

$$\max\{2\frac{n}{q}\cdot r,\ \text{TSP}\} \le R \le 2\lceil \frac{n}{q} \rceil r + (1-\frac{1}{q})\cdot \text{TSP}$$

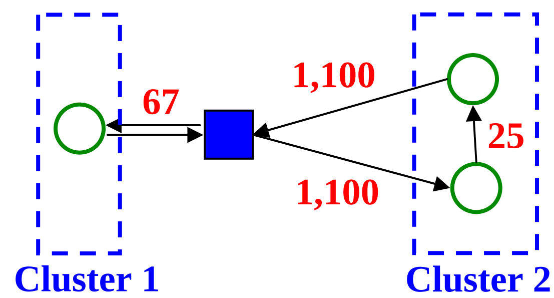
  - *n* – # of customers in the cluster
  - *q* – capacity, max. # of customers that can be visited in one trip or volume in terms of # of customers with unit demand
  - *r* – average distance from customers to the plant
  - TSP – traveling salesman distance to visit all customers once

- Examples
  - Cluster 1: *q*=1, TSP=0, *r* = 67

$$2\lceil \frac{n}{q} \rceil r + (1-\frac{1}{q})\cdot \text{TSP} = 2r = 2\times 67 = \boxed{134\text{km}}$$

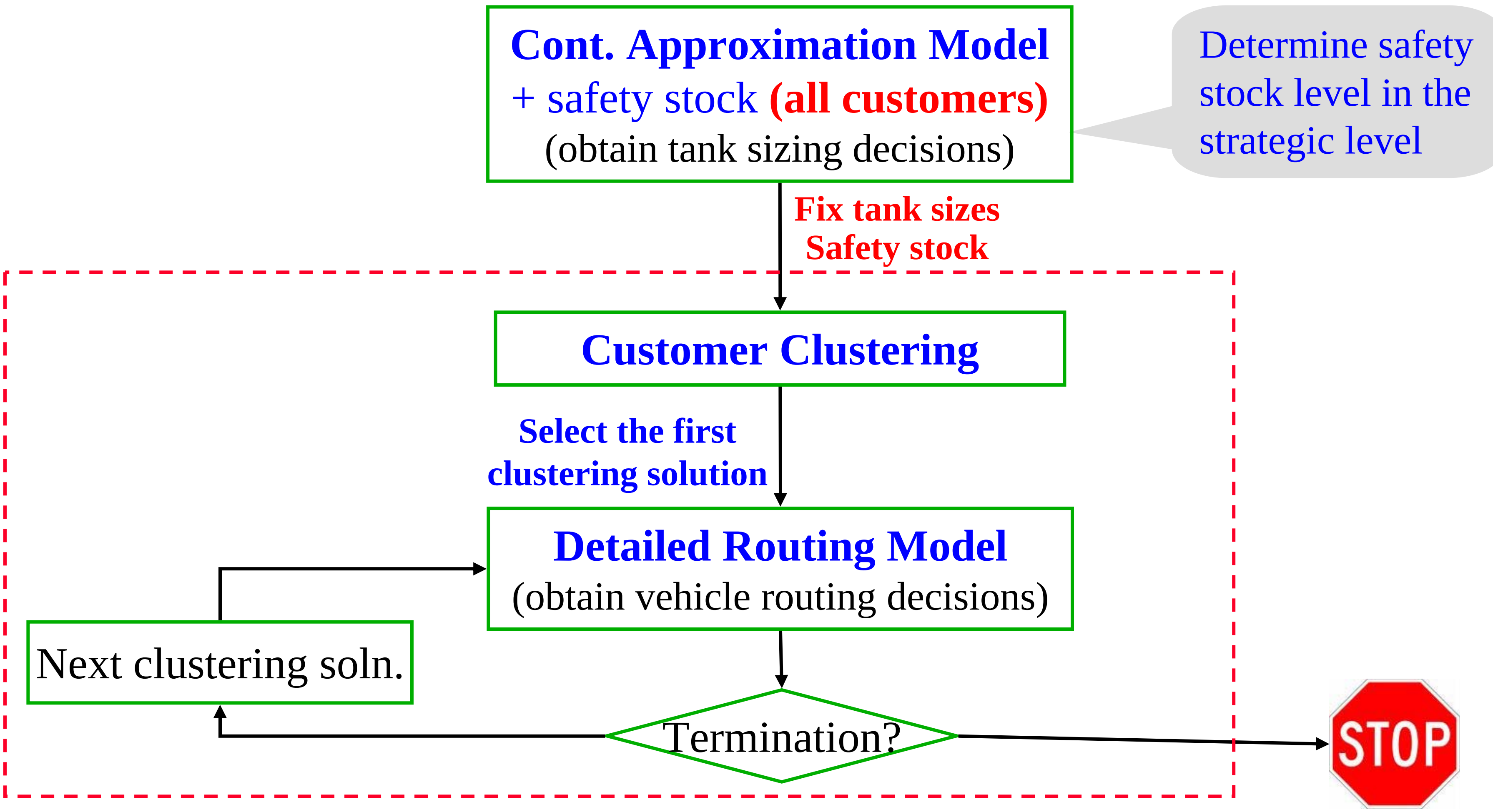
  - Cluster 2: *q*=1, same as Cluster 1, $R = \boxed{4,400\text{km}}$
  - Cluster 2: *q*=2, TSP=50, *r* = 1,100

$$2\lceil \frac{n}{q} \rceil r + (1-\frac{1}{q})\cdot \text{TSP} = 2r + \frac{\text{TSP}}{2} = \boxed{2,225\text{km}}$$



* *M Haimovich, AHG Rinnooy Kan, "Bounds and heuristics for CRP", Math. of Oper. Res., 1985, 10(4), 527-541*

Improved CAM Approach



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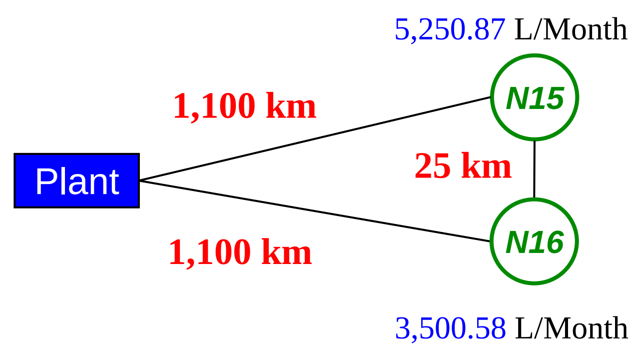
Stochastic Case: Example 3

- Problem Size:
  - A cluster with 2 customers, *N15* is new
  - 6 available tank size, 4 types of trucks
  - Demands follow normal dist. ($\sigma=\mu$)



- Fix safety stock to 15% of tank size
  - Total cost: $16,792; Tank size: 13,000 L for *N15*, no change for others
  - Safety stocks: *N15*: 1,950 L, *N16*: 1,950 L
  - Service level: *N15*: ~100%, *N16*: ~100%,
- Safety stock optimization (97.75% service level)
  - Total cost: $16,646; Tank size: 13,000 L for *N15*, no change for others
  - Safety stocks: *N15*: 1,189L, *N16*: 793L
- What is the implications of different total costs and tank sizes?

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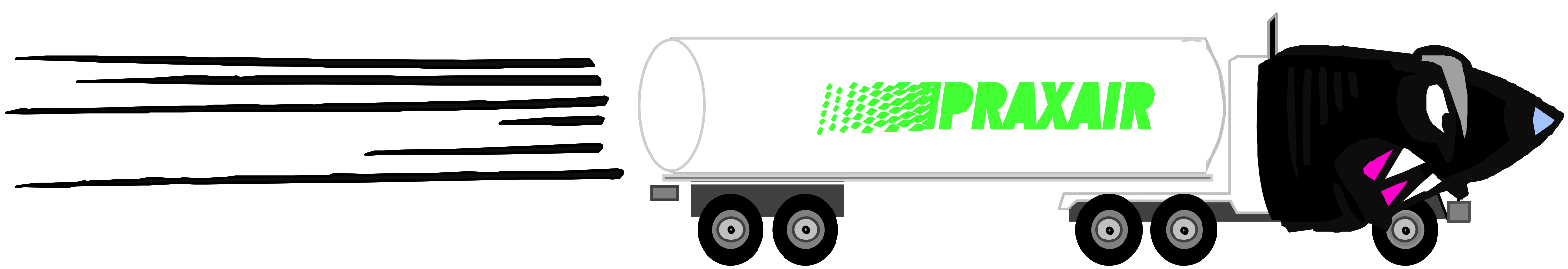
Conclusion / Future Work

- Conclusion
  - Two approaches to reduce the computational efforts for the integrated vehicle routing – tank sizing problem
  - Integrate safety stock optimization in the continuous approximation approach to account for demand fluctuation
- Future Work
  - Refine the models and validate the assumptions
  - Consider uncertainties such as adding or losing customers
  - Quantify the economic benefits of considering uncertainties



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