Next Generation Modeling and Optimization of Integrated Site Designs

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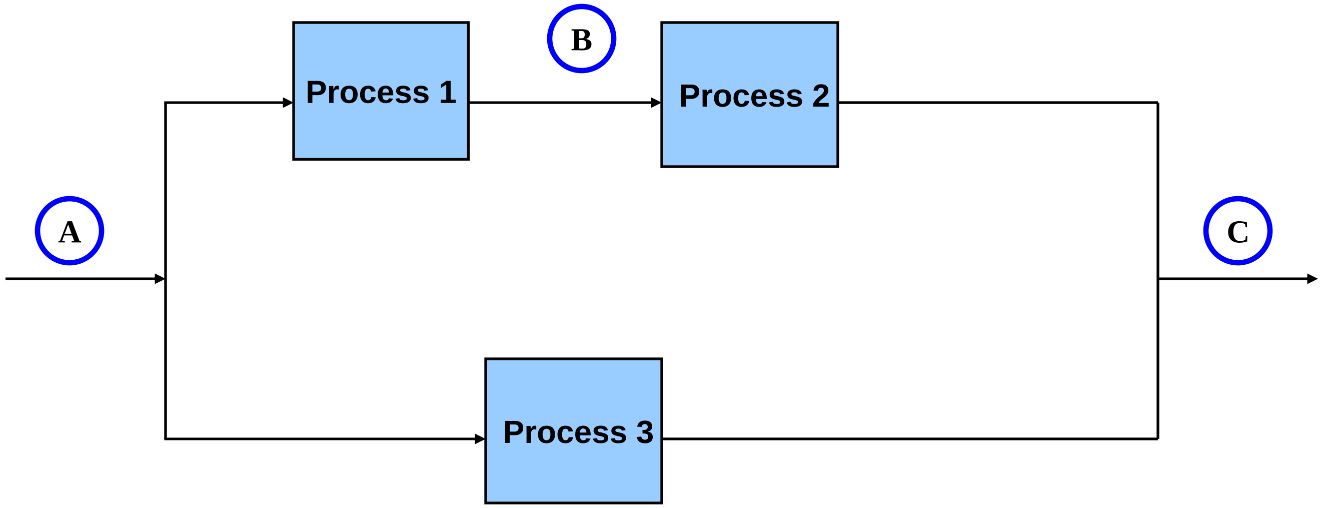
Objective:

Present a systematic method for optimizing the long term availability of an integrated site.

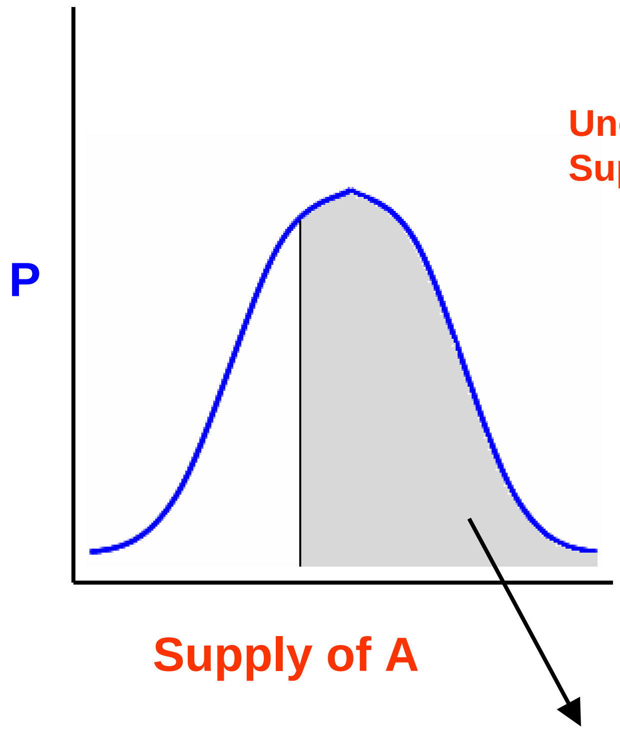
Model an integrated site as a network of processes

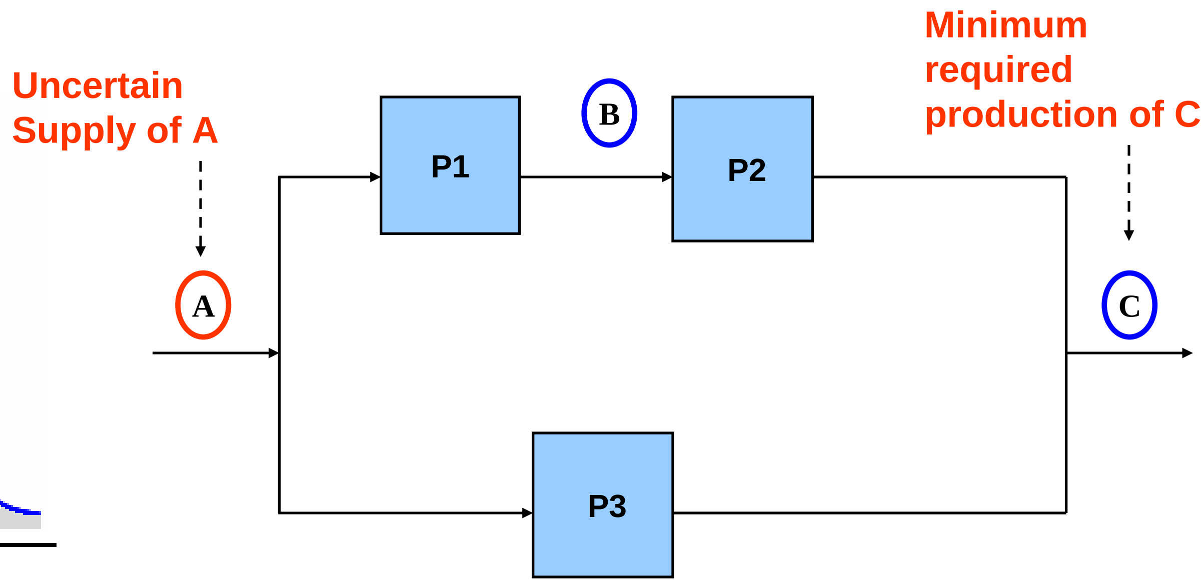


Processes in the network are subject to:

- **Continuous uncertain (e.g. supply of A, demand of C)**
- **Discrete uncertainties (e.g. failure of process 3)**

Stochastic flexibility: Ability to operate under cont. uncertainties



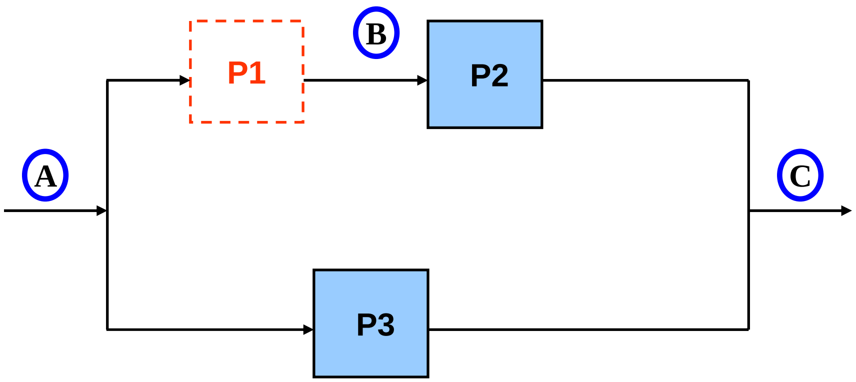


Stochastic Flexibility (SF) represents area of feasible operation under probability distribution

e.g. SF = 0.67

Discrete uncertainties modeled using discrete states

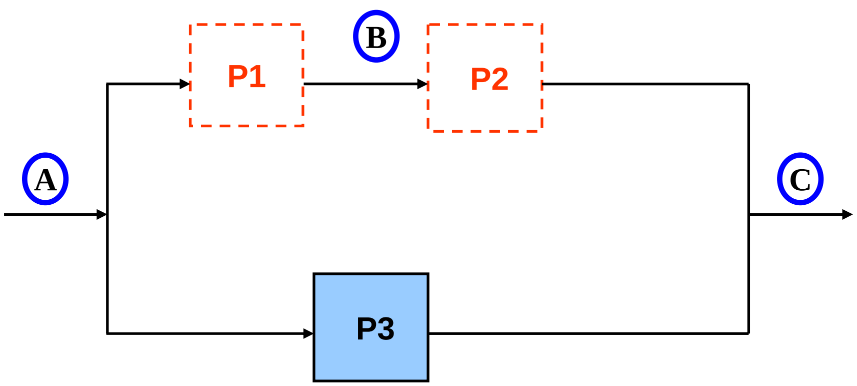
A discrete state is described by sets of failed and active processes



State 1

Active Processes = { P2, P3 }

Failed Processes = { P1}

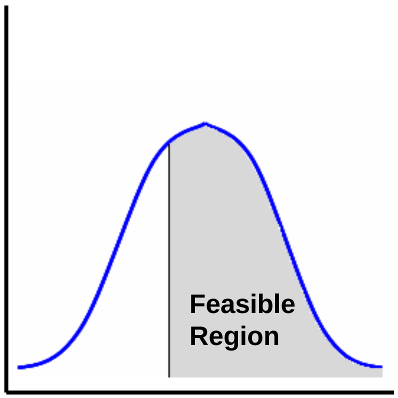


State 2

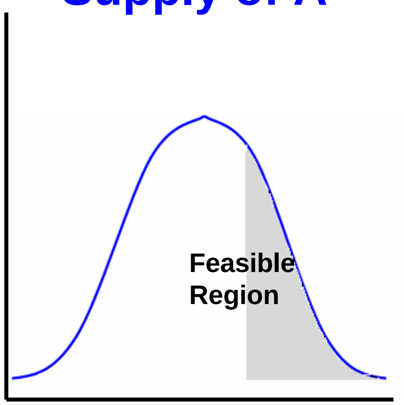
Active Processes = { P3 }

Failed Processes = { P1, P2 }

Expected Stochastic Flexibility E(SF) considers discrete states



Supply of A



Define:

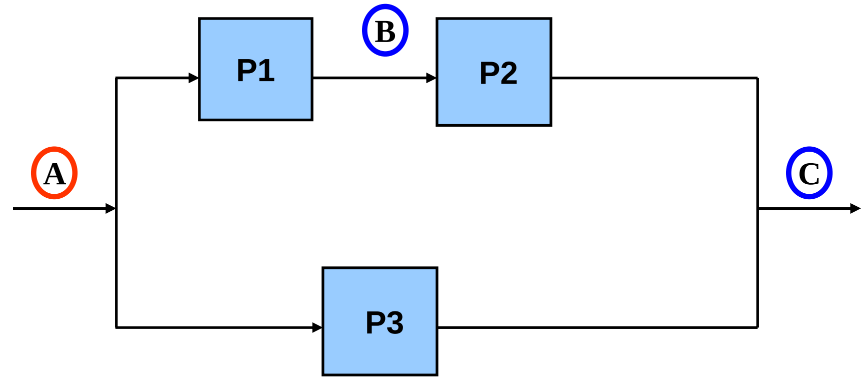
p_1: probability of finding system in State 1

SF_1 : Stochastic Flexibility in State 1

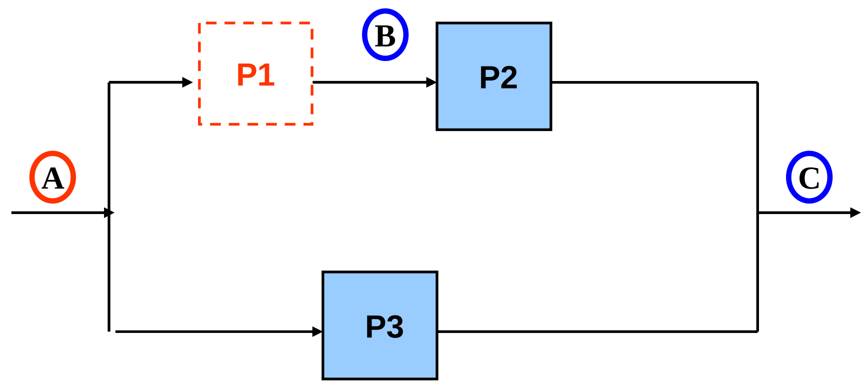
p_2: probability of finding system in State 2

SF_2 : Stochastic Flexibility in State 2

State 1



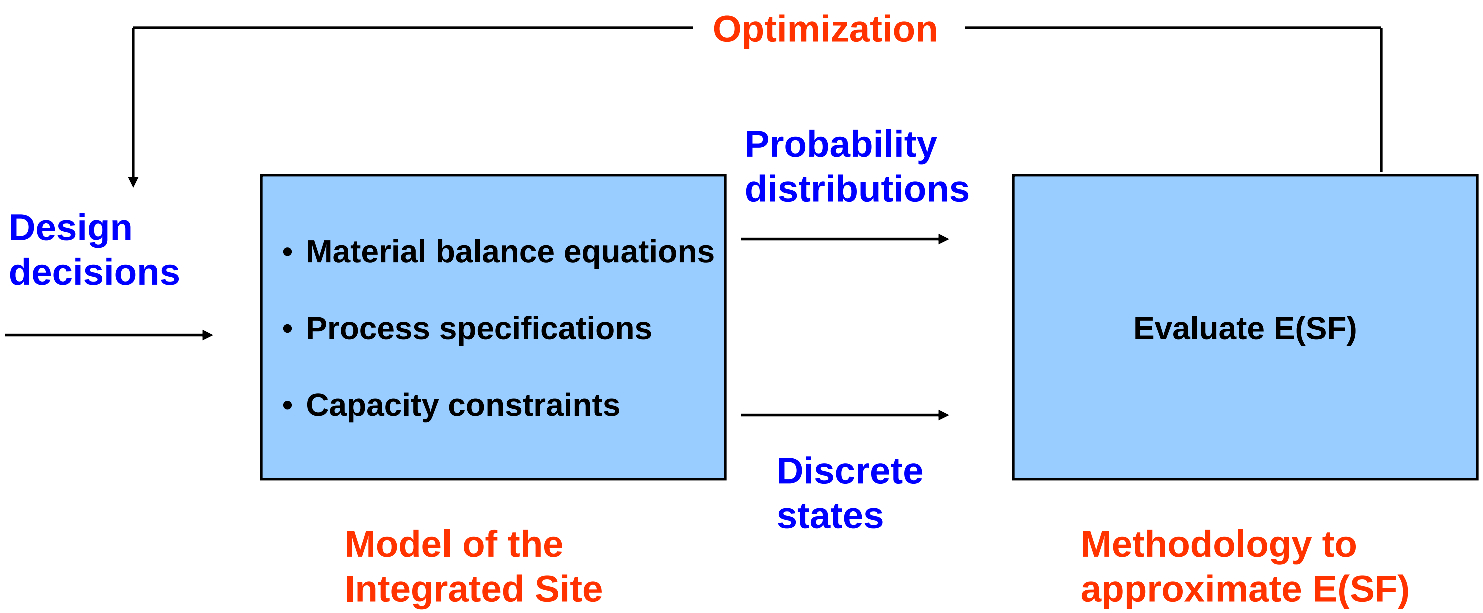
State 2



$$E(SF) = p_1SF_1 + p_2SF_2$$

Use E(SF) and process model to optimize availability

Mathematical programming approach requires a process model and methodology for evaluating E(SF)



Maximizing E(SF) = Maximizing probability of feasible operation

= Maximizing long term availability

Problem definition

Given:

- A network of processes and their capacities
- A set of products produced in the network
- A set of possible failures, failure rates & repair rates
- An external demand of products and external supply of raw materials
- A set of uncertain parameters and their probability distributions
- A maximum amount of resources available for design decisions

Determine:

- Location of storage tanks
- Maximum storage capacity and steady state inventory levels
- Process capacity expansions

In order to maximize the expected stochastic flexibility

Process Model in general form

Material balance equations — $h(x, d, \theta, \alpha, y) = 0$

Process capacity

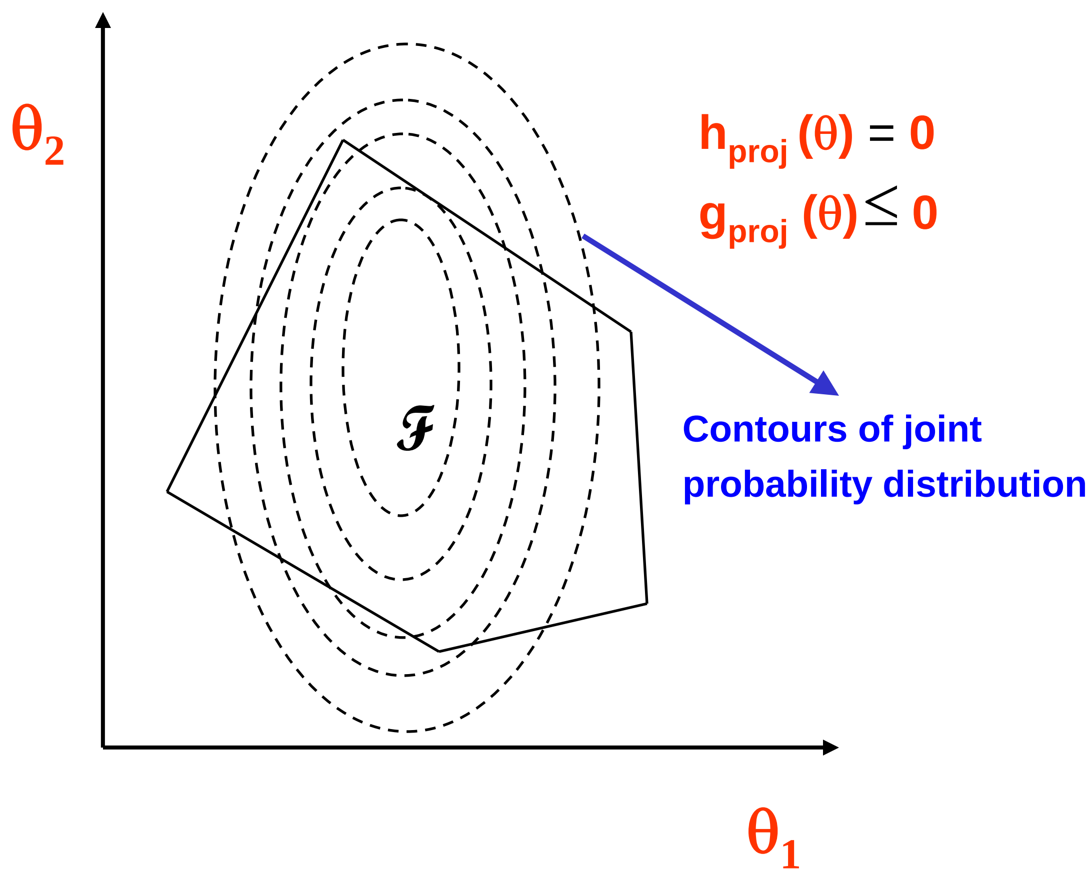
Intermediate storage capacity — $g(x, d, \theta, \alpha, y) \leq 0$

Process specifications

- **x: continuous process variables (flows)**
- **d: integrated design variables (intermediate storage, etc.)**
- **θ: uncertain parameters (demand, supply, etc.)**
- **α: deterministic parameters (unit ratio coefficients, etc.)**
- **y: 0 – 1parameters; 1 – active process, 0 – failed process**

Projection of g( ) and h( ) onto uncertain parameter space

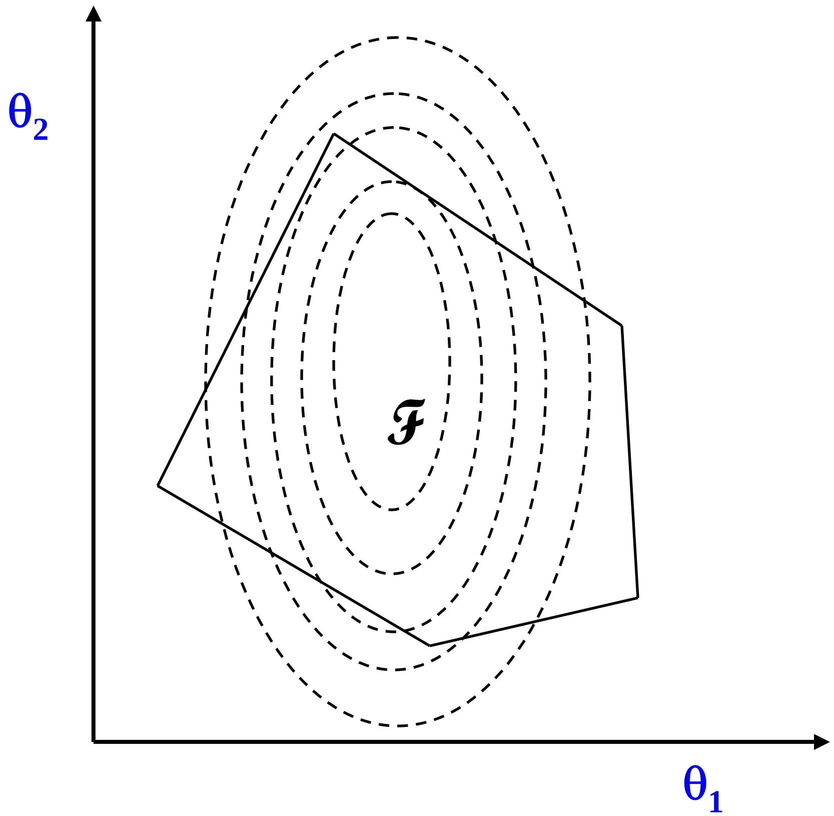
Consider fixed *y* (one discrete state)
Consider fixed *d* (fixed integrated design)
Project *x* onto θ space
Consider two uncertain parameters θ_1 θ_2

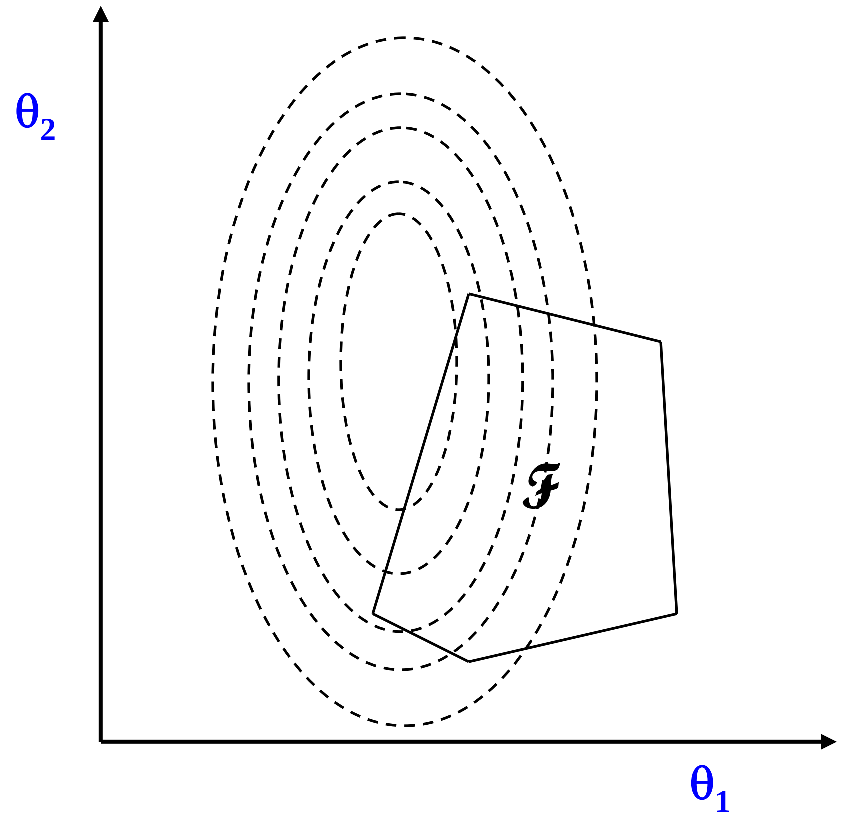


Stochastic Flexibility (SF) :

Area under joint distribution contained in $\mathcal{F}$

Impact of discrete states on SF



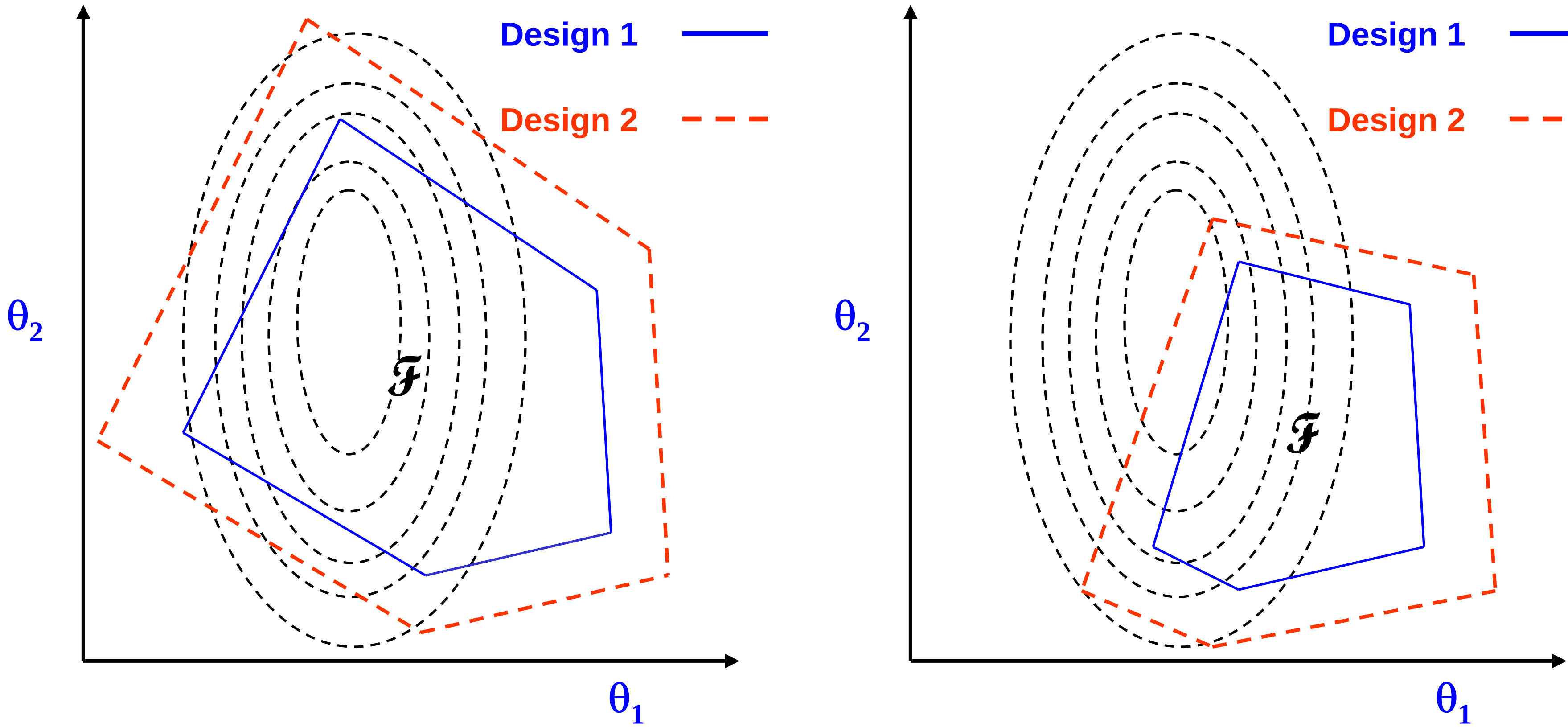


Discrete State A → **Discrete State B**

Equipment failure

$$SF_A > SF_B$$

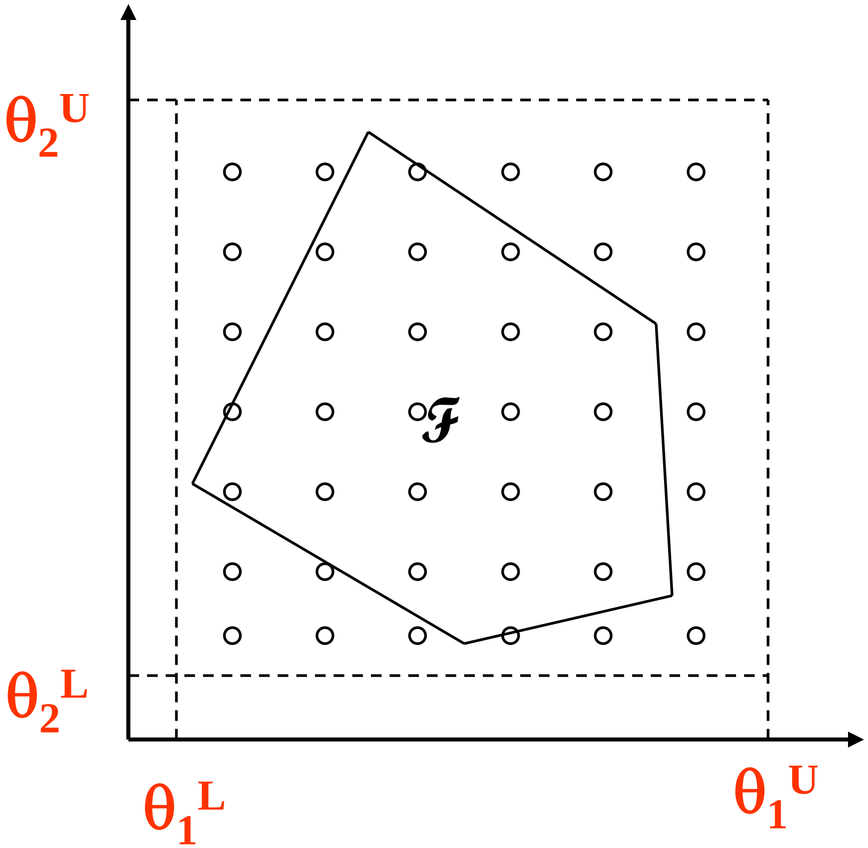
Impact of changes on integrated site design



Discrete State A → **Discrete State B**

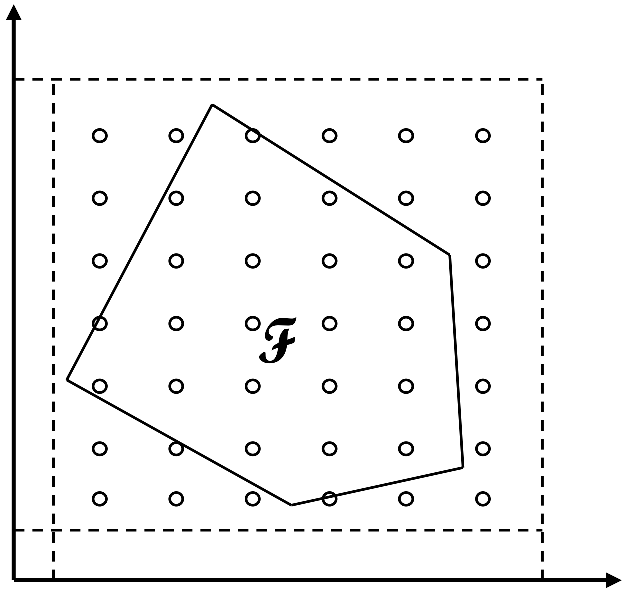
Equipment failure

Maximization of E(SF): Discretization of θ



- Determine upper and lower bounds (e.g. μ +/- 4σ)
- Use (Gaussian) quadrature points
- Use k_i as the index of quadrature points for θ_i

Maximization of E(SF): Assigning probability to each point



Assume two uncertain parameters

k_1 refers to the *k-th* discrete value in θ_1

k_2 refers to the *k-th* discrete value in θ_2

Each discrete point corresponds to a pair (k_1,k_2)

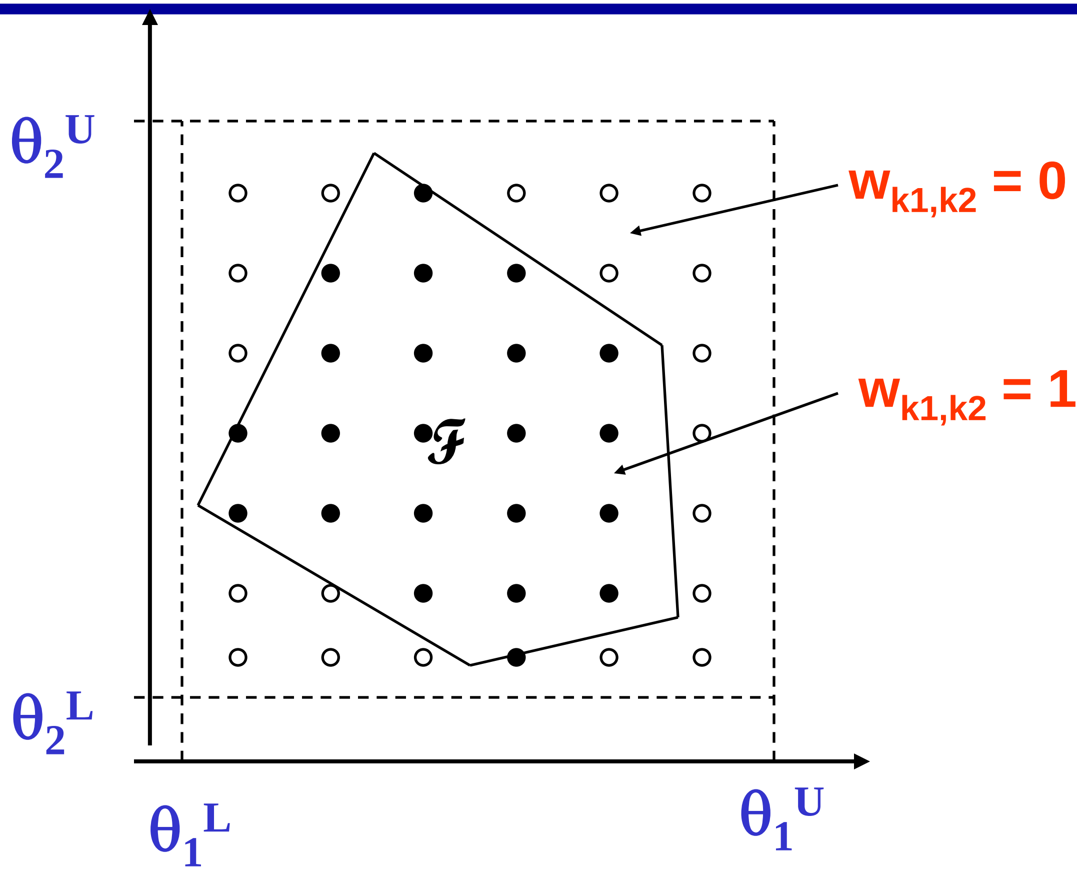
Use the following quadrature formula to assign a probability γ_{k1k2} to each point (k_1,k_2)

$$\gamma_{k1k2} = \omega_{k1}\omega_{k2}\frac{\theta_1^U - \theta_1^L}{2}\frac{\theta_2^U - \theta_2^L}{2} \times j(\theta_{1_{k1}}, \theta_{2_{k2}})$$

$j(\theta_1,\theta_2)$ is the joint probability distribution

ω_{ki} is the weight of the *k-th* quadrature point of the *i-th* uncertain parameter

Maximization of E(SF): Determine feasibility for each point



Process model equations for each (k_1,k_2) point:

$$h_{k1k2}(\,) = 0$$

$$g_{k1k2}(\,) \leq 0$$

⇓

$$h_{k1k2}(\,) = 0$$

(1) $g_{k1k2}(\,) \leq u_{k1k2}$

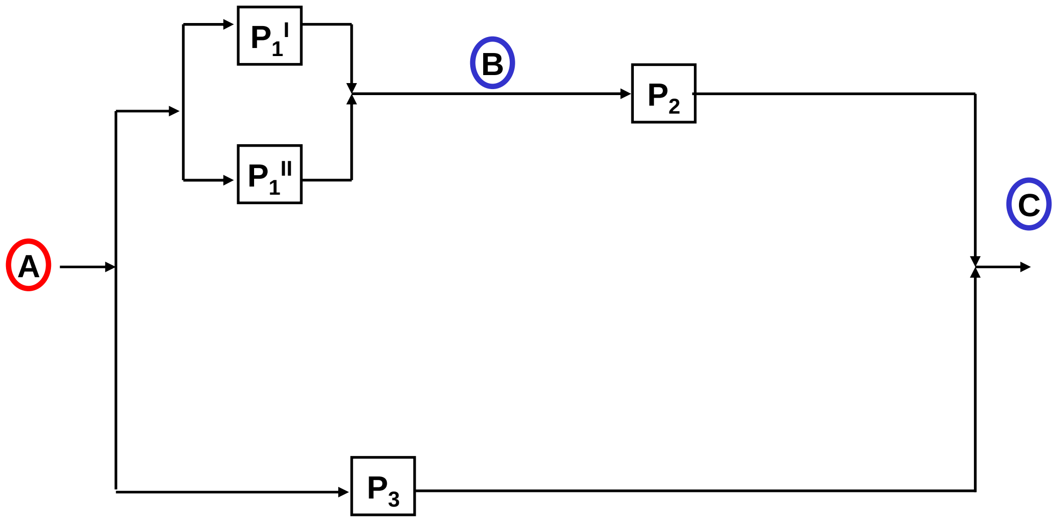
(2) $u_{k1k2} \leq M(1 - w_{k1k2})$

Introduce slack variable *u* for right hand side of constraints (1)

u can be positive only if a binary *w* is equal to zero (2)

w = 1 indicates feasible point *w* = 0 indicates infeasible point

Case Study from Straub & Grossmann 1990



Uncertain supply of A

Uncertain demand of C

Plants subject to random failures

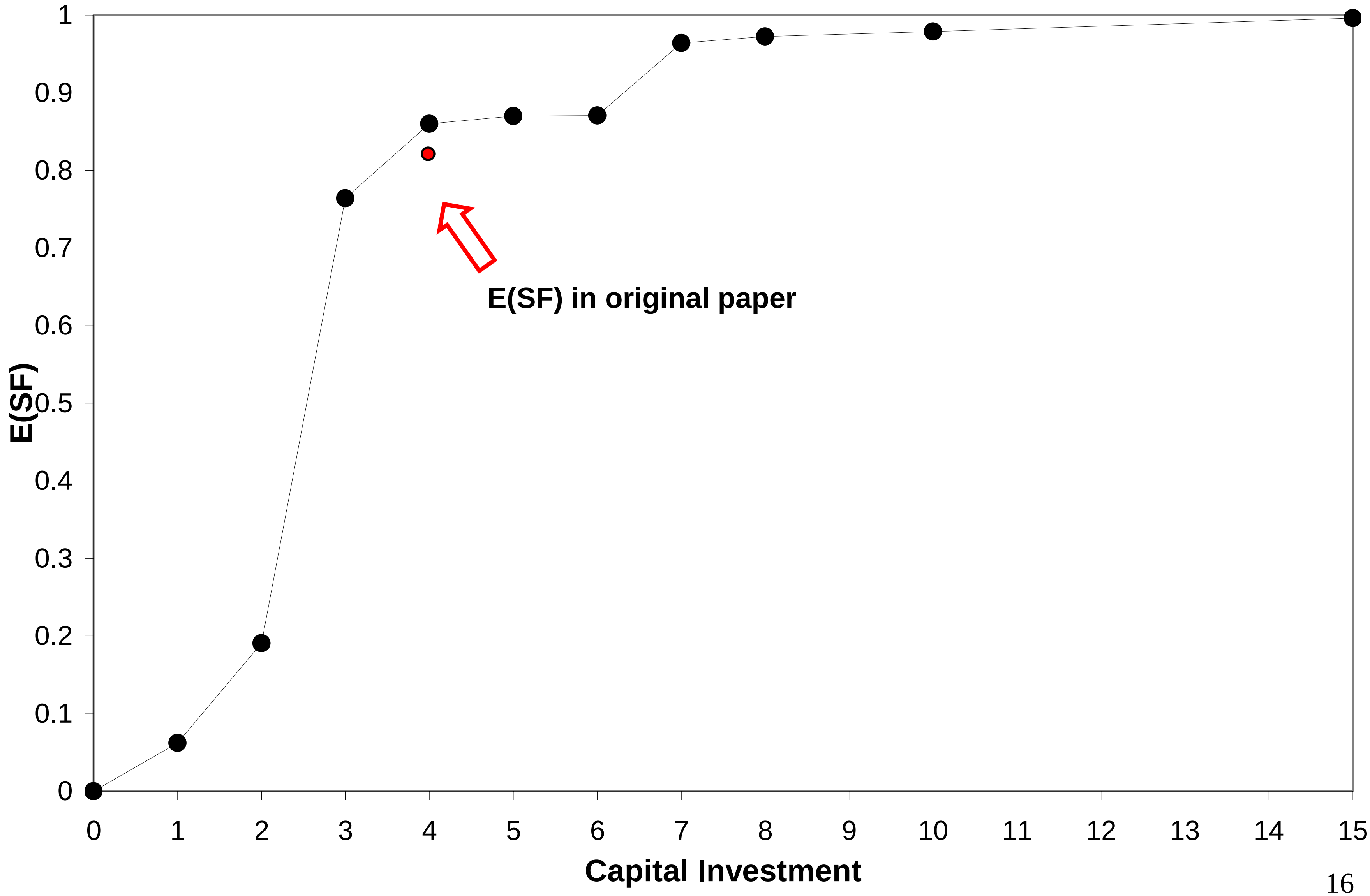
Base case (Grossmann and Straub 1990):

- All plants installed
- No storage
- No capacity expansions

E(SF) = 0.804 present approach

= 0.813 Original paper

Case Study from Straub & Grossmann 1990



Comments

Algorithmic techniques such as upper and lower bounding are required to deal with large number of discrete states.

Optimization results will be validated using Stochastic modeling.

The long term goal of the project is to come up with a general tool applicable in other areas such as supply chain.