Global Supply Chain Planning under Demand and Freight Rate Uncertainty

Fengqi You
Ignacio E. Grossmann

Nov. 13, 2007

Sponsored by The Dow Chemical Company (John Wassick)
Motivation

• Global Chemical Supply Chain
  - Costs several billion dollars annually
  - Planning under uncertain environment

• Objective:
  - Developing Models and Algorithms for Global Multiproduct Chemical Supply Chains Planning under Uncertainty
Supply Chain Tactical Planning
Problem Statement

• Given
  - Minimum and initial inventory
  - Inventory holding cost and throughput cost
  - Transport times of all the transport links
  - Uncertain customer demands and transport cost

• Determine
  - Transport amount, inventory and production levels

• Objective: Minimize Cost
Outline

• **Stochastic Programming Model**
  - Two-stage stochastic programming model
  - Simulation-optimization framework

• **Model Extensions for Robustness and Risk**
  - Robust optimization
  - Risk management

• **Algorithms**
  - Multi-cut L-shaped method
Uncertain Parameters

- Demand and Freight Rate Uncertainty
  - Normal distribution
  - Forecast value as mean, variances come from historical data
    » Demand uncertainty has 3 level variances
    » Freight rate uncertainty has 2 level variances
Scenario Planning

- Discretize the probability distribution function
  - Approximate all the uncertainties to discrete distribution (scenarios)
  - Each scenario represents a possible outcome
  - Generate scenarios by Monte Carlo sampling
    » Assign each scenario the same probability (i.e. For $N$ sampling, $P_s = 1/N$)
    » Combine statistical methods for “good” approximation
Decision Stages under Uncertainty

- **Here-and-now**
  - Decisions \((x)\) are taken before uncertainty \(\omega\) resolves.

- **Wait-and-see**
  - Decisions \((y_\omega)\) are taken after uncertainty \(\omega\) resolves as “corrective action” - recourse.
Two-stage Stochastic Programming

• First stage decisions
  • Here-and-now: decisions for the first month (production, inventory, shipping)

• Second stage decisions
  • Wait-and-see: decisions for the remaining 11 months

Minimize $E [\text{cost}]$
Multi-period Planning Model

- Objective Function:
  - Min: Total Expected Cost

- Constraints:
  - Mass balance for plants
  - Mass balance for DCs
  - Mass balance for customers
  - Minimum inventory level constraint
  - Capacity constraints for plants
Objective Function

\[ E[\text{Cost}] = \text{Cost}_1 + \sum_s P_s \cdot \text{Cost}_2 s \]

First stage cost
\[ \text{Cost}_1 = \sum_k \sum_{k'} \sum_j \sum_t h_{k,j} I_{k,j,t} \]

Inventory Costs

Probability of each scenario
\[ + \sum_k \sum_{k'} \sum_j \sum_t \gamma_{k,k',j} F_{k,k',j,t} \]

Freight Costs

Second stage cost
\[ + \sum_k \sum_{k'} \sum_j \sum_t \gamma_{k,l,j} S_{k,l,j,t} \]

Throughput Costs

Demand Unsatisfied
\[ + \sum_k \sum_{k'} \sum_j \sum_t \delta_{k,j} F_{k,k',j,t} \]

\[ + \sum_k \sum_{k'} \sum_j \sum_t \delta_{k,j} S_{k,l,j,t} \]

\[ + \sum_{l} \sum_j \sum_t \eta_{l,j} S_{l,j,t} \]
Simulation-optimization Framework

Rolling Horizon Strategy

- Inter-facility shipment from the previous time periods considered as pipeline inventory
- Facility-customer shipment considers as part of demand realization
- Inventory level in the previous time period considered as the initial inventory for $t=1$
- Consider uncertainty reduction as time period moving forward

Jan | Feb | Mar | Apr | May | Jun | Jul | Aug | Sep | Oct | Nov | Dec

revealed | uncertain

$\sigma/\mu = 5\%$
$\sigma/\mu = 10\%$
$\sigma/\mu = 20\%$
Robust Optimization

**Objective:** To find out the *optimal* solution that yields similar results under the *uncertain* environment – *Robust solution!*
Robust Optimization using Variance

• Goal Programming Formulation
  
  - New objective function: Minimize $E[Cost] + \rho \cdot V[Cost]$
  - Different $\rho$ can lead to different solution (multi-objective optimization)

\[
E[Cost] + \rho \cdot V[Cost] = Cost1 + \sum_s p_s \cdot Cost2_s + \rho \cdot \sum_s p_s \left( \sum_{s'} p_{s'} \cdot Cost2_{s'} \right) - Cost2_s\]^2
\]

- Expected Cost
- Weighted coefficient
- Expected Variance of each scenario
Robust Optimization via Variability Index

- **First Order Variability index**
  - Convert NLP to LP by replacing two norm to one norm

\[
\text{Min: } E[Cost] + \rho \cdot \sum_s p_s (E[Cost] - Cost_s)^2
\]

\[
\text{Min: } E[Cost] + \rho \cdot \sum_s p_s |E[Cost] - Cost_s|
\]
Robust Optimization

Variability Index (cont)

- **Linearize the absolute value term**
  - Introducing a *first order* non-negative variability index $\Delta$

\[
\begin{align*}
\text{Min: } & \ E[\text{Cost}] + \rho \cdot \sum_s p_s \cdot (E[\text{Cost}] - \text{Cost}_s + 2\Delta_s) \\
\text{s.t. } & \Delta_s \geq \text{Cost}_s - E[\text{Cost}] \\
& \Delta_s \geq 0
\end{align*}
\]

\[
\begin{align*}
\text{Min: } & \ E[\text{Cost}] + \rho \cdot \sum_s p_s |E[\text{Cost}] - \text{Cost}_s|
\end{align*}
\]
Risk Management (cont)

- **Risk**: The probability of exceeding certain target cost level $\Omega$

\[
Risk(x, \Omega) = \Pr [cost(x) > \Omega]
\]
Risk Management (cont)

• Risk Management for scenario planning
  
  ♦ Calculation by

  \[ \text{Risk}(x, \Omega) = \Pr[\text{Cost}(x) > \Omega] = \sum_s p_s Z_s(x, \Omega) \]

  ♦ Binary variables

  \[ Z_s(x, \Omega) = \begin{cases} 
  1 & \text{if Cost}_s > \Omega \\
  0 & \text{otherwise} 
\end{cases} \]
Risk Management Model Formulation

Risk Objective $\rightarrow$ Min: $Risk(x, \Omega) = \sum_s p_s Z_s$

Economic Objective $\rightarrow$ Min: $E[Cost] = Cost_1 + \sum_s P_s \cdot Cost_2s$

s.t.

Risk Management Constraints

\begin{align*}
Cost_1 + Cost_2s & \leq \Omega + M \cdot Z_s \\
Cost_1 + Cost_2s & \geq \Omega - M \cdot (1 - Z_s) \\
Ax & = b \\
W_s y_s & = h_s - T_s x \\
x & \geq 0, y_s \geq 0, z_s \in \{0, 1\}
\end{align*}
Downside Risk

- Definition: **Positive Profit Deviation**
  - Binary variables are not required, **pure LP (MILP -> LP)**

\[
DRisk(x, \Omega) = \sum_s p_s \delta_s(x, \Omega)
\]

\[
\delta_s(x, \Omega) \geq \text{cost}_s - \Omega, \forall s
\]

\[
\delta_s(x, \Omega) \geq 0, \forall s
\]
Algorithm: Multi-cut L-shaped Method

Standard L-shaped Method

Min \[ c^T x + p_1 q_1^T y_1 + p_2 q_2^T y_2 + \cdots + p_s q_s^T y_s \]

s.t. \[ Ax + T_1 x + W_1 y_1 + T_2 x + W_2 y_2 + \cdots + T_s x + W_s y_s = b \]

\[ = b \rightarrow \text{Master problem} \]

\[ = h_1 \]
\[ = h_2 \]
 \[ = : \]
 \[ = : \]
 \[ = : \]

\[ = h_s \]

\[ x \geq 0, \ y_1 \geq 0, \ y_2 \geq 0, \ \cdots \ y_s \geq 0 \]
Algorithm: Multi-cut L-shaped Method

**Expected Recourse Function**

- The expected recourse function $Q(x)$ is convex and piecewise linear
- Each optimality cut supports $Q(x)$ from below
Algorithm: Multi-cut L-shaped Method

Multi-cut L-shaped Method

Solve master problem to get a lower bound (LB)

Add cut
\[ e_{l,s} = \pi_s^T T_s \]
\[ d_{l,s} = \pi_s^T h_s \]

Solve the sub problem to get an upper bound (UB)

min \[ c^T x + \sum_s p_s \theta_s \]

s.t. \[ Ax = b \]
\[ \theta_s \geq e_{l,s} x + d_{l,s} \]
\[ x \geq 0 \]

min \[ q_s^T y \]

s.t. \[ W y = h_s - T_s x \]
\[ y \geq 0 \]

UB – LB < Tol ?

No

Yes

STOP
Conclusion

• Current Work
  - Develop a two-stage stochastic programming approach for global supply chain planning under uncertainty. Simulation studies show that **5.70%** cost saving in average can be achieved
  - Present two robust optimization models and two risk management model. Develop an efficient solution algorithm to solve the large scale stochastic programming problem

• Future Work
  - Multi-site capacity planning under uncertainty
Questions?