Global Supply Chain Planning under Demand and Freight Rate Uncertainty

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Sponsored by The Dow Chemical Company (John Wassick)

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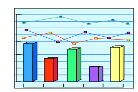
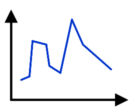
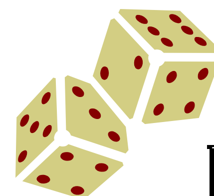
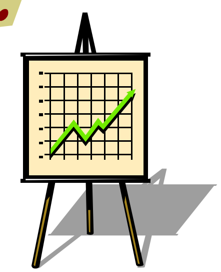
Motivation

- Global Chemical Supply Chain
  - Costs several billion dollars annually
  - Planning under uncertain environment



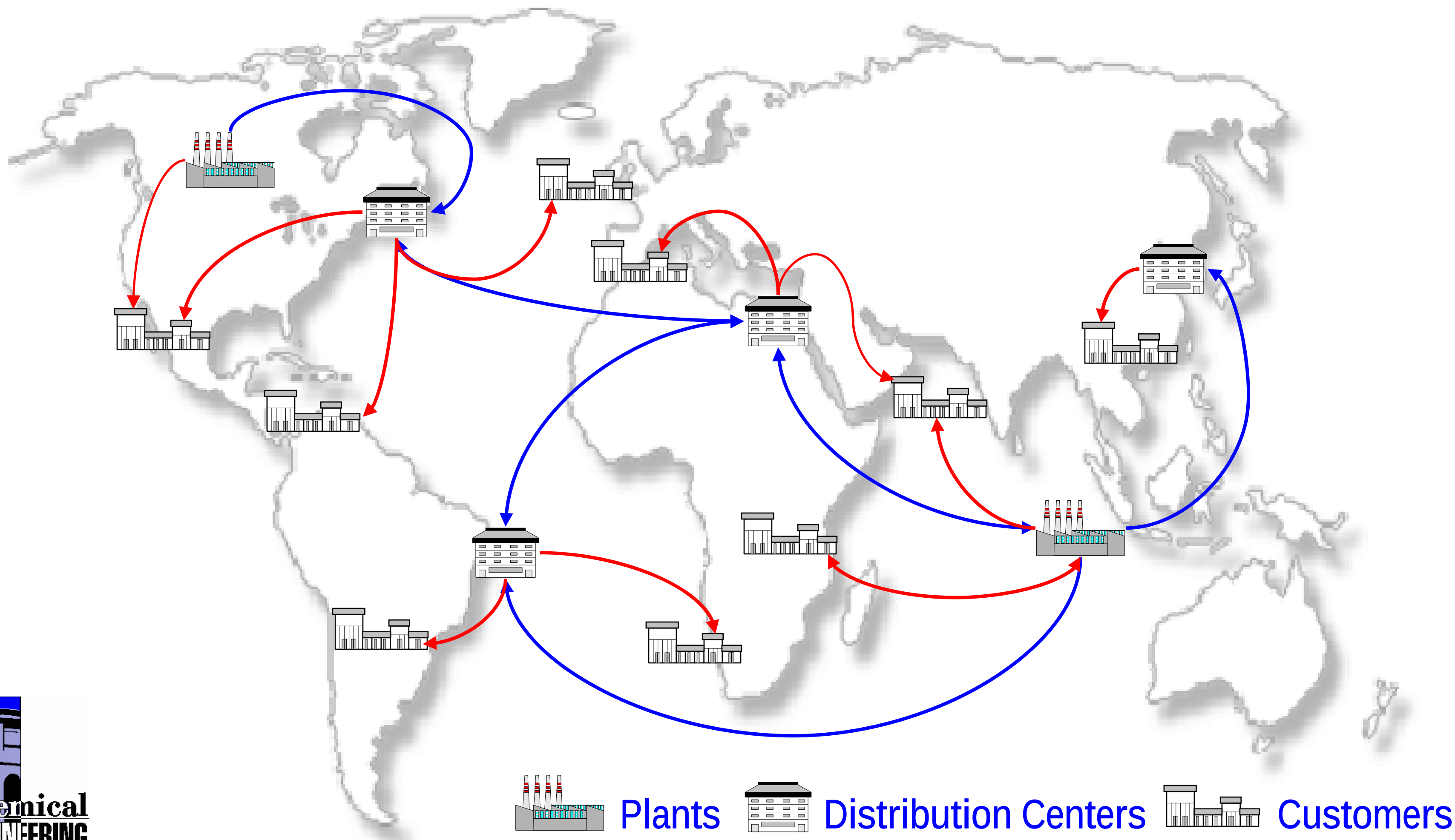
Changes of customer orders



Fluctuations of gas prices

- Objective:
  - Developing Models and Algorithms for Global Multiproduct Chemical Supply Chains Plannning under Uncertainty

Supply Chain Tactical Planning



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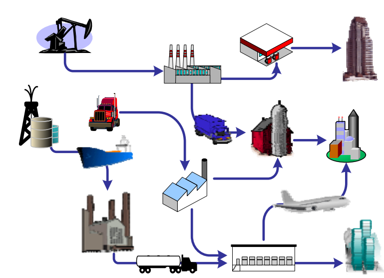
Problem Statement

- Given
  - Minimum and initial inventory
  - Inventory holding cost and throughput cost
  - Transport times of all the transport links
  - Uncertain customer demands and transport cost
- Determine
  - Transport amount, inventory and production levels

• Objective: Minimize Cost



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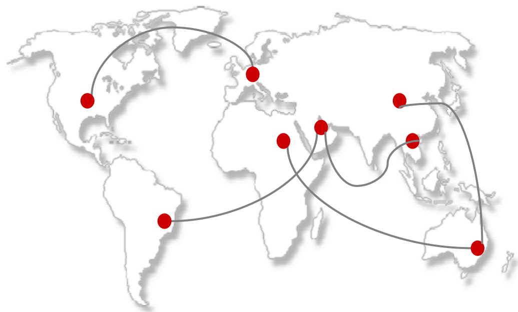
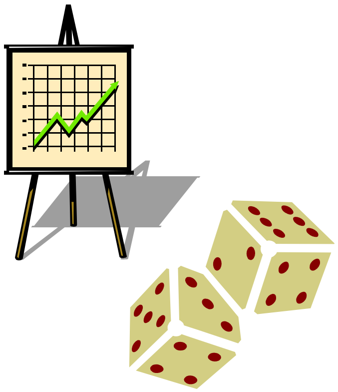
Outline

- Stochastic Programming Model
  - Two-stage stochastic programming model
  - Simulation-optimization framework
- Model Extensions for Robustness and Risk
  - Robust optimization
  - Risk management
- Algorithms
  - Multi-cut L-shaped method





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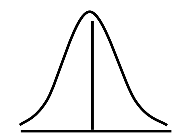
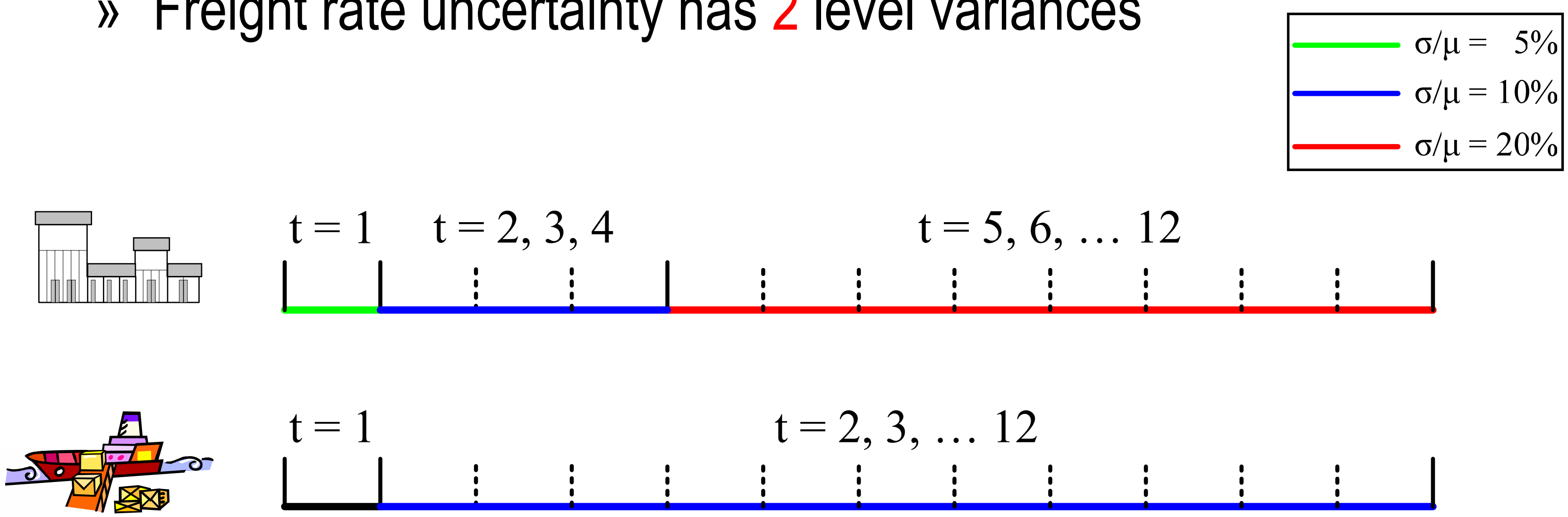
Uncertain Parameters

- Demand and Freight Rate Uncertainty
  - Normal distribution
  - Forecast value as mean, variances come from historical data
    - » Demand uncertainty has 3 level variances
    - » Freight rate uncertainty has 2 level variances





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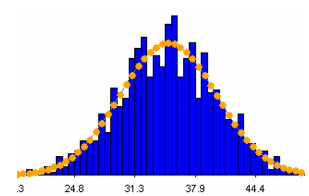
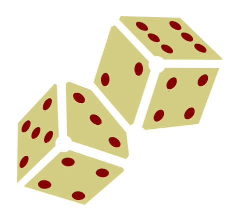
Scenario Planning

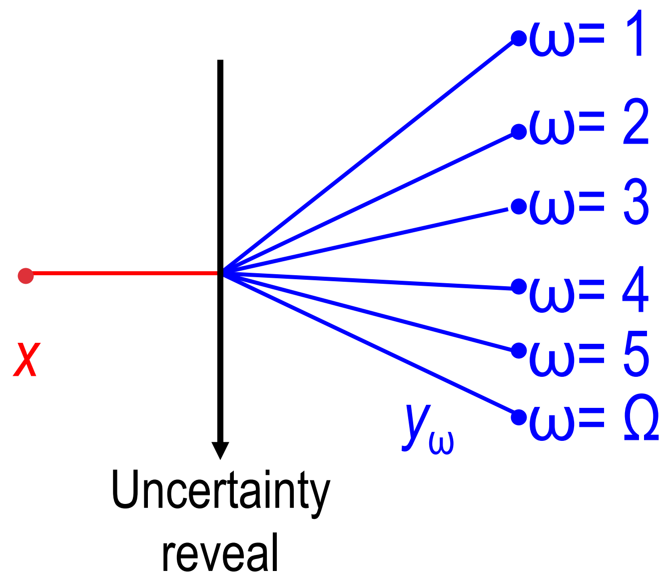
- Discretize the probability distribution function
  - Approximate all the uncertainties to discrete distribution (scenarios)
  - Each scenario represents a possible outcome
  - Generate scenarios by Monte Carlo sampling
    - » Assign each scenario the same probability (i.e. For *N* sampling, $P_s = 1/N$)
    - » Combine statistical methods for "good" approximation





Decision Stages under Uncertainty

- Here-and-now
  - Decisions (x) are taken before uncertainty ω resolute
- Wait-and-see
  - Decisions (y_ω) are taken after uncertainty ω resolute as "*corrective* action" - *recourse*

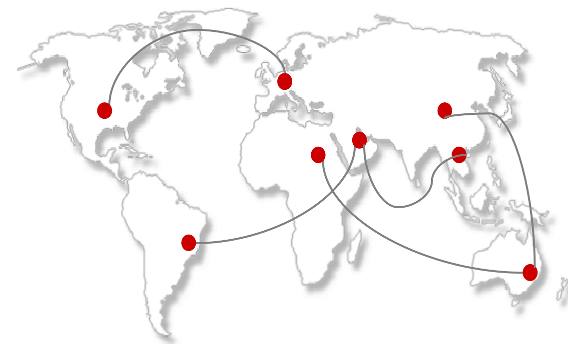


Two-stage Stochastic Programming

- First stage decisions
  - Here-and-now: decisions for the first month (production, inventory, shipping)
- Second stage decisions
  - Wait-and-see: decisions for the remaining 11 months

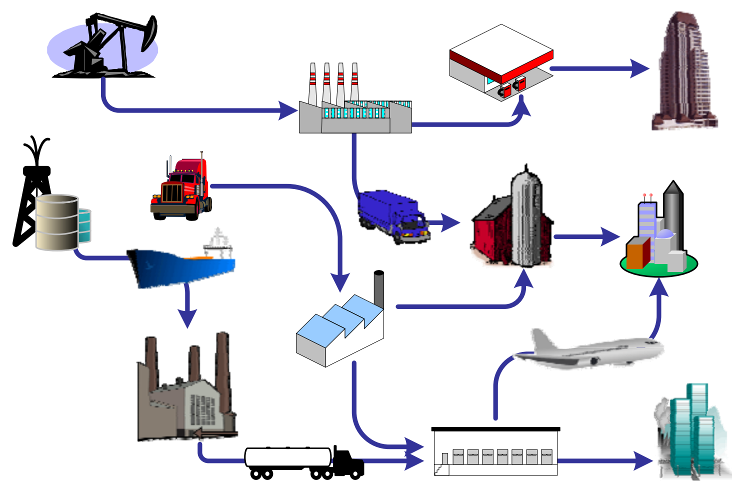
Minimize *E* [cost]

Multi-period Planning Model



- Objective Function:
  - Min: Total Expected Cost

- Constraints:
  - Mass balance for plants
  - Mass balance for DCs
  - Mass balance for customers
  - Minimum inventory level constraint
  - Capacity constraints for plants



Objective Function

$$E[Cost] = Cost1 + \sum_s P_s \cdot Cost2_s$$

First stage cost | **Probability of each scenario** | **Second stage cost**

$$Cost1 = \sum_k \sum_j \sum_t h_{k,j} I_{k,j,t}$$
$$+ \sum_k \sum_{k'} \sum_j \sum_t \gamma_{k,k',j} F_{k,k',j,t} + \sum_k \sum_l \sum_j \sum_t \gamma_{k,l,j} S_{k,l,j,t}$$
$$+ \sum_k \sum_{k'} \sum_j \sum_t \delta_{k,j} F_{k,k',j,t} + \sum_k \sum_l \sum_j \sum_t \delta_{k,j} S_{k,l,j,t}$$

$$Cost2_s = \sum_k \sum_j \sum_t h_{k,j} I_{k,j,t,s}$$
$$+ \sum_k \sum_{k'} \sum_j \sum_t \gamma_{k,k',j,s} F_{k,k',j,t,s} + \sum_k \sum_l \sum_j \sum_t \gamma_{k,l,j,s} S_{k,l,j,t,s}$$
$$+ \sum_k \sum_{k'} \sum_j \sum_t \delta_{k,j} F_{k,k',j,t,s} + \sum_k \sum_l \sum_j \sum_t \delta_{k,j} S_{k,l,j,t,s}$$
$$+ \sum_l \sum_j \sum_t \eta_{l,j} SF_{l,j,t,s}$$

- Inventory Costs
- Freight Costs
- Throughput Costs
- Demand Unsatisfied

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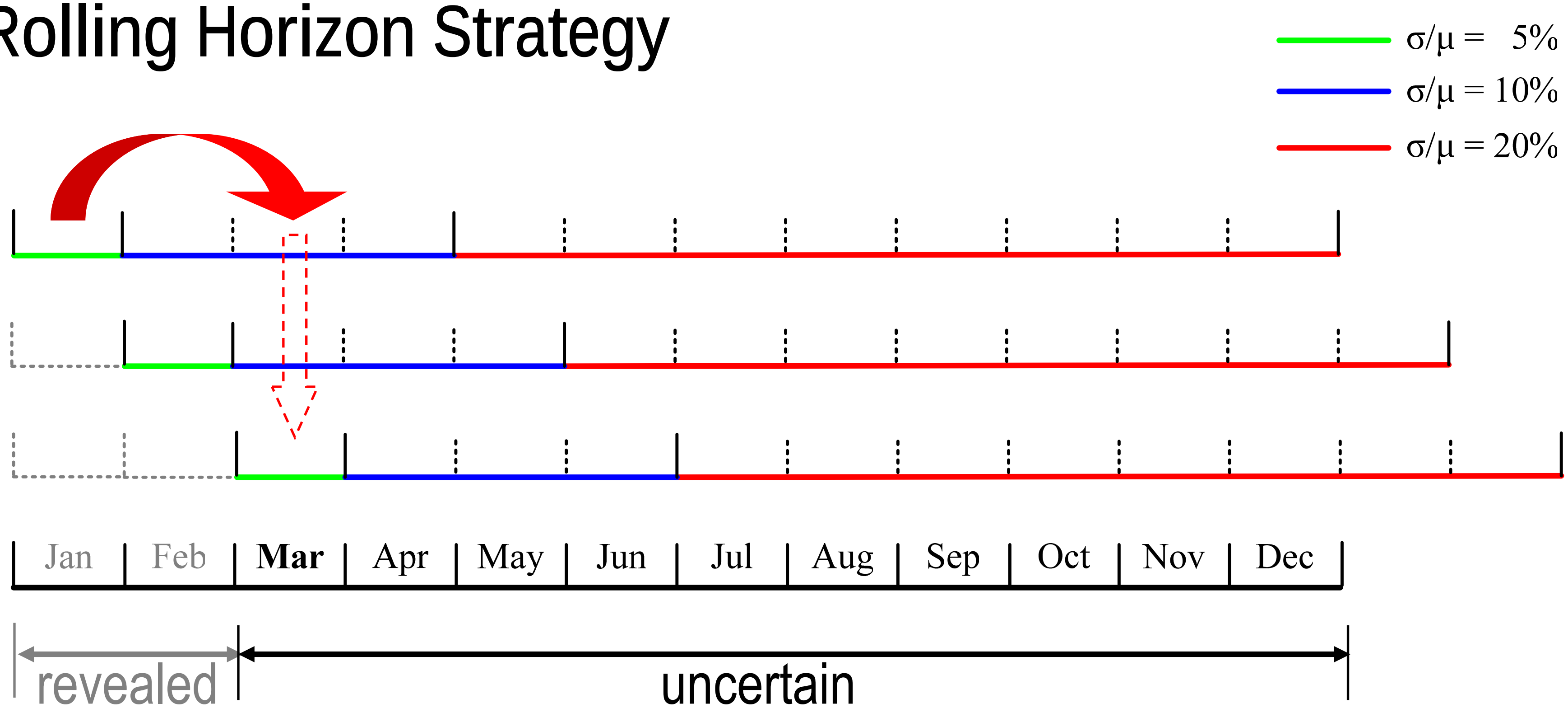
Rolling Horizon Strategy



- Inter-facility shipment from the previous time periods considered as pipeline inventory
- Facility-customer shipment considers as part of demand realization
- Inventory level in the previous time period considers as the initial inventory for t =1
- Consider uncertainty reduction as time period moving forward

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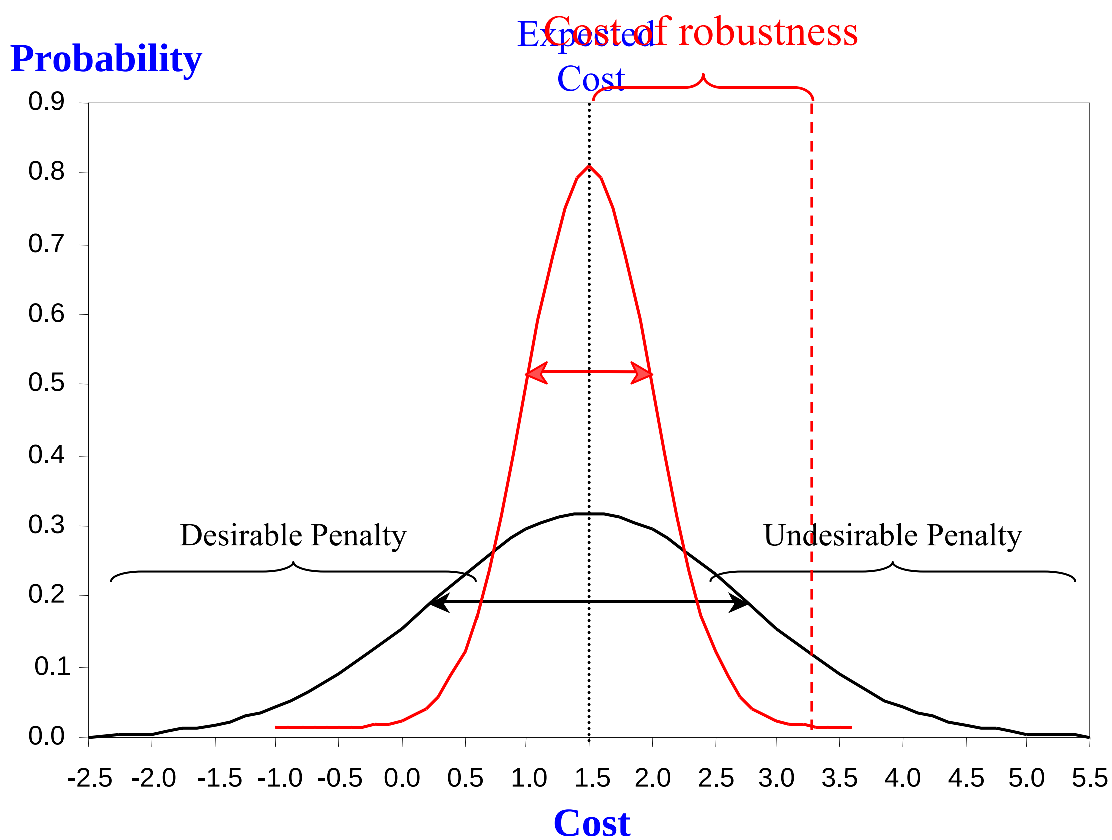
Robust Optimization



Objective: To find out the *optimal* solution that yields similar results under the *uncertain* environment – *Robust solution!*

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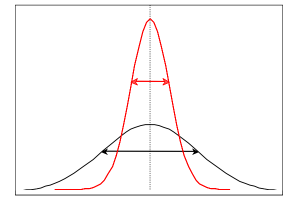
Robust Optimization using Variance



- Goal Programming Formulation
  - New objective function: Minimize $E[Cost] + \rho \cdot V[Cost]$
  - Different ρ can lead to different solution (multi-objective optimization)

$$E[Cost] + \rho \cdot V[Cost]$$
$$= \underbrace{Cost1 + \sum_{s} p_s \cdot Cost2_s}_{\text{Expected Cost}} + \rho \cdot \underbrace{\sum_{s} p_s[(\sum_{s'} p_{s'} \cdot Cost2_{s'}) - Cost2_s]^2}_{\text{Expected Variance of each scenario}}$$

Expected Cost

Weighted coefficient

Expected Variance of each scenario

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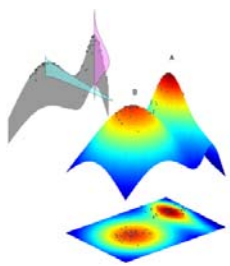
Robust Optimization via Variability Index

- First Order Variability index
  - Convert NLP to LP by replacing two norm to one norm

$$\text{Min: } E[Cost] + \rho \cdot \sum_s p_s (E[Cost] - Cost_s)^2$$

$$\Downarrow$$

$$\text{Min: } E[Cost] + \rho \cdot \sum_s p_s |E[Cost] - Cost_s|$$



Variability Index *(cont)*

- Linearize the absolute value term
  - Introducing a first order non-negative variability index *Δ*

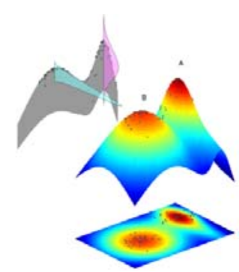
$$\text{Min: } E[Cost] + \rho \cdot \sum_s p_s \cdot (E[Cost] - Cost_s + 2\Delta_s)$$

$$\text{s.t. } \Delta_s \geq Cost_s - E[Cost]$$

$$\Delta_s \geq 0$$

⇕

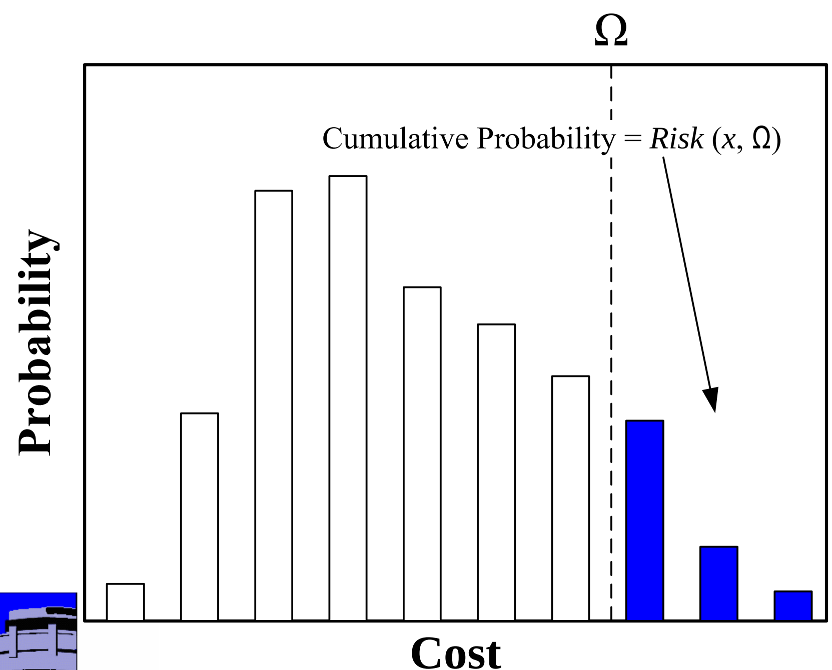
$$\text{Min: } E[Cost] + \rho \cdot \textstyle\sum_s p_s |E[Cost] - Cost_s|$$



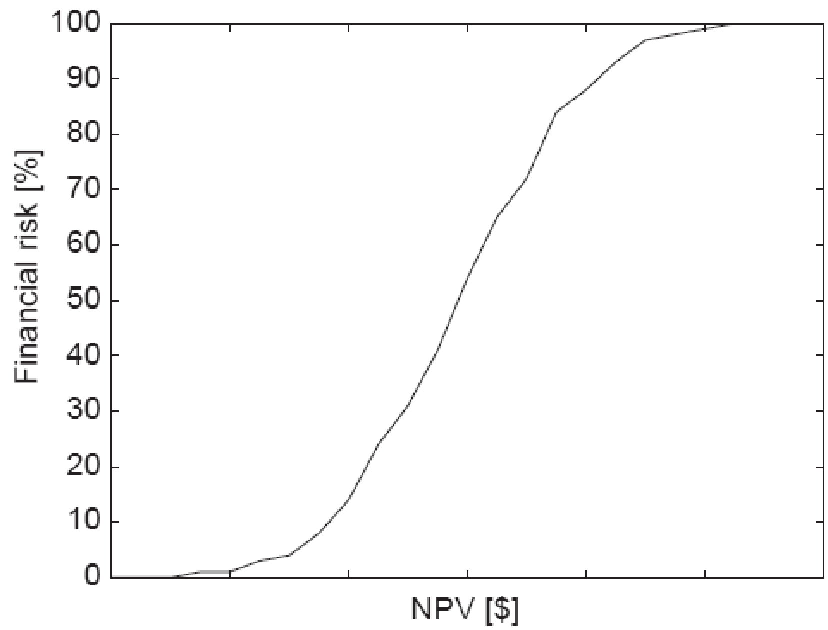
Risk Management *(cont)*

- Risk: The probability of exceeding certain target cost level Ω

$$Risk(x, \Omega) = \mathbf{Pr}\ [cost(x) > \Omega]$$



Histogram of Frequencies



Cumulative Risk Curve

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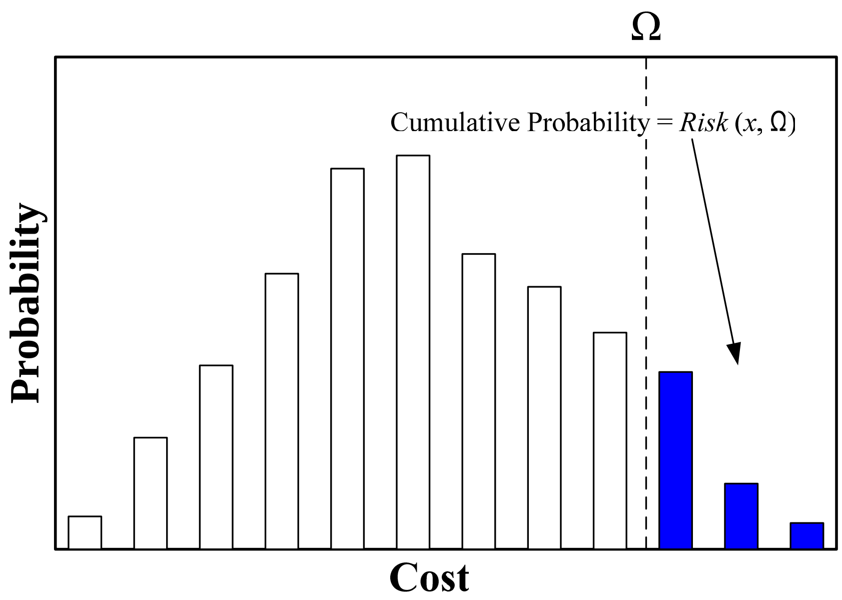
Risk Management *(cont)*

- Risk Management for scenario planning
  - Calculation by

$$\begin{aligned} \text{Risk}(x,\Omega) &= \mathbf{Pr}[Cost(x) > \Omega] \\ &= \sum_s p_s Z_s(x,\Omega) \end{aligned}$$



  - Binary variables

$$Z_s(x,\Omega) = \begin{cases} 1 & \text{if } Cost_s > \Omega \\ 0 & \text{otherwise} \end{cases}$$

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Risk Management Model Formulation

Risk Objective → $\text{Min: } Risk(x,\Omega) = \sum_s p_s Z_s$

Economic Objective → $\text{Min: } E[Cost] = Cost1 + \sum_s P_s \cdot Cost2_s$

Multi-objective

s.t.

Risk Management Constraints

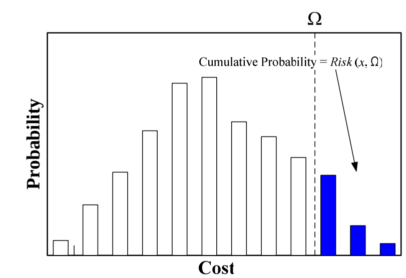
$$Cost1 + Cost2_s \leq \Omega + M \cdot Z_s$$

$$Cost1 + Cost2_s \geq \Omega - M \cdot (1 - Z_s)$$

$$Ax = b$$

$$W_s y_s = h_s - T_s x$$

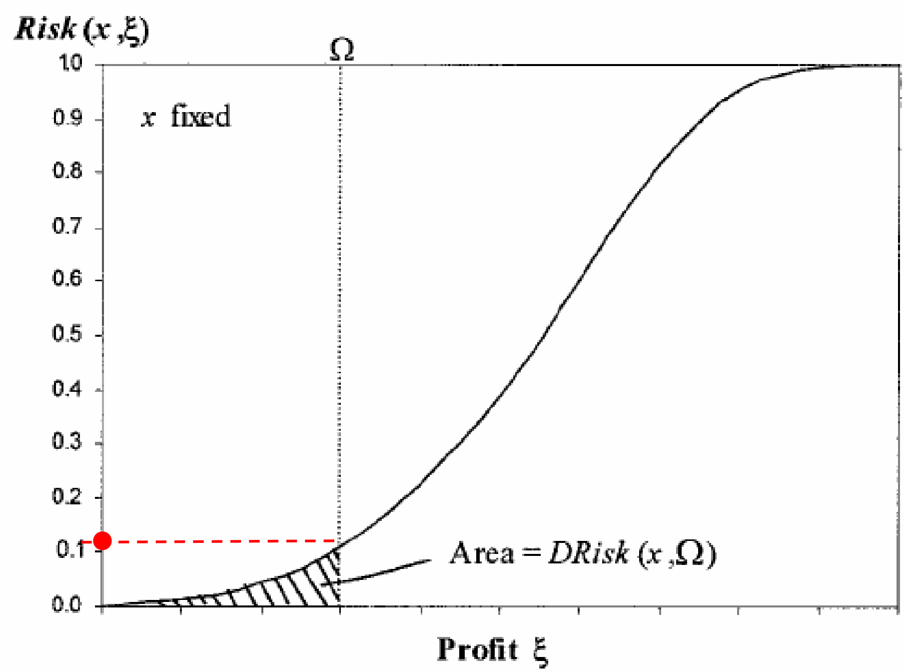
$$x \geq 0, y_s \geq 0, z_s \in \{0,1\}$$



Downside Risk

- Definition: Positive Profit Deviation
  - Binary variables are not required, pure LP (MILP -> LP)

$$
\begin{aligned}
DRisk(x,\Omega) &= \sum_s p_s \delta_s(x,\Omega) \\
\delta_s(x,\Omega) &\geq \text{cost}_s - \Omega, \forall s \\
\delta_s(x,\Omega) &\geq 0, \forall s
\end{aligned}
$$



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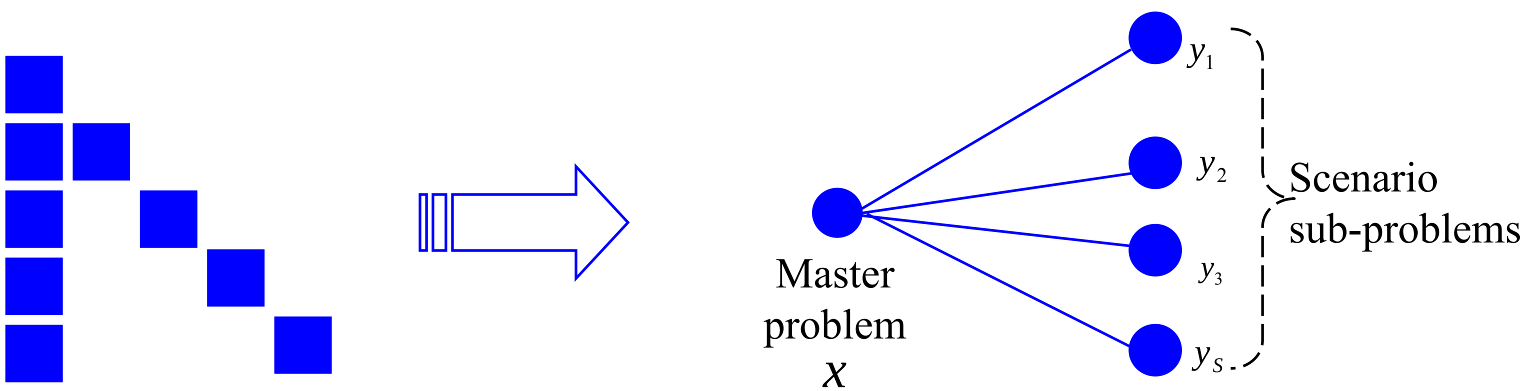
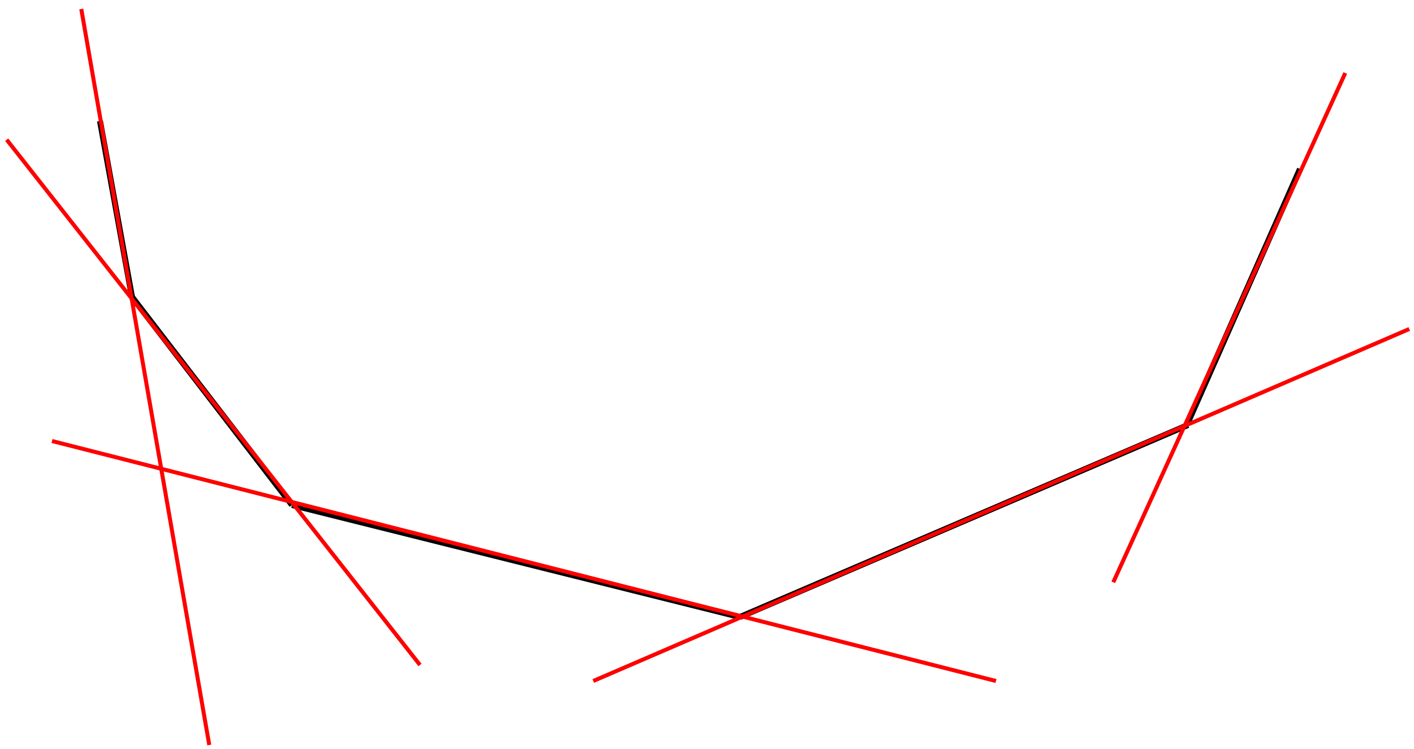
Standard L-shaped Method

$$
\begin{array}{llllllll}
\text{Min} & c^T x & + & p_1 q_1^T y_1 + p_2 q_2^T y_2 & \cdot \quad \cdot \quad \cdot & + \, p_s q_s^T y_s & & \\
\text{s.t.} & Ax & & & & & = b & \rightarrow \text{Master problem} \\
& T_1 x & + & W_1 y_1 & & & = h_1 & \\
& T_2 x & + & \quad W_2 y_2 & & & = h_2 & \\
& \vdots & + & & \cdot & & = \vdots & \\
& \vdots & + & & \quad \cdot & & = \vdots & \text{Scenario sub-problems} \\
& \vdots & + & & \qquad \cdot & & = \vdots & \\
& T_s x & + & & & W_s y_s & = h_s & \\
& & & \multicolumn{5}{l}{x \geq 0, \; y_1 \geq 0, \; y_2 \geq 0, \; \cdot \; \cdot \; \cdot \; y_s \geq 0}
\end{array}
$$



Expected Recourse Function



- The expected recourse function Q(x) is convex and piecewise linear
- Each optimality cut supports Q(x) from below

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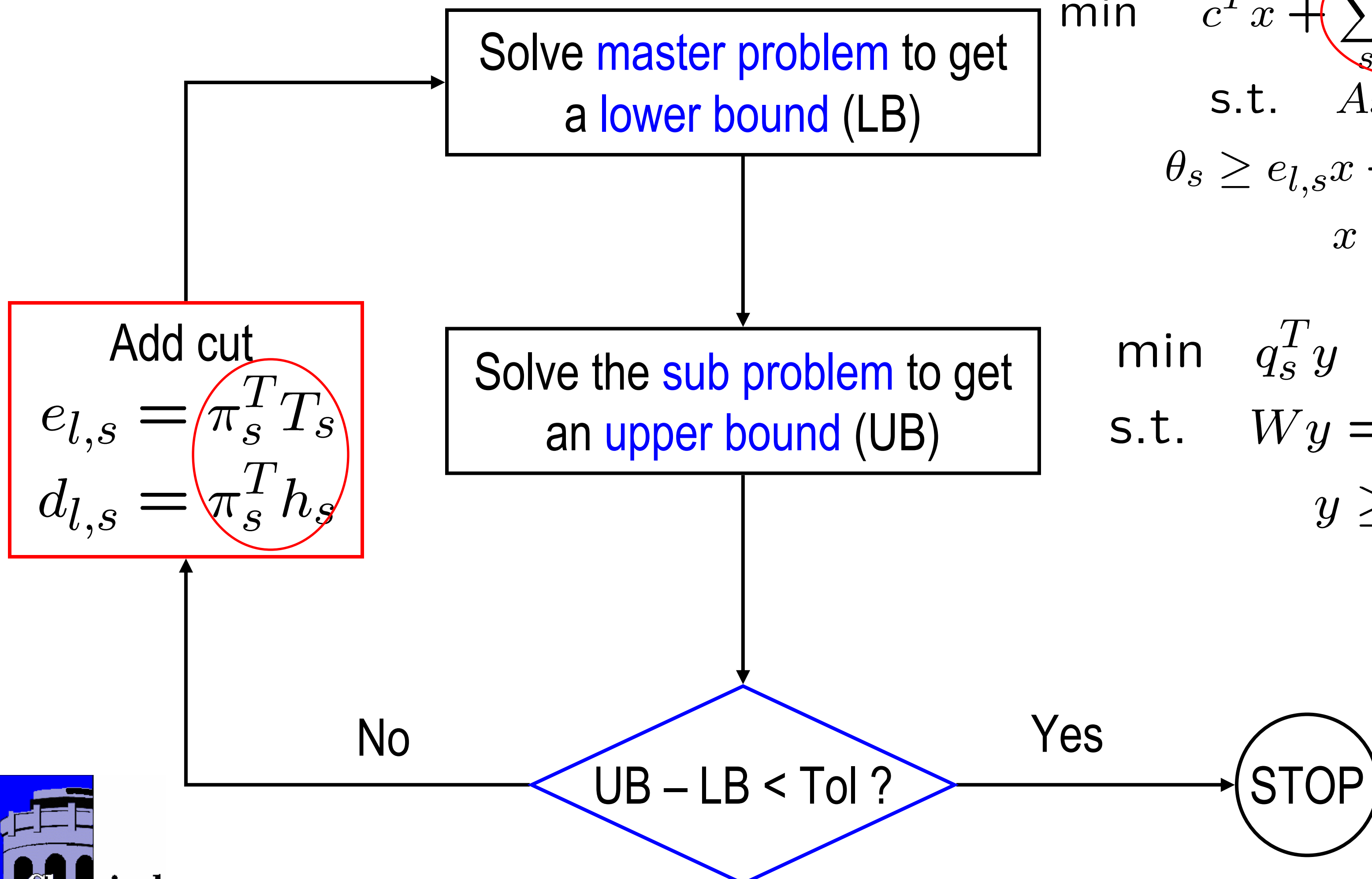
Multi-cut L-shaped Method



Master problem:

$$\begin{aligned} \min \quad & c^T x + \sum_s p_s \theta_s \\ \text{s.t.} \quad & Ax = b \\ & \theta_s \geq e_{l,s} x + d_{l,s} \\ & x \geq 0 \end{aligned}$$

Sub problem:

$$\begin{aligned} \min \quad & q_s^T y \\ \text{s.t.} \quad & Wy = h_s - T_s x \\ & y \geq 0 \end{aligned}$$

Conclusion

- Current Work
  - Develop a two-stage stochastic programming approach for global supply chain planning under uncertainty. Simulation studies show that 5.70% cost saving in average can be achieved
  - Present two robust optimization models and two risk management model. Develop an efficient solution algorithm to solve the large scale stochastic programming problem
- Future Work
  - Multi-site capacity planning under uncertainty

Questions?