Scheduling of Crude Oil Movements at Refinery Front-end

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ExxonMobil

Enterprise-wide Optimization Project

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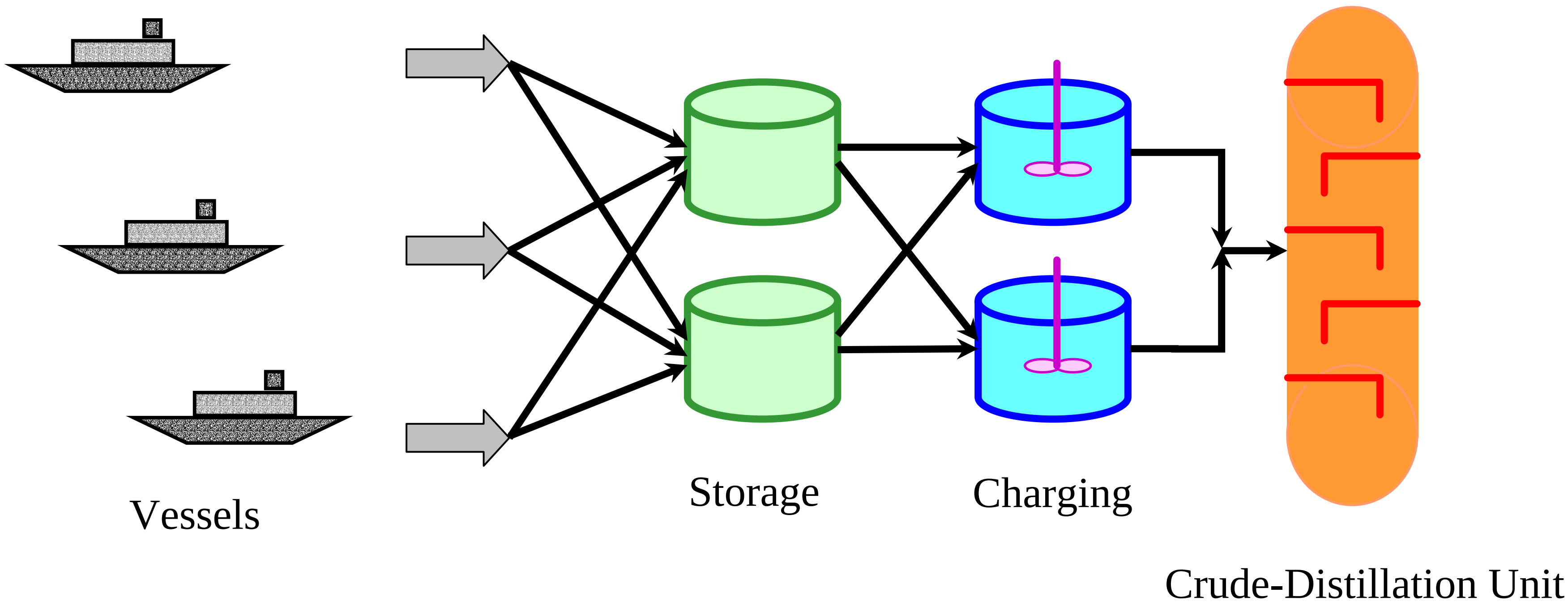
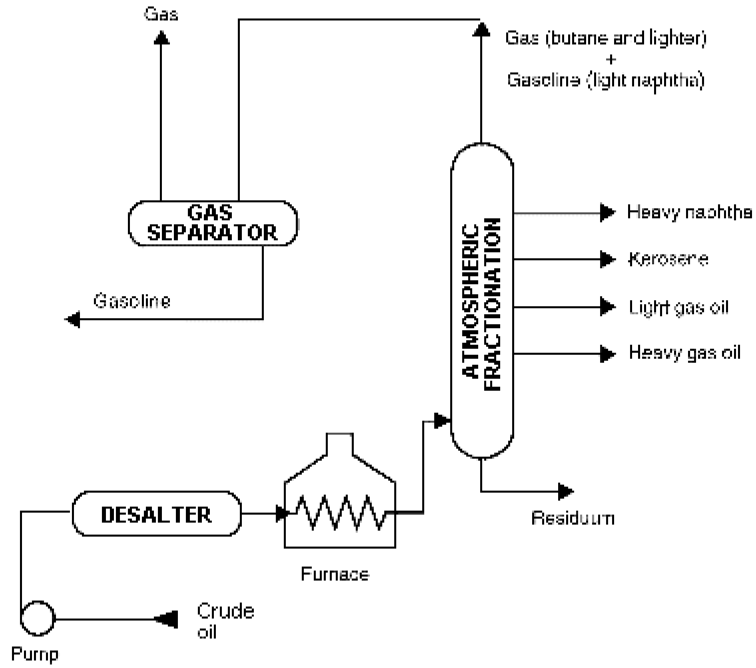
Motivation

- Scheduling and Planning of flow of crude oil is key problem in petrochemical refineries
- Large cost savings can be realized with an optimum schedule for the movement of crude oil

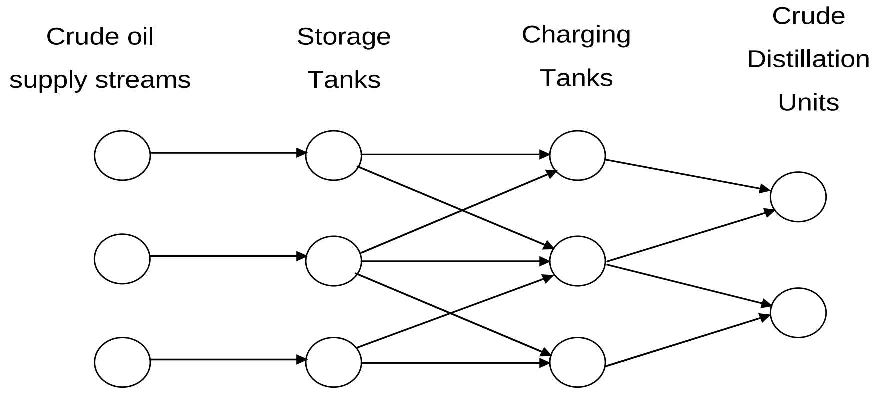


How to coordinate discharge of vessels with loading to storage?
How to synchronize charging tanks with crude-oil distillation?

Crude Distillation Unit



Problem Statement



Given:

(a) Maximum and minimum inventory levels for a tank
(b) Initial total and component inventories in a tank
(c) Upper and lower bounds on the fraction of key components in the crude inside a tank
(d) Times of arrival of crude oil in the supply streams
(e) Amount of crude arriving in the supply streams
(f) Fractions of various components in the supply streams
(g) Bounds on the flowrates of the streams in the network
(h) Time horizon for scheduling

Determine:

(i) Total and component inventory levels in the tanks at various points of time
(ii) Volumes of total and component flows from one unit to another in a certain time interval
(iii) Start and end times of the flows in each stream of the system.

Objective: Minimize Cost

MILP Model: ***Lee, Pinto, Grossmann, Park (1996)***

Assumptions MINLP Model

1. Perfect mixing takes place in each tank.
2. Negligible change in specific gravities on mixing.
3. Discrete flows of volumes into and from a tank.
4. Simultaneous inputs into and outputs from a tank are not allowed.
5. Each distillation unit can be charged by at most one charging tank at a point of time.
6. Each charging tank can charge at most one distillation unit at a point of time.
7. All the distillation units have to be operated continuously throughout the entire time horizon.

Scheduling Model

- Continuous time formulation by Furman et al. (2006)
  - State Task Network Representation
- Based on time events where inputs and outputs to an unit can take place in the same time event
  - No simultaneous input into and from a tank
- Formulation reduces number of binary variables required in the scheduling model

Optimization model :

min *cost objective* (P)

s.t. Tank constraints

Distillation unit constraints

Supply stream constraints

Variable bounds

Variables in the model :

$I_{b,t}^{tot}$ – Total inventory in tank *b* at end of time event *t*

$I_{b,t}^{j}$ – Inventory of component *j* in tank *b* at end of time event *t*

$V_{s,t}^{tot}$ – Total flow in stream *s* in time event *t*

$V_{s,t}^{j}$ – Flow of component *j* in stream *s* in time event *t*

$T_{s,t}^{1}$ – Start time of flow in stream *s* in time event *t*

$T_{s,t}^{2}$ – End time of flow in stream *s* in time event *t*

$w_{s,t}$ – Binary variable pertaining to existence of flow in stream *s* in time event *t*

Constraints in Model

Tank constraints

- Overall mass balance
- Individual component balance ⇐ Non-linear equations containing Bilinearities
- Logic constraints
    - o Related to the existence of a flow into or from a tank in a time event ***t***
- Duration constraints
    - o To bound the flow of a stream into/from a tank in a particular time event ***t***
- Simple sequencing constraints
- Inventory bounds
- Bounds on component fractions inside a tank
- Input and output restraints over whole horizon

Constraints in Model (Contd …)

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Distillation unit constraints

- ❑ Continuous time operation constraint
- ❑ Allocation constraints
    - o Only one CDU can be charged by a charging tank in a time event ***t***
    - o Only one charging tank can charge a CDU in a time event ***t***
- ❑ Crude-mix demand constraints

Supply stream constraints

- ❑ Overall mass balances
- ❑ Component mass balances
- ❑ Start and end timing constraints

Non-convex MINLP

Objective function :

- Minimize a cost objective similar to the one by Jia and Ierapetritou (2003)

min *total cost* = *waiting cost for supply streams*
+ *unloading cost of supply streams*
+ *inventory cost for each tank over scheduling horizon*
+ *changeover cost for charging CDUs with different charging tanks*

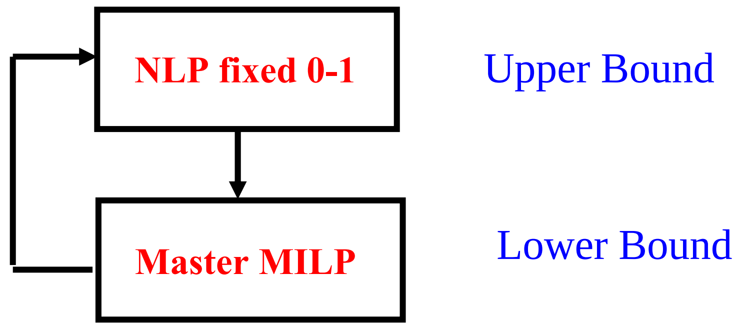
Overall model ≡ **(P)** ⟹ Non-convex MINLP

Convex relaxation of (P)
(obtained by linearizing non-linear equations in *Tank constraints* and introducing McCormick convex envelopes (1976) for bilinear terms) ≡ **(R)** ⟹ MILP

Global Optimization of MINLP

- Large-scale non-convex MINLPs such as (P) are very difficult to solve
  - Global optimization solvers fail to converge to solution in tractable computational times (e.g. BARON)

- Special Outer-Approximation algorithm proposed to solve problem to global optimality



  - ***Guaranteed to converge*** to global optimum within tolerance of lower and upper bounds

Upper Bound : Feasible solution of (P) obtained by fixing the binary variables to the values obtained from the solution of the relaxation and solving the resulting NLP

Lower Bound : Obtained by solving a convex relaxation (R) of the non-convex MINLP model with Lagrangean Decomposition based cuts added to it

Upper Bound

Local solution of NLP- non-rigorous

Lower bound on Global Optimum

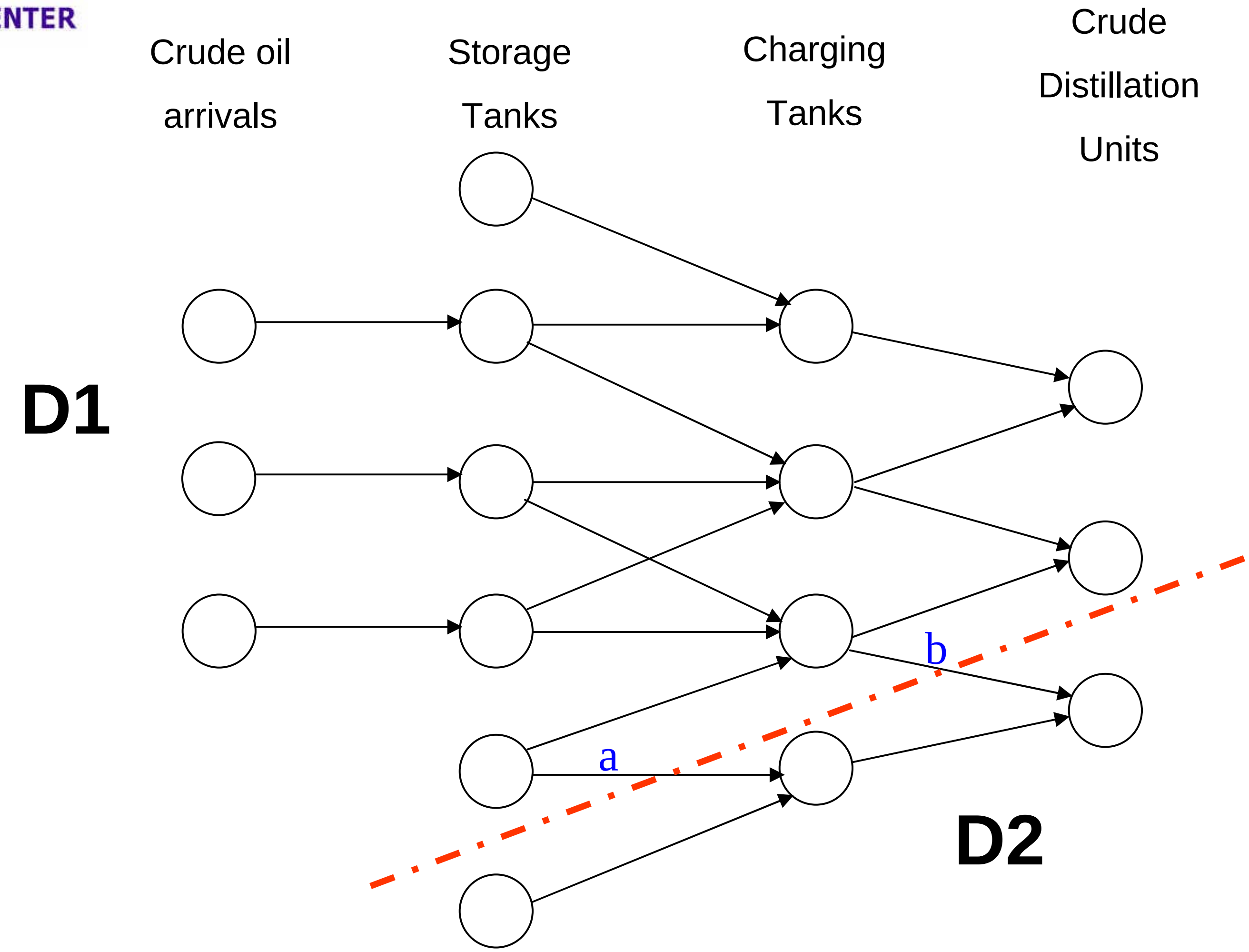
Convex relaxation (R) is a large MILP and is also difficult to solve

Generate cuts to add to relaxation to strengthen it and reduce solution times

Karuppiah and Grossmann (2006)

Cut generation performed by a *spatial decomposition* of the network structure

Spatial Decomposition of the network



- Network is split into two decoupled sub-structures D1 and D2
  - Physically interpreted as cutting some pipelines (Here ***a*** and ***b***)
  - Set of split streams denoted by p ∊ {a , b }

Decomposition of the model

- Create two copies of the variables pertaining to the split streams $\{V_{p,t}^{tot}, V_{p,t}^{j}, T_{p,t}^{1}, T_{p,t}^{2}, w_{p,t}\}$ and get two sets of *duplicate variables* : $\{V_{p,t}^{tot,1}, V_{p,t}^{j,1}, T_{p,t}^{1,1}, T_{p,t}^{2,1}, w_{p,t}^{1}\}$

  and

  $\{V_{p,t}^{tot,2}, V_{p,t}^{j,2}, T_{p,t}^{1,2}, T_{p,t}^{2,2}, w_{p,t}^{2}\}$

  The equations involving the split streams in model (R) are re-written in terms of the newly created variables

  These *duplicate variables* are related by equality constraints which are added to (R) to get model (RP):

$$V_{p,t}^{tot,1} = V_{p,t}^{tot,2} \qquad \forall p,t$$

$$V_{p,t}^{j,1} = V_{p,t}^{j,2} \qquad \forall j,p,t$$

$$T_{p,t}^{1,1} = T_{p,t}^{1,2} \qquad \forall p,t$$

$$T_{p,t}^{2,1} = T_{p,t}^{2,2} \qquad \forall p,t$$

$$w_{p,t}^{1} = w_{p,t}^{2} \qquad \forall p,t$$

Non-anticipativity constraints

- Non-anticipativity constraints in (RP) are multiplied by Lagrange multipliers and transferred to objective function to bring model to a decomposable form which is decomposed into sub-models (LD1) and (LD2)

Decomposed Sub-models

Sub-problem involves duplicate variables $\{V_{p,t}^{tot,1}, V_{p,t}^{j,1}, T_{p,t}^{1,1}, T_{p,t}^{2,1}, w_{p,t}^{1}\}$

$\min z_1 =$ *waiting cost for supply streams + unloading cost of supply streams + inventory cost for tanks in D1 over scheduling horizon + changeover cost for charging CDUs in D1 with different charging tanks +*

$$\sum_p \sum_t \lambda_{p,t}^{Vtot} V_{p,t}^{tot,1} + \sum_j \sum_p \sum_t \lambda_{j,p,t}^{V} V_{p,t}^{j,1} + \sum_p \sum_t \lambda_{p,t}^{T1} T_{p,t}^{1,1} + \sum_p \sum_t \lambda_{p,t}^{T2} T_{p,t}^{2,1} + \sum_p \sum_t \lambda_{p,t}^{w} w_{p,t}^{1}$$

s.t.
Tank constraints
Distillation unit constraints
Supply stream constraints
Variable bounds

(LD1)

Globally optimize to get solution z_1^*

Sub-problem involves duplicate variables $\{V_{p,t}^{tot,2}, V_{p,t}^{j,2}, T_{p,t}^{1,2}, T_{p,t}^{2,2}, w_{p,t}^{2}\}$

$\min z_2 =$ *inventory cost for tanks in D2 over scheduling horizon + changeover cost for charging CDUs in D2 with different charging tanks +*

$$-\sum_p \sum_t \lambda_{p,t}^{Vtot} V_{p,t}^{tot,2} - \sum_j \sum_p \sum_t \lambda_{j,p,t}^{V} V_{p,t}^{j,2} - \sum_p \sum_t \lambda_{p,t}^{T1} T_{p,t}^{1,2} - \sum_p \sum_t \lambda_{p,t}^{T2} T_{p,t}^{2,2} - \sum_p \sum_t \lambda_{p,t}^{w} w_{p,t}^{2}$$

s.t.
Tank constraints
Distillation unit constraints
Variable bounds

(LD2)

Globally optimize to get solution z_2^*

Cut Generation

- Using solutions z_1^* and z_2^* we develop the following cuts :

$z_1^* \leq$ *waiting cost for supply streams + unloading cost of supply streams + inventory cost for tanks in D1 over scheduling horizon + changeover cost for charging CDUs in D1 with different charging tanks +*

$$\sum_p \sum_t \lambda^{Vtot}_{p,t} V^{tot}_{p,t} + \sum_j \sum_p \sum_t \lambda^{V}_{j,p,t} V^{j}_{p,t} + \sum_p \sum_t \lambda^{T1}_{p,t} T^{1}_{p,t} + \sum_p \sum_t \lambda^{T2}_{p,t} T^{2}_{p,t} + \sum_p \sum_t \lambda^{w}_{p,t} w_{p,t}$$

Lagrange Multipliers

$z_2^* \leq$ *inventory cost for tanks in D2 over scheduling horizon + changeover cost for charging CDUs in D2 with different charging tanks +*

$$-\sum_p \sum_t \lambda^{Vtot}_{p,t} V^{tot}_{p,t} - \sum_j \sum_p \sum_t \lambda^{V}_{j,p,t} V^{j}_{p,t} - \sum_p \sum_t \lambda^{T1}_{p,t} T^{1}_{p,t} - \sum_p \sum_t \lambda^{T2}_{p,t} T^{2}_{p,t} - \sum_p \sum_t \lambda^{w}_{p,t} w_{p,t}$$

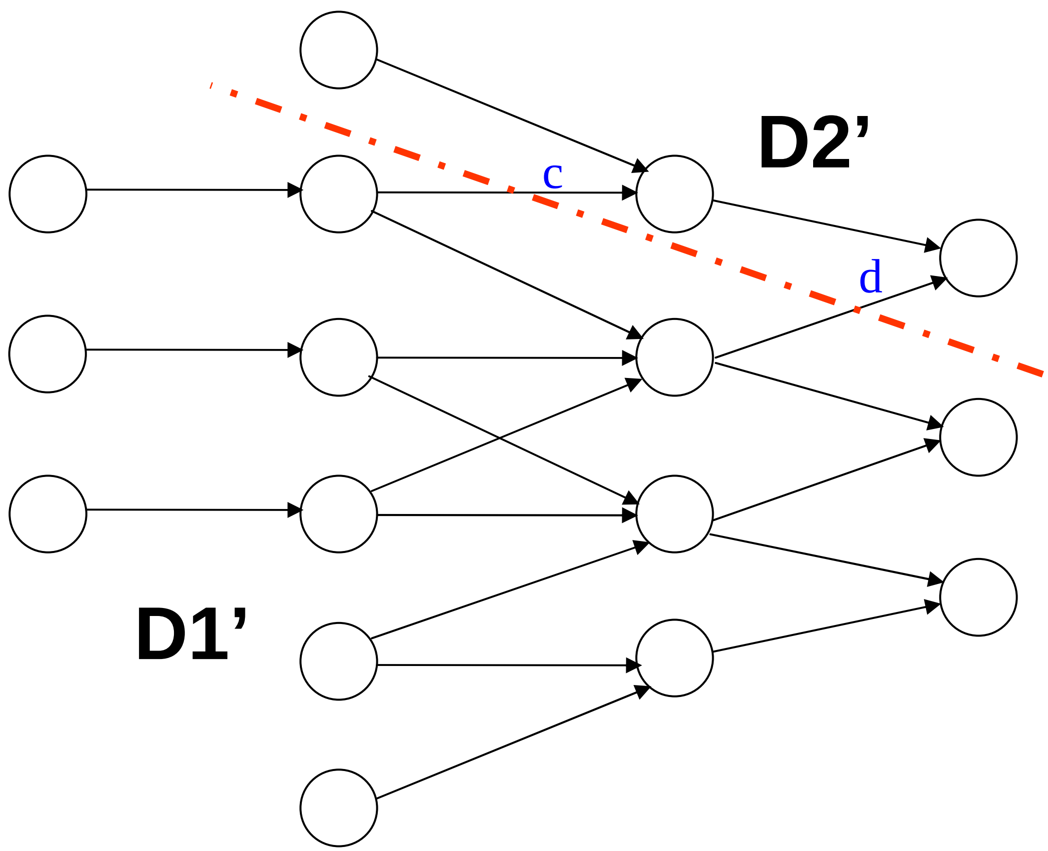
- Add above cuts to (R) to get (R') which is solved to obtain a valid lower bound on global optimum of (P)

Remark: Update Lagrange multipliers and generate more cuts to add to (R)

Advantages of Cut Generation

- Lower bound obtained is stronger (or as strong) than one from conventional Lagrangean decomposition or LP relaxation of (R)

- Alternative decomposition schemes can be used to generate more cuts to tighten relaxation (R)



Proposed Algorithm

Outer-Approximation based algorithm:

Step1: Preprocessing – Bounds on the variables in the model are determined by physical inspection of the network structure and using the numerical data given

Step2: *Lower Bound Generation* – Generate a valid lower bound on the solution by solving (R')

Step3: *Upper bound* – Fix integer variables in (P) to the values obtained from solution of (R') and solve resulting non-convex NLP denoted by (P-NLP)

Step4: *Integer Cut* – Making use of the integer solution of (R'), add an integer cut to model (R') to preclude the current combination of integer variables from re-appearing in future iterations

Step5: *Convergence* – Iterate between solving models (R') and (P-NLP) till the lower bound exceeds the upper bound or the relaxation gap between the lower and upper bounds is less than a specified tolerance

Illustrative Example

3 Supply streams – 6 Storage Tanks – 4 Charging Tanks – 3 Distillation units

Scheduling Horizon	15 hours		
Number of Input sources	3		
	Arrival Time	Incoming Volume of crude	Fraction of key component
IN1	1	60	0.03
IN2	6	60	0.05
IN3	11	60	0.065

Number of Charging Tanks	3		
	Capacity	Initial Inventory	Initial Fraction of key component (min – max)
Tank1	80	5	0.0317 (0.03 – 0.035)
Tank2	80	30	0.0483 (0.043 – 0.05)
Tank3	80	30	0.0633 (0.06 – 0.065)
Tank4	80	30	0.075 (0.071 – 0.08)

Number of Storage Tanks	6		
	Capacity	Initial Inventory	Initial fraction of key component (min – max)
Tank1	10 – 90	60	0.031 (0.025 – 0.038)
Tank2	10 – 110	10	0.03 (0.02 – 0.04)
Tank3	10 – 110	50	0.05 (0.04 – 0.06)
Tank4	10 – 110	40	0.065 (0.06 – 0.07)
Tank5	10 – 90	30	0.075 (0.07 – 0.08)
Tank6	10 – 90	60	0.075 (0.07 – 0.08)

Number of CDUs : 3
Waiting cost for supply streams (*Csea*): 5
Unloading cost for supply streams (*Cunload*): 7
Tank inventory costs (*Cinv(b)*): storage tanks – 0.05;
charging tanks – 0.06
Changeover cost for charged oil switch (*Cset*): 30

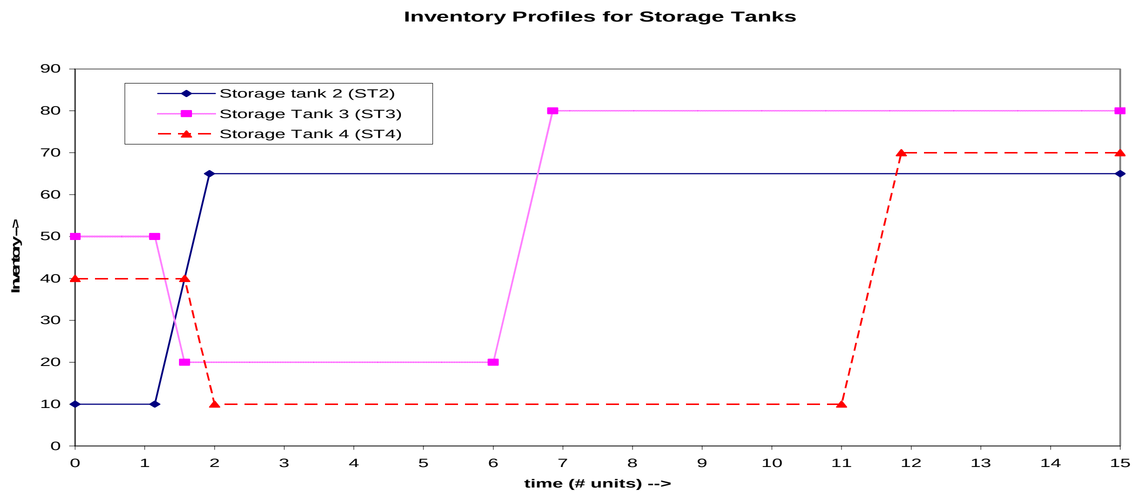
Demand of mixed oils by CDUs :

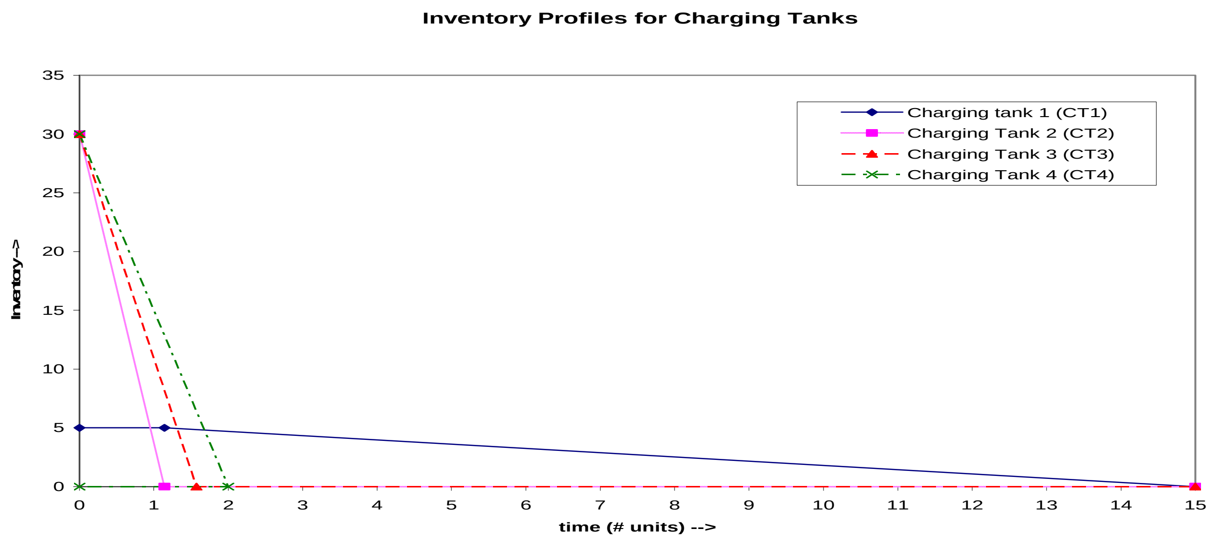
oil mix 1	60
oil mix 2	60
oil mix 3	60
oil mix 4	60

Bounds on flowrates in the streams: Lower Bound – 1.5, Upper Bound – 70

Optimal Crude Flow Schedule

Gantt chart of optimal schedule





Optimal Crude Flow Schedule

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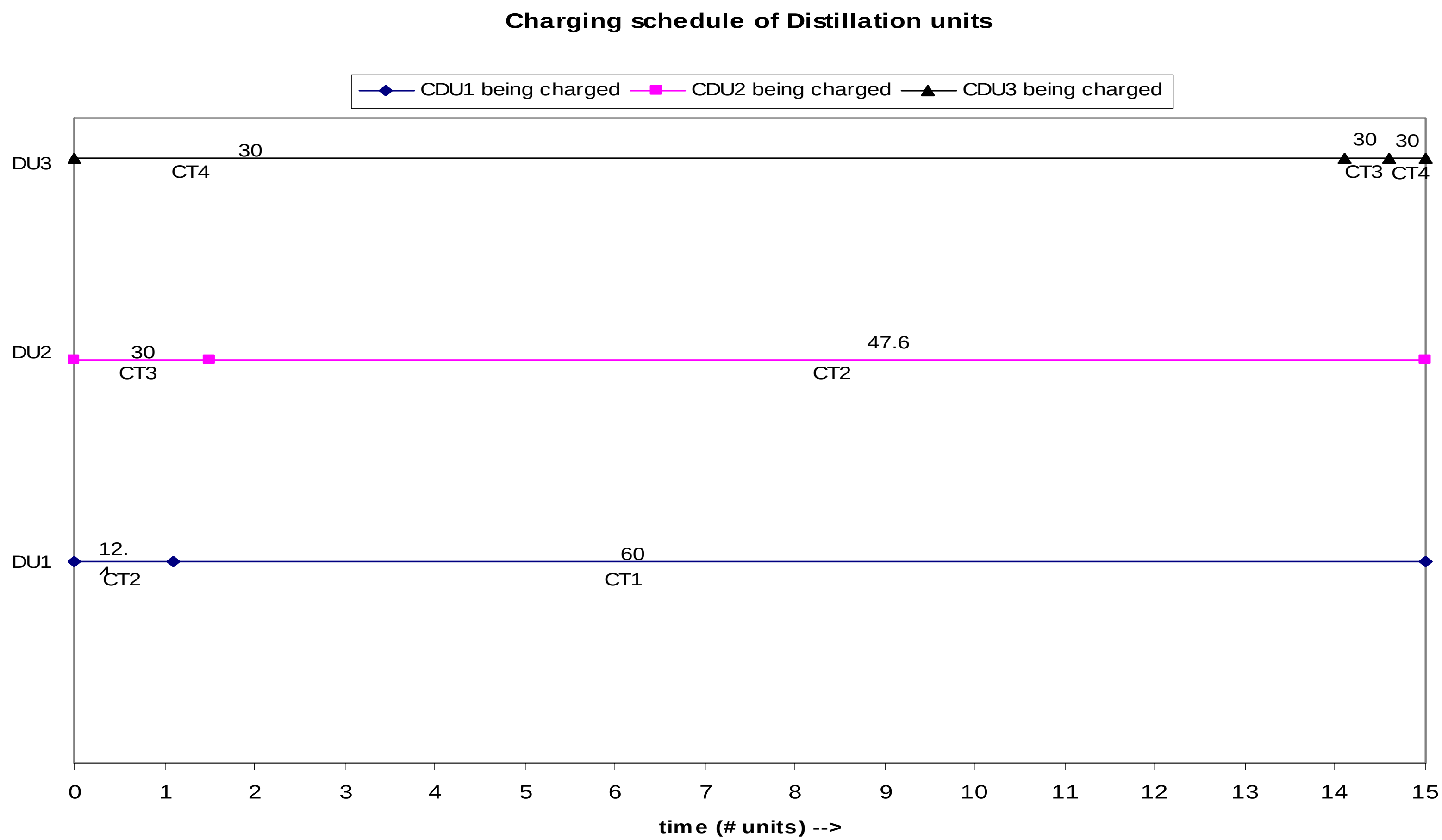

Preliminary Computational Results

MINLP 57 binary variables, 439 continuous variables and 1564 constraints

Sub-optimal solutions obtained (using GAMS/ DICOPT) : 447 or 463 vs 440.94 (global)

Proposed Algorithm :

Solvers Used : **MILP** ⇨ CPLEX 9.0, **NLP** ⇨ CONOPT3

Cut generation time* = 161.4 s

		Solution	Time* (sec)
Lower bound :	On solving (R) (without cuts)	440.93	7123.5
	On solving (R') (with proposed cuts)	440.93	2342.9
Upper bound :		440.94	

Total time* taken to solve problem = **2504.3 s**

Lower and Upper bounds converge within 1 % tolerance at 1st iteration of algorithm

BARON (Sahinidis, 1996) could not find global solution in more than 10 hours*

* Pentium IV, 2.8 GHz , 512 MB RAM

Future work

1. Consider addition of RLT constraints to strengthen master problem
2. Consider global solution of NLP subproblems
3. Increase model accuracy
4. Extend time horizon
5. Integration with downstream refinery