GDP Formulation of a segmented CDU Swing Cut Model for Refinery Planning

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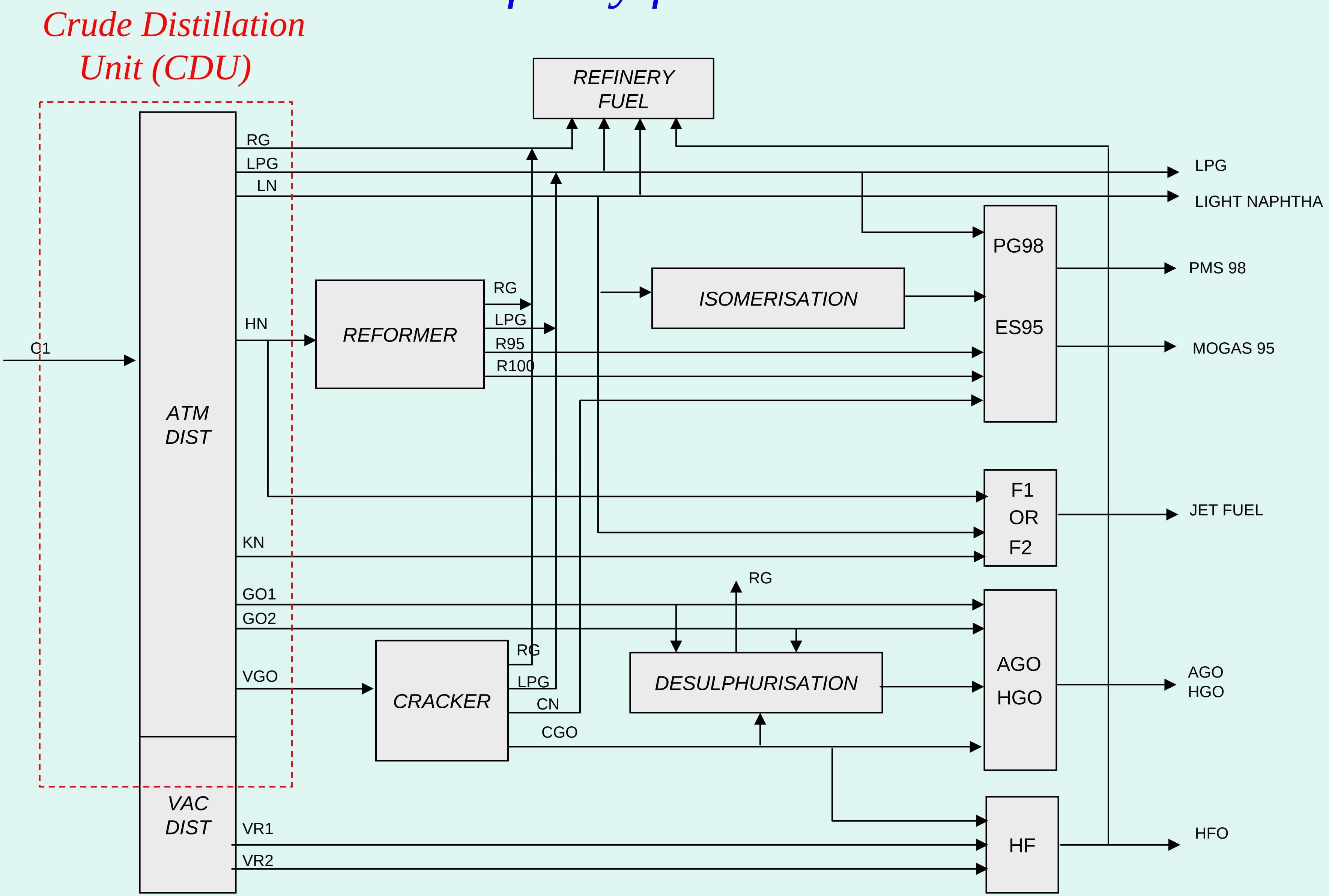
Motivation

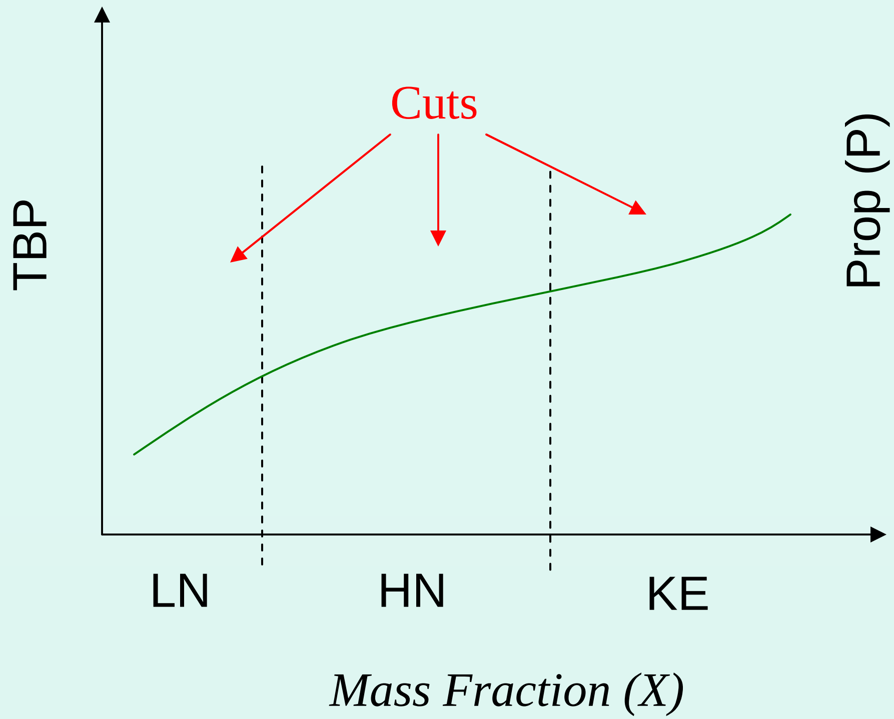
- **Refinery planning** is an active area in process systems that strongly **relies** on the accuracy of the **CDU model** that is used
- **Rigorous models** of CDU units may lead to **high computational efforts** preventing them from being applied on a regular basis (Barsamian, 2001)
- **Current approaches** such as fixed yield structure representation or Swing Cut Models (Zhang et al.,2001), although computationally efficient they are **not able to capture the actual behavior of CDUs** leading to planning solutions that may be suboptimal.

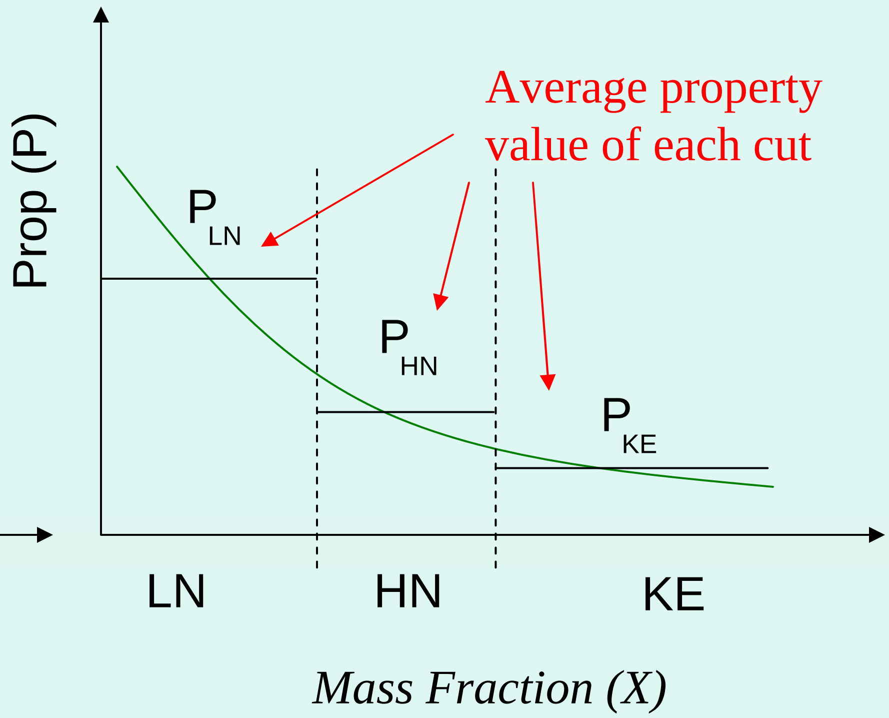
Goal: *Find a methodology that considers both,* ***accuracy*** *in quality estimation and* ***computational efficiency***

Refinery flow chart



Traditional Fixed Yield Model





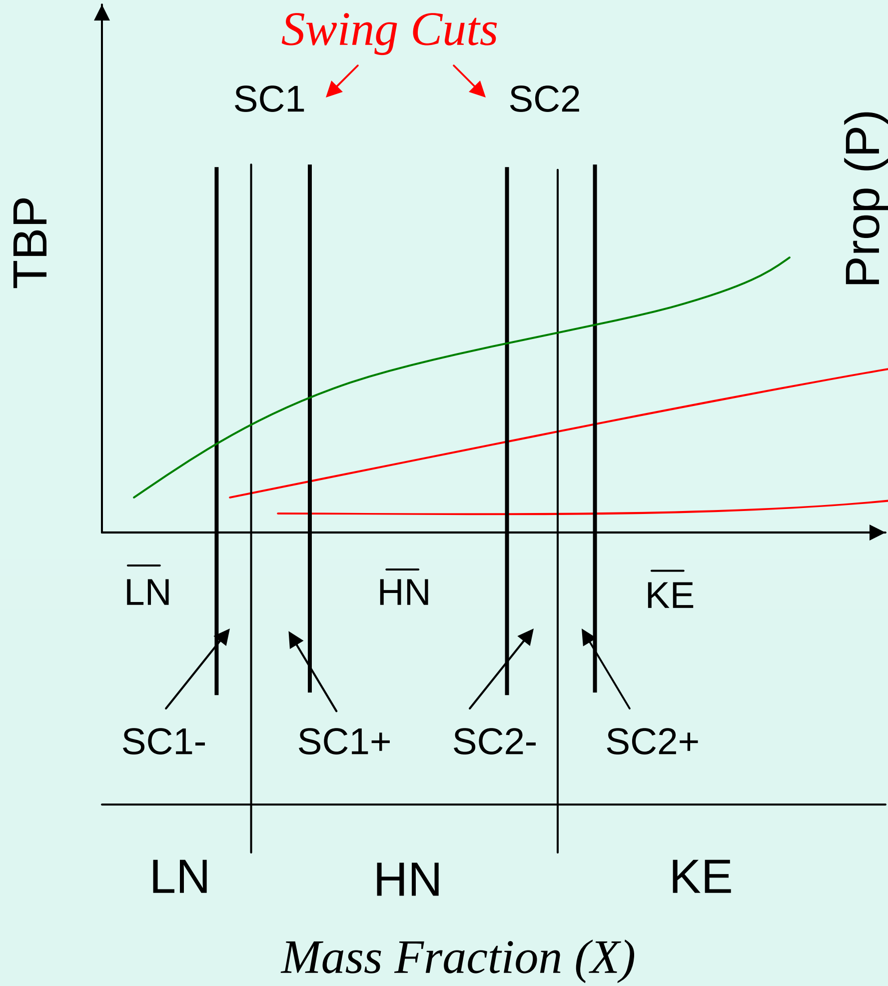
*The **mass fraction** of each cut and their **properties** are **fixed** !*

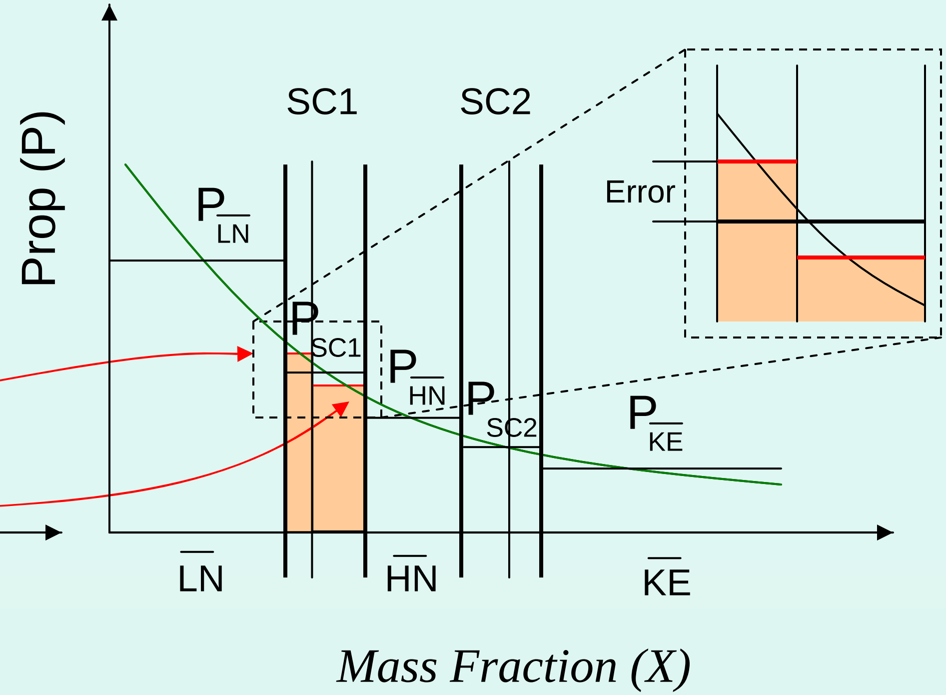
Example : HN cut definition

X_{HN} , P_{HN}

*Since the **actual temperature range** of each cut is **not fixed** (e.g. the HN cut is defined by an IBP: 270 F and EP : 325 – 400 F). The **yield** of each cut is a **natural variable** that should be obtained from the solution of the optimization problem. The **Swing Cut Model** considers this aspect.*

Traditional Swing Cut Model





Example : HN cut definition

$$X_{HN} = X_{\overline{HN}} + X_{SC1+} + X_{SC2-}$$

$$X_{HN}P_{HN} = P_{\overline{HN}}X_{\overline{HN}} + P_{SC1}X_{SC1+} + P_{SC2}X_{SC2-}$$

*It is clear that the **property value** of the the swing cut fraction **SC1+** is **different from** the value of the fraction **SC1-** . However, in the **traditional swing cut** model a **constant value for SC1** is considered.*

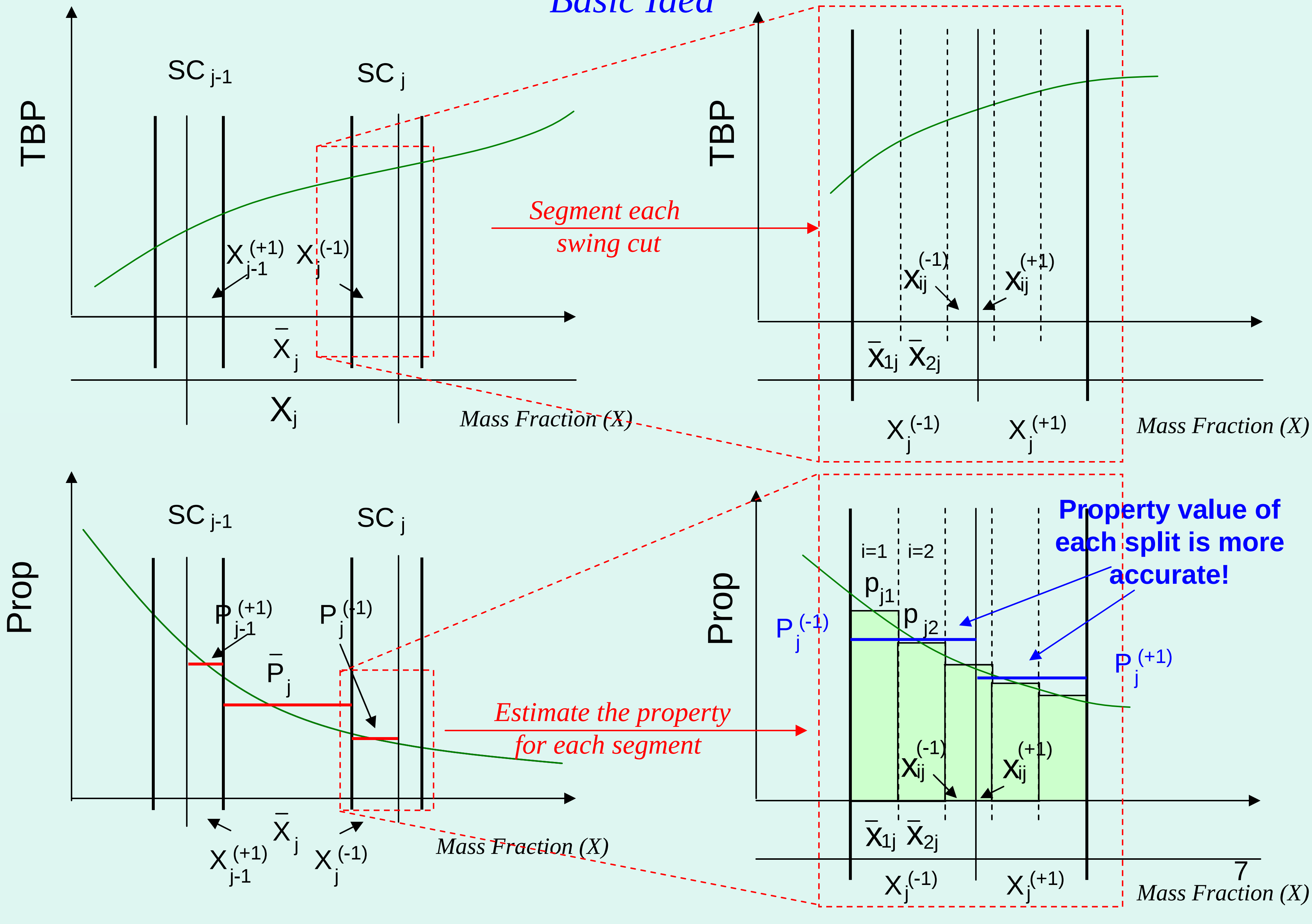
Summary of current approaches and goal of this work

- ***Fixed yield*** *structure representation model does* ***not*** *allow a* ***variable yield*** *and hence the* ***property values*** *of each cut is* ***predefined****.*
- ***Swing cut models*** *consider a variable yield, however, the* ***property estimations*** *are* ***inaccurate****.*

Objective: Develop a **GDP formulation** of the **swing cut model** by considering a **segmentation** of the swing cuts

Segmented Swing Cut Model

Basic Idea



Segmented Swing Cut Model

Formulation

Swing Cut Split Definition

$$\bigvee_{i \in I_j} \left[\begin{array}{c} Y_{ij} \\ X_j^{(-1)} = \sum_{k=1}^{k=i-1} \bar{x}_{kj} + x_{ij}^{(-1)} \\ X_j^{(+1)} = \sum_{k=i+1}^{k=|I_j|} \bar{x}_{kj} + (\bar{x}_{ij} - x_{ij}^{(-1)}) \\ X_j^{(-1)} P_j^{(-1)} = \sum_{k=1}^{k=i-1} \bar{x}_{kj}\, p_{kj} + x_{ij}^{(-1)} p_{ij} \\ X_j^{(+1)} P_j^{(+1)} = \sum_{k=i+1}^{k=|I_j|} \bar{x}_{kj}\, p_{kj} + (\bar{x}_{ij} - x_{ij}^{(-1)})\, p_{ij} \\ 0 \le x_{ij}^{(-1)} \le \bar{x}_{ij} \end{array} \right] \qquad j = 1,2,\ldots,|J|$$

Cut Property Definition

$$X_j P_j = \bar{X}_j \bar{P}_j + X_j^{(-1)} P_j^{(-1)} + X_{j-1}^{(+1)} P_{j-1}^{(+1)} \qquad j = 2,\ldots,|J|$$

$$X_1 P_1 = \bar{X}_1 \bar{P}_1 + X_1^{(-1)} \bar{P}_1^{(-1)}$$

$$X_{|J|+1} P_{|J|+1} = \bar{X}_{|J|+1} \bar{P}_{|J|+1} + X_{|J|}^{(+1)} P_{|J|}^{(+1)}$$

Bilinear

Cut Yield Definition

$$X_j = \bar{X}_j + X_j^{(-1)} + X_{j-1}^{(+1)} \qquad j = 2,\ldots,|J|$$

$$X_1 = \bar{X}_1 + X_1^{(-1)}$$

$$X_{|J|+1} = \bar{X}_{|J|+1} + X_{|J|}^{(+1)}$$

Sets:

J : Set of pseudo-cuts in which the segmentation takes place

I_j : Set of segments in the pseudo-cut j

Variables:

$X_j^{(-1)}$: Mass fraction of the left hand side of the pseudo-cut j after splitting

$X_j^{(+1)}$: Mass fraction of the right hand side of the pseudo-cut j after splitting

$P_j^{(-1)}$: Property value of the left hand side of the pseudo-cut j after splitting

$P_j^{(+1)}$: Property value of the right hand side of the pseudo-cut j after splitting

X_j : Mass fraction of the actual cut j

P_j : Property value of the actual cut j

$x_{ij}^{(+1)}$: Mass fraction of the right hand side of the segment i in the segmented pseudo-cut j after splitting

$x_{ij}^{(-1)}$: Mass fraction of the left hand side of the segment i in the segmented pseudo-cut j after splitting

Parameters:

$\bar{x}_{ij}$: Mass fraction of the segment i in the pseudo-cut j

p_{ij} : Property value of the segment i in the pseudo-cut j

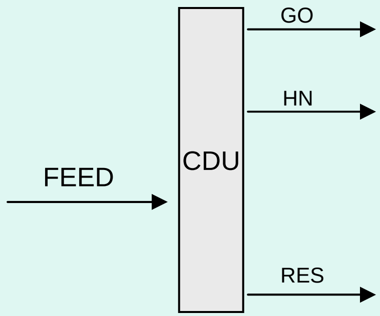
$\bar{X}_j$: Mass fraction of the non-segmented pseudo-cut j

$\bar{P}_j$: Property value of the non-segmented pseudo-cut j

Illustrative Example

Optimization of CDU production

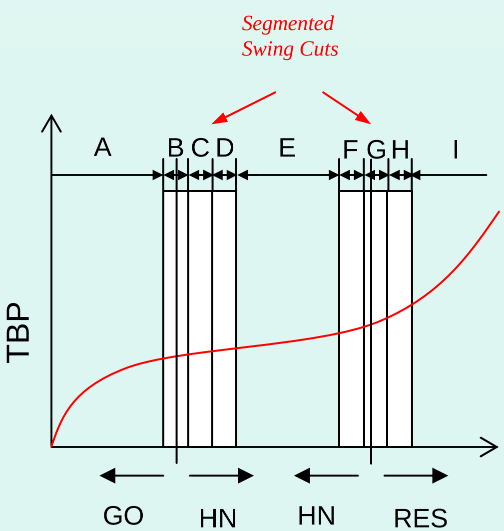
$$Max\ SP^*_{GO} X_{GO} + SP^*_{HN} X_{HN} + SP^*_{RES} X_{RES}$$ ← *Maximize profit*



Swing cut segmentation

$$\begin{bmatrix} Y_{BCD1} \\ X_{GO(BCD)} = X^{(-1)}{}_{(BCD)} \\ X_{HN(BCD)} = \bar{X}_C + \bar{X}_D + (\bar{X}_B - X^{(-1)}{}_{(BCD)}) \\ XP^{(-)}{}_{(BCD)} = P_B X^{(-1)}{}_{(BCD)} \\ XP^{(+)}{}_{(BCD)} = P_C \bar{X}_C + P_D \bar{X}_D + P_B(\bar{X}_B - X^{(-1)}{}_{(BCD)}) \\ 0 \le X^{(-1)}{}_{(BCD)} \le \bar{X}_B \end{bmatrix} \begin{bmatrix} Y_{BCD2} \\ X_{GO(BCD)} = X^{(-1)}{}_{(BCD)} + \bar{X}_B \\ X_{HN(BCD)} = \bar{X}_D + (\bar{X}_C - X^{(-1)}{}_{(BCD)}) \\ XP^{(-)}{}_{(BCD)} = P_B X^{(-1)}{}_{(BCD)} \\ XP^{(+)}{}_{(BCD)} = P_D \bar{X}_D + P_C(\bar{X}_C - X^{(-1)}{}_{(BCD)}) \\ 0 \le X^{(-1)}{}_{(BCD)} \le \bar{X}_C \end{bmatrix} \begin{bmatrix} Y_{BCD3} \\ X_{GO(BCD)} = \bar{X}_B + \bar{X}_C + X^{(-1)}{}_{(BCD)} \\ X_{HN(BCD)} = \bar{X}_D - X^{(-1)}{}_{(BCD)} \\ XP^{(-)}{}_{(BCD)} = P_C \bar{X}_C + P_B \bar{X}_B + P_D(\bar{X}_D - X^{(-1)}{}_{(BCD)}) \\ XP^{(+)}{}_{(BCD)} = P_D(\bar{X}_D - X^{(-1)}{}_{(BCD)}) \\ 0 \le X^{(-1)}{}_{(BCD)} \le \bar{X}_D \end{bmatrix}$$

$$\begin{bmatrix} Y_{FGH1} \\ X_{HN(FGH)} = X^{(-1)}{}_{(FGH)} \\ X_{RES(FGH)} = \bar{X}_G + \bar{X}_H + (\bar{X}_F - X^{(-1)}{}_{(FGH)}) \\ XP^{(-)}{}_{(FGH)} = P_F X^{(-1)}{}_{(FGH)} \\ XP^{(+)}{}_{(FGH)} = P_G \bar{X}_G + P_H \bar{X}_H + P_F(\bar{X}_F - X^{(-1)}{}_{(FGH)}) \\ 0 \le X^{(-1)}{}_{(FGH)} \le \bar{X}_F \end{bmatrix} \begin{bmatrix} Y_{FGH2} \\ X_{HN(FGH)} = X^{(-1)}{}_{(FGH)} + \bar{X}_F \\ X_{RES(FGH)} = \bar{X}_H + (\bar{X}_G - X^{(-1)}{}_{(FGH)}) \\ XP^{(-)}{}_{(FGH)} = P_F \bar{X}_F + P_G X^{(-1)}{}_{(FGH)} \\ XP^{(+)}{}_{(FGH)} = P_H \bar{X}_H + P_G(\bar{X}_G - X^{(-1)}{}_{(FGH)}) \\ 0 \le X^{(-1)}{}_{(FGH)} \le \bar{X}_G \end{bmatrix} \begin{bmatrix} Y_{FGH3} \\ X_{HN(FGH)} = \bar{X}_F + \bar{X}_G + X^{(-1)}{}_{(FGH)} \\ X_{HN(FGH)} = \bar{X}_H - X^{(-1)}{}_{(FGH)} \\ XP^{(-)}{}_{(FGH)} = P_F \bar{X}_F + P_G \bar{X}_G + P_H(\bar{X}_H - X^{(-1)}{}_{(FGH)}) \\ XP^{(+)}{}_{(FGH)} = P_H(\bar{X}_H - X^{(-1)}{}_{(FGH)}) \\ 0 \le X^{(-1)}{}_{(FGH)} \le \bar{X}_H \end{bmatrix}$$



$$X_{HN} = \bar{X}_{HN(E)} + X_{HN(BCD)} + XP_{HN(FGH)} \ ; \ X_{GO} = \bar{X}_{GO(A)} + X_{GO(BCD)} \ ; \ X_{RES} = \bar{X}_{RES(I)} + X_{RES(FGH)}$$

$$X_{GO}P_{GO} = \bar{X}_{GO(A)} P_{GO(A)} + XP^{(-)}{}_{(BCD)} \ ; X_{HN}P_{HN} = \bar{X}_{HN(E)} P_{HN(E)} + XP^{(+)}{}_{(BCD)} + XP^{(-)}{}_{(FGH)} \ ; X_{RES}P_{RES} = \bar{X}_{RES(I)} P_{RES(I)} + XP^{(+)}{}_{(FGH)}$$

$$P_{GO} \le P_{GO}{}^* ; \ P_{HN} \le P_{HN}{}^* \ ; \ P_{RES} \le P_{RES}{}^* \longrightarrow X_{GO}P_{GO} \le X_{GO}P_{GO}{}^* \ ; X_{HN}P_{HN} \le X_{HN}P_{HN}{}^* \ ; X_{RES}P_{RES} \le X_{RES}P_{RES}{}^*$$

$$X_{GO} + X_{HN} + X_{RES} = 1$$

$$Y_{BCD1} \vee Y_{BCD2} \vee Y_{BCD3} = True$$
$$Y_{FGH1} \vee Y_{FGH2} \vee Y_{FGH3} = True$$

Under this set of constraints *the bilinear terms can be substituted by the correspondent linear expressions, leading to a* ***Linear GDP***

In general bilinear terms cannot always be eliminated!

Remarks and Conclusions

- *A **more accurate representation** of the quality of the swing cut can be obtained if a segmentation is performed.*
- *A segmentation of the swing cut leads to a **GDP that is still linear** (and hence its MIP reformulation) under the same assumptions of the two previous methods presented.*
- *If a particular value representing the **quality** of the product is **necessary** in the model **individually**, such as Pj (e.g. splitting and blending operations downstream) it is clear that a **bilinear GDP arises.***
- ***Strategies** to solve Bilinear GDPs efficiently **have been developed** (Ruiz & Grossmann, 2007)*