Centralized versus Distributed Manufacturing: A Continuous Location-Allocation Problem

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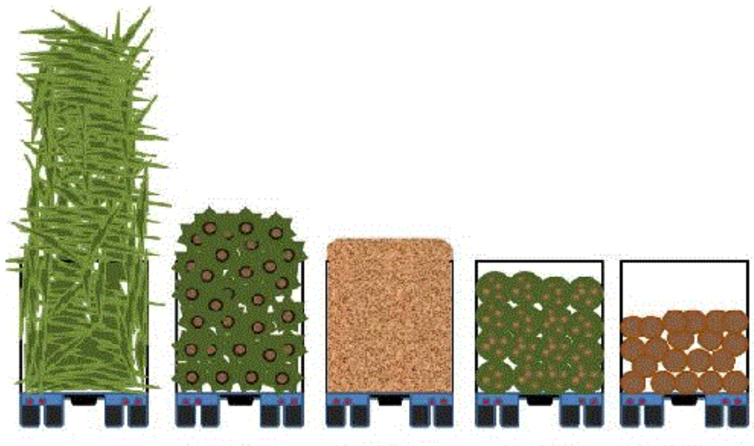
Motivation: Rethinking of traditional manufacturing

Distributed Manufacturing – geographically dispersed network of facilities

- Exploit new technology and modularity
- Attend new requirements of the market
- Logistical aspects

Potential applications

- Biomass supply chain (ethanol production)
- Shale gas supply chain (gas processing plants)
- Electric power generation (distributed power)



Motivation: Rethinking of traditional manufacturing

Tradeoff: Capital cost vs. Transportation Cost

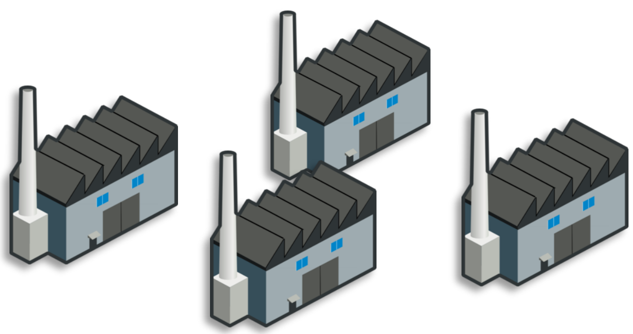
- Potential advantages of Distributed Manufacturing
- Economy of scale favors large-scale production

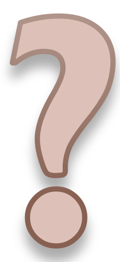
Need for a general framework that captures the tradeoff and design best network

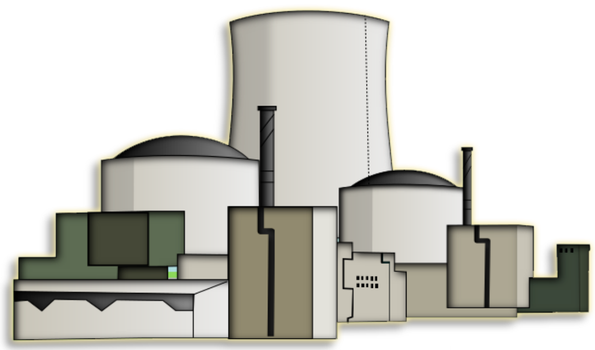
- Evaluate cost of centralized versus distributed manufacturing
- Address higher level planning problems

Problem formulated as Capacitated Multi-facility Weber problem[1]

- Determine location in continuous 2-D space for new facilities in relation to the location of existing facilities

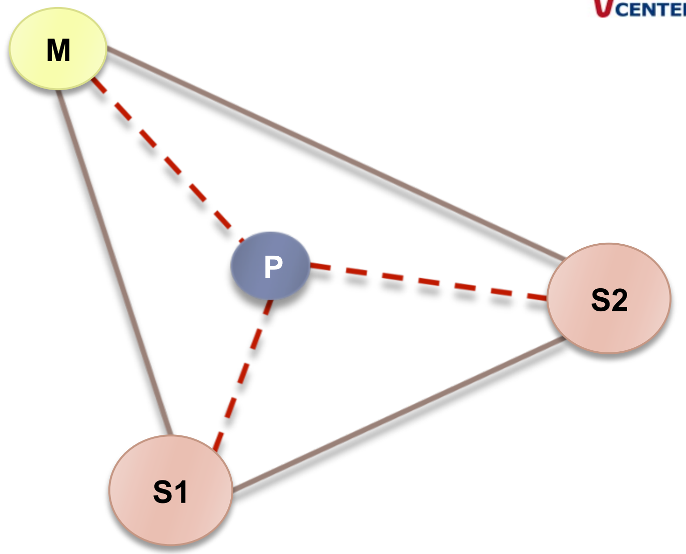






1 Brimberg, P. Hansen, N. Mladonovic, and S. Salhi, "A survey of solution methods for the continuous location allocation problem,", 2008.

Background: The Weber Problem



The Original Weber Problem (1909)[2]

- 2 suppliers, 1 market, and 1 facility
- Fixed points not colinear
- Euclidean distances
- Find facility location in 2-D space

Capacitated Multi-facility Weber Problem

- Facilities to be installed have maximum capacity
- Cooper (1972) was the first to attempt this problem[3]
  - Exact method: can only be applied for very small-problems
  - Heuristic method: Alternate the solution of the transportation and location problems until convergence. Do not guarantee optimality
- Sherali, Al-Lougani, Subramanian (2002) developed a Branch-and-Bound Algorithm[4]
- Several heuristic methods[5]

2 A. Weber and C. J. Friedrich, *Theory of the Location of Industries*, 1929.
3 L. Cooper, "The Transportation-Location Problem," *1972*.
4 H. D. Sherali, I. Al-Loughani, and S. Subramanian, "Global Optimization Procedures for the Capacitated Euclidean and l p Distance Multifacility Location-allocation Problems," , 2002
5 J. Brimberg, P. Hansen, N. Mladonovic, and S. Salhi, "A survey of solution methods for the continuous location allocation problem,", 2008.

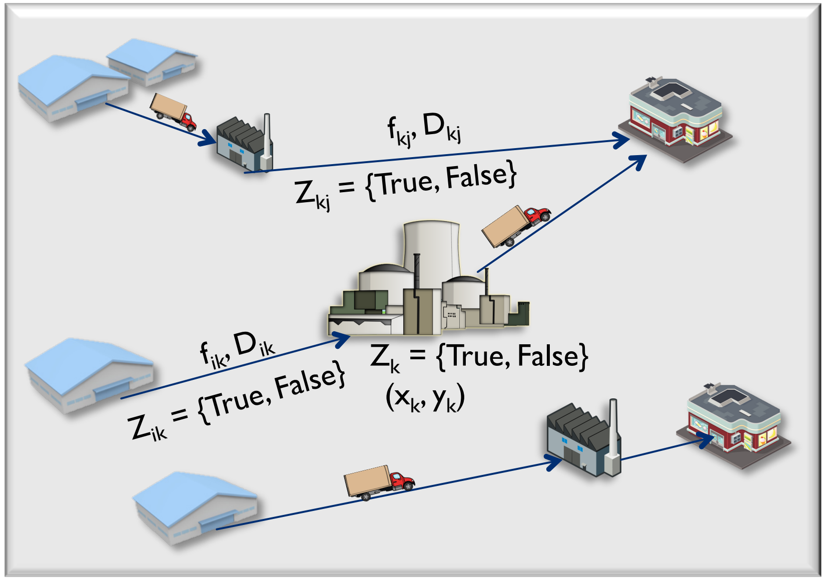
Problem statement

Given:

- A set of suppliers i, a set of consumer markets j, and their respective fixed location, availability and demand
- M potential distributed and N potential centralized set of k single-product facilities, and their corresponding maximum capacity and conversion rate (unknown location)
- Investment, operating and transportation costs

Find:

- Number, type and 2-D location of facilities to design a manufacturing network that minimizes the cost



Continuous variables: x_k, y_k, f_{ik}, f_{kj}, f_k, D_{ik}, D_{kj}
Boolean variables: Z_k, Z_{ik}, Z_{kj}

General Disjunctive Programming (GDP) Formulation

$$\text{Min } Cost = \sum_k Cost_k + \sum_i \sum_k Cost_{ik} + \sum_k \sum_j Cost_{kj}$$

Total cost

$$\begin{bmatrix} Z_k \\ Cost_k = ff_k + vf_k \cdot f_k \\ 0 \le f_k \le mc_k \\ 0 \le x_k \le x_k^U \\ 0 \le y_k \le y_k^U \end{bmatrix} \vee \begin{bmatrix} \neg Z_k \\ Cost_k = 0 \\ f_k = 0 \\ x_k = 0 \\ y_k = 0 \end{bmatrix} \quad ,\forall k \in K$$

Choice of facility

$$\begin{bmatrix} Z_{ik} \\ Cost_{ik} = RM_i \cdot f_{ik} + ft_{ik} + vt_{ik} \boxed{f_{ik} \cdot D_{ik}} \\ 0 \le f_{ik} \le f_{ik}^U \\ R \le D_{ik} \le D_{ik}^U \end{bmatrix} \vee \begin{bmatrix} \neg Z_{ik} \\ Cost_{ik} = 0 \\ f_{ik} = 0 \end{bmatrix} \quad ,\forall\, i \in I,\ k \in K$$

Choice of link supplier/ facility

$$\begin{bmatrix} Z_{kj} \\ Cost_{kj} = ft_{kj} + vt_{kj} \boxed{f_{kj} \cdot D_{kj}} \\ 0 \le f_{kj} \le f_{kj}^U \\ R \le D_{kj} \le D_{kj}^U \end{bmatrix} \vee \begin{bmatrix} \neg Z_{kj} \\ Cost_{kj} = 0 \\ f_{kj} = 0 \end{bmatrix} \quad ,\forall\, k \in K, j \in J$$

Choice of link facility/ market

$$D_{ik} \ge \sqrt{(x_i - x_k)^2 + (y_i - y_k)^2} \quad ,\forall\, i \in I,\ k \in K$$

Distance supplier/facility

$$D_{kj} \ge \sqrt{(x_j - x_k)^2 + (y_j - y_k)^2} \quad ,\forall\, k \in K, j \in J$$

Distance facility/market

$$Z_k \Leftrightarrow \vee_i Z_{ik} \quad ,\forall k \in K$$

$$Z_k \Leftrightarrow \vee_j Z_{kj} \quad ,\forall k \in K$$

Logic constraints

$$\sum_k f_{ik} \le a_i \quad ,\forall i \in I$$

Availability of raw-material

$$\sum_i f_{ik} \cdot cv_k = f_k \quad ,\forall k \in I$$

$$f_k = \sum_j f_{kj} \quad ,\forall k \in K$$

Mass balances

$$\sum_k f_{kj} = d_j \quad ,\forall j \in J$$

Market demand

$$Z_k, Z_{ik}, Z_{kj} \in \{True, False\} \quad ,\forall i = i \in K, \quad k \in K, \quad j \in J$$

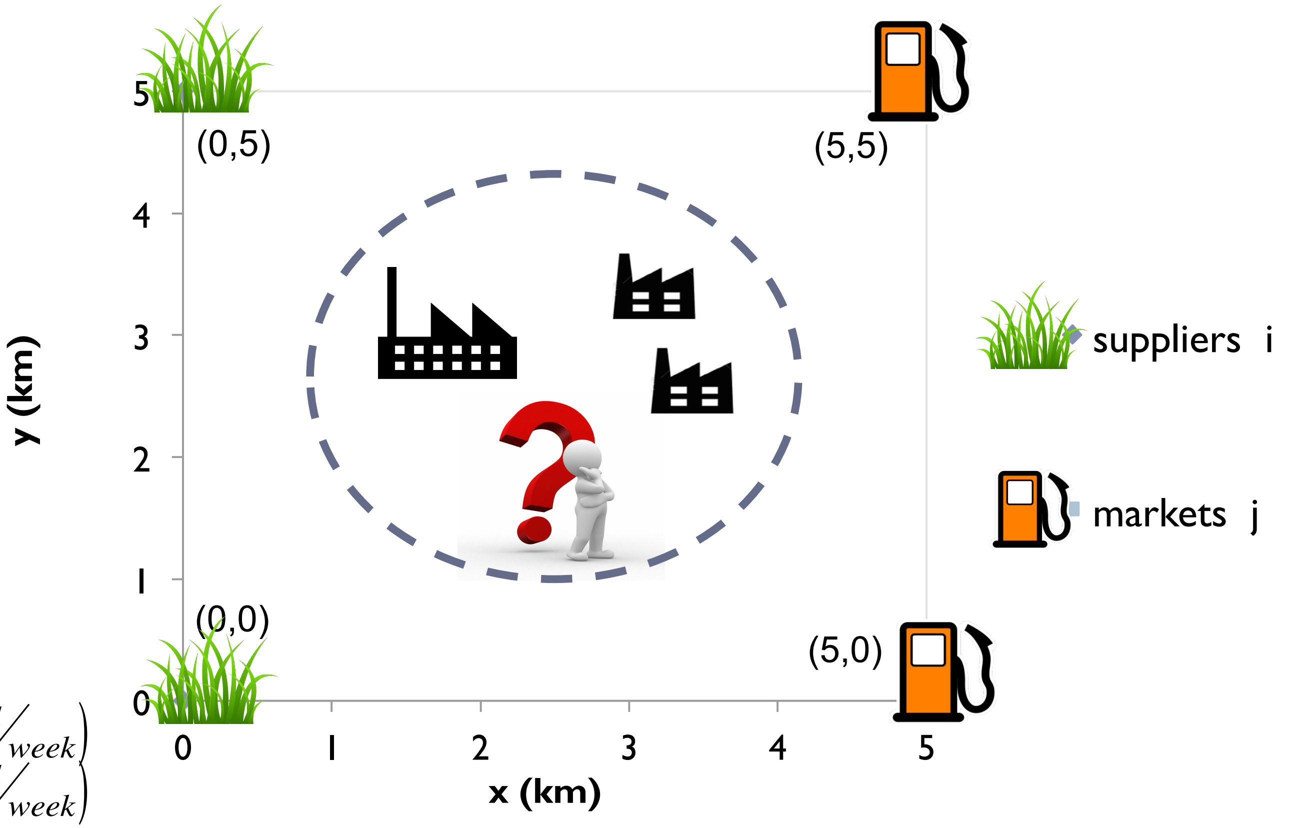
Illustrative example: ethanol production

Problem:

- 2 switchgrass suppliers
- 120 $\frac{tons}{week}$ of switchgrass available per supplier
    - Supplier 1: $\frac{\$2{,}000}{ton}$
    - Supplier 2: $\frac{\$2{,}200}{ton}$

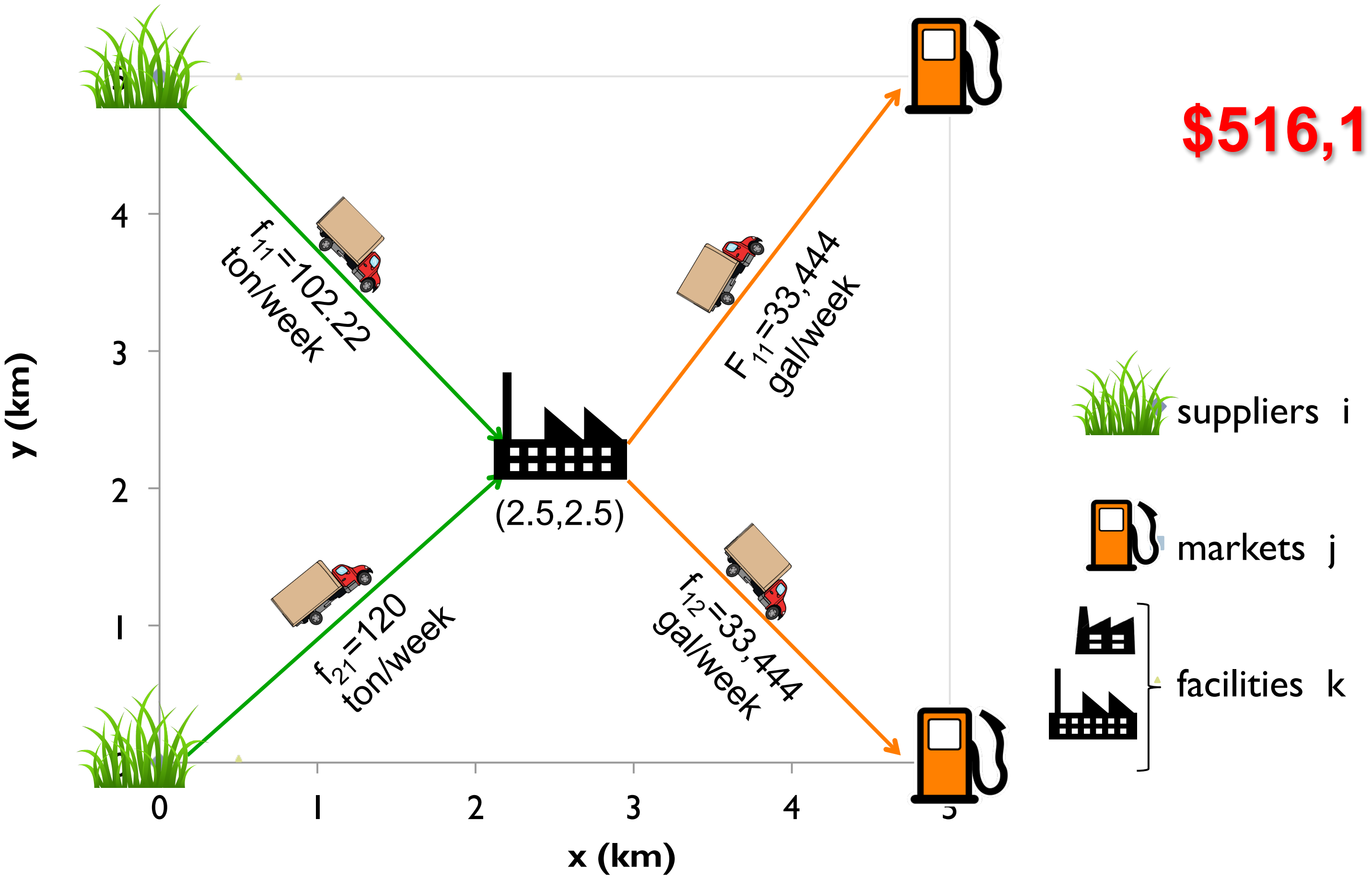
- 2 markets
- Demand of 33,444 $\frac{gal}{week}$ of ethanol per market

- 3 potential facilities $(cv_k = 90\%)$
    - 2 distributed $\left(mc_m = 41{,}792 \frac{gal}{week}\right)$
    - 1 centralized $\left(mc_n = 83{,}610 \frac{gal}{week}\right)$



Illustrative example: ethanol production

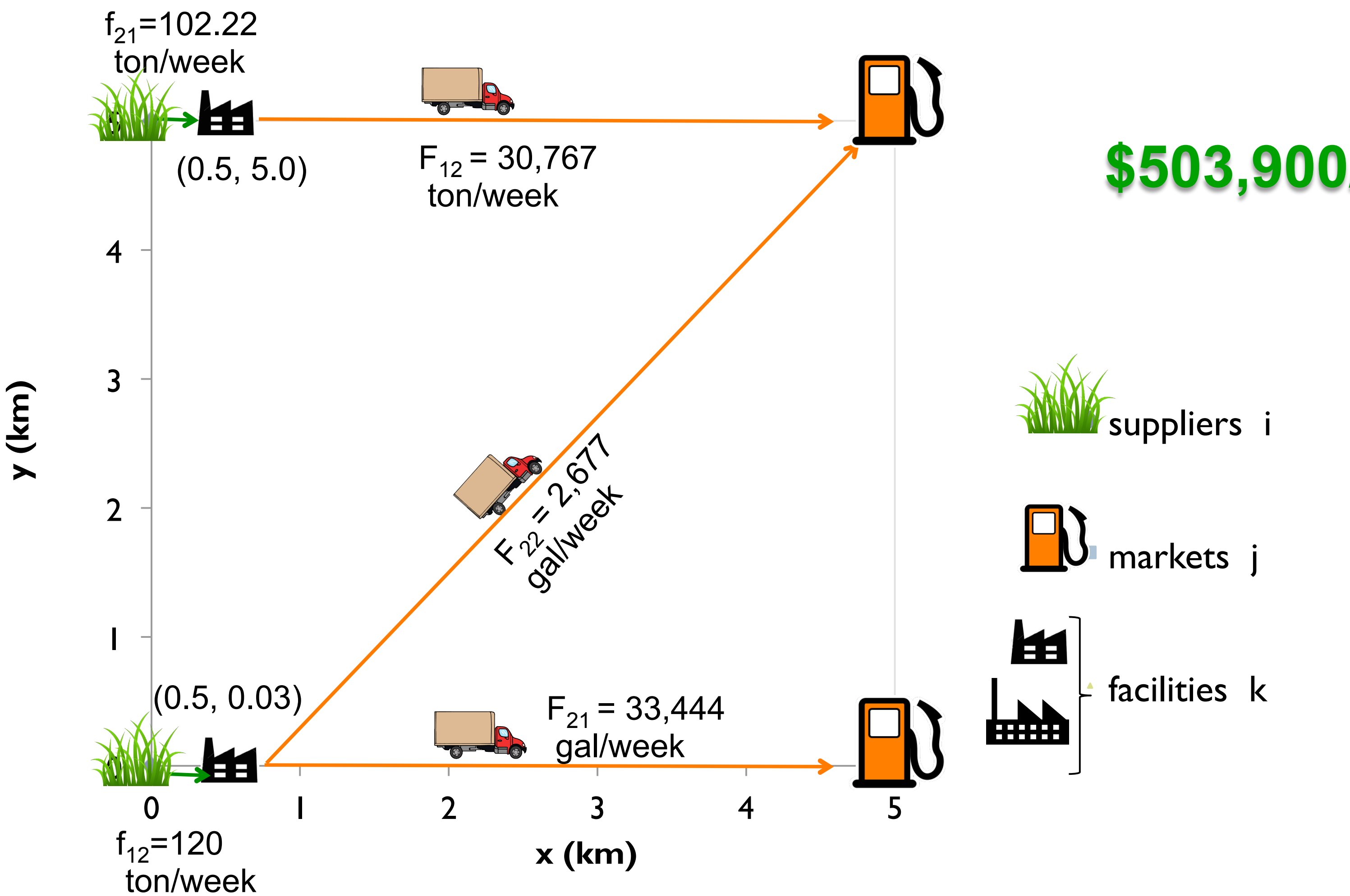
Intuitive answer: 1 centralized facility



$516,100/week

Illustrative example: ethanol production

Optimal network: 2 distributed facilities



Illustrative example

Computational results

Global Optimization:

METHOD	Cost $\left(\$10^3/_{week}\right)$	CPU time (s)
BARON	503.9	703.2
SCIP	503.9	656.7
Multiparametric Disagreggation[6]	503.9	1,045.6
Bilevel decomposition	503.9	**36.1**

Convex relaxation (lower bound)

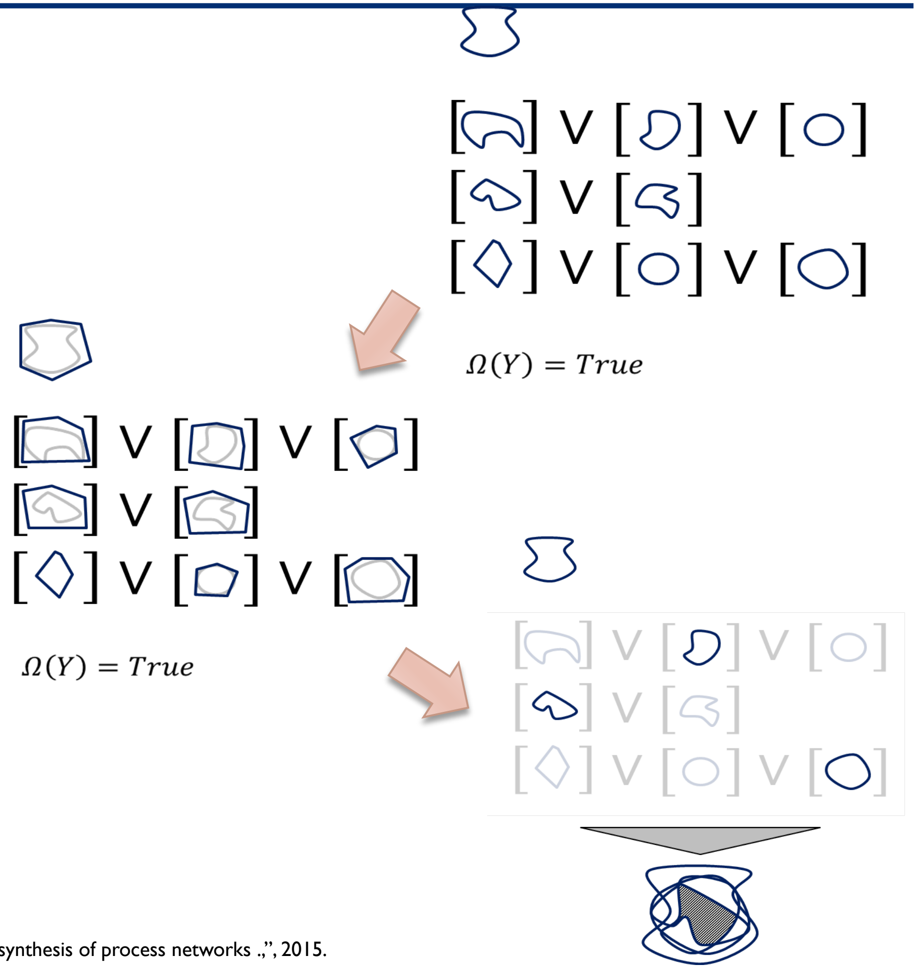
METHOD	Cost $\left(\$10^3/_{week}\right)$	CPU time (s)
McCormick	482.4	0.3
Piecewise McCormick[7] (16 partitions)	503.8	2065.5
Logarithmic Piecewise McCormick[8] (16 partitions)	503.8	**35.9**

6 S. Kolodziej, P. M. Castro, and I. E. Grossmann, "Global optimization of bilinear programs with a multiparametric disaggregation technique," *J*, 2013
7 P. M. Castro, "Tightening piecewise McCormick relaxations for bilinear problems,", 2014
8 R. Misener, J. P. Thompson, and C. a. Floudas, "Apogee: Global optimization of standard, generalized, and extended pooling problems via linear and logarithmic partitioning schemes," 2011

Bilevel decomposition: Background

Global Logic-Based Outer Approximation (GLBOA)[9]

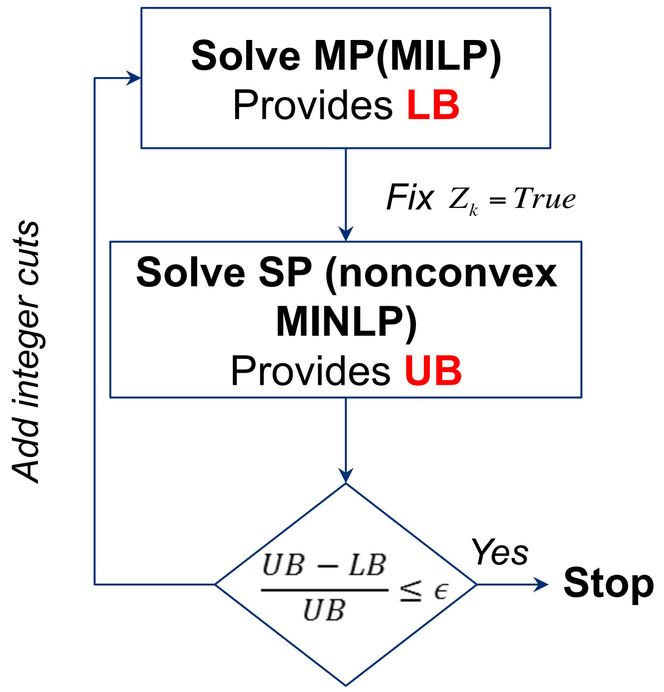
- Non-convex GDP
- Master problem (MP): linear relaxation of the nonconvex GDP
  - ➢ Lower Bound
- Subproblem (SP): lower dimensional nonconvex NLP in which the Boolean variables are fixed in the GDP
  - ➢ Upper bound
- Every time MP is resolved, an integer cut is added to exclude fixed discrete variables already used



9 F. Trespalacios and I. E. Grossmann, "Cutting planes for improved global logic-based outer-approximation for the synthesis of process networks .", 2015.

Bilevel decomposition algorithm

Algorithm



Description

Master problem: linear GDP relaxation provides a lower bound, and the selection of facilities to fix

- Bilinear terms are approximated using Logarithmic Piecewise McCormick[10]
- Distance constraints, which are convex, are linearized for a given discretization of space

Subproblem: For the fixed alternative of facilities, the MINLP is solved with global solver to obtain an upper bound

- Potential links, which involve discrete variables, are still to be determined.

Integer cuts are added to the (MP)

10 R. Misener, J. P. Thompson, and C. a. Floudas, "Apogee: Global optimization of standard, generalized, and extended pooling problems via linear and logarithmic partitioning schemes," 2011

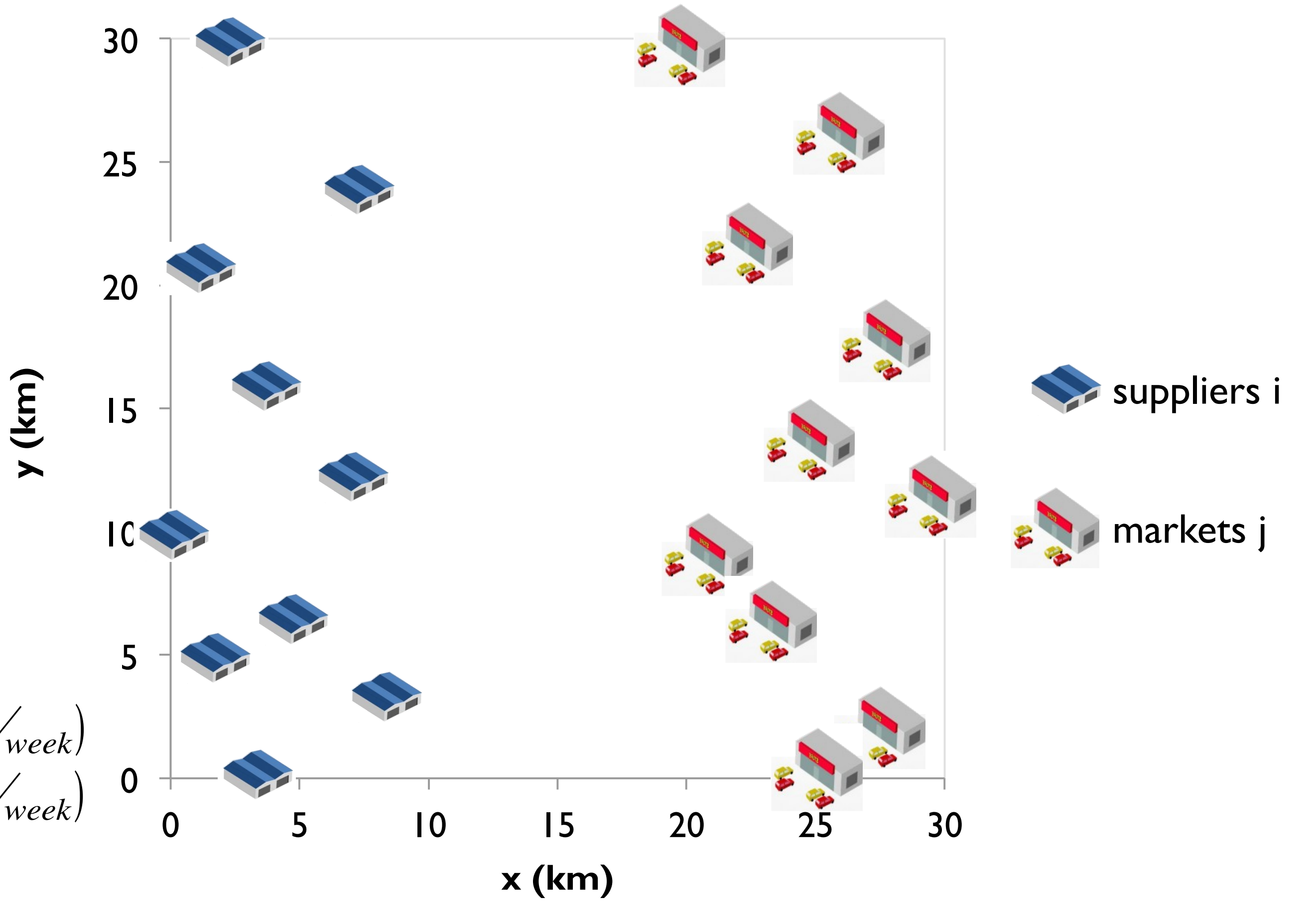
Large-scale problems: Example 1

Problem

- 10 suppliers
- 120 $^{tons}/_{week}$ of raw material available per supplier

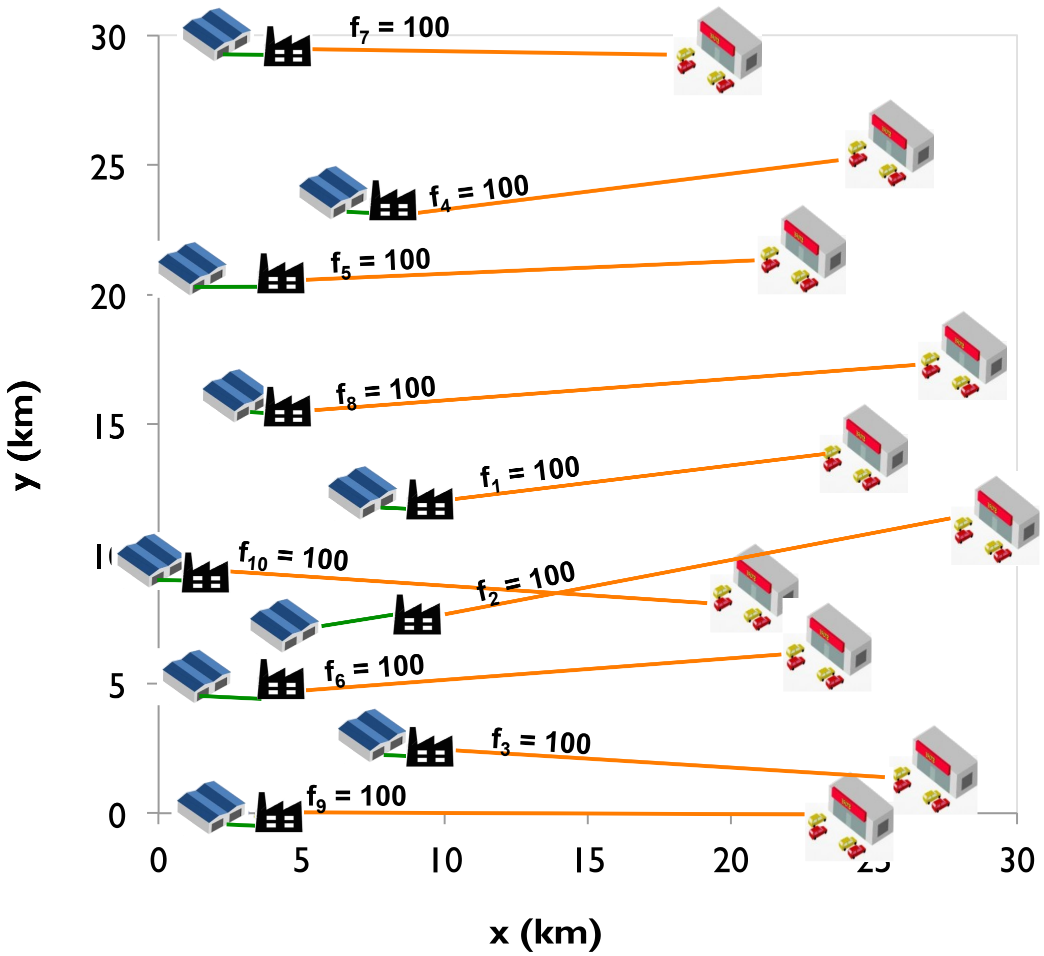
- 10 markets
- Demand of 100 $^{tons}/_{week}$ per market

- 12 potential facilities $(cv_k = 90\%)$
    - 10 distributed $\left(mc_m = 100\,^{tons}/_{week}\right)$
    - 2 centralized $\left(mc_n = 1000\,^{tons}/_{week}\right)$

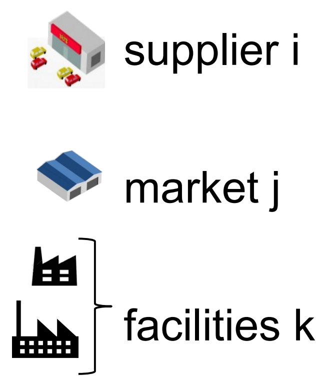


Large-scale problems: Example 1

Optimal network found: 10 distributed facilities



$28,991,000 /week



Large-scale problems: Example 1

Computational results

Global Optimization:

METHOD	Cost $\left(\$10^3/_{week}\right)$	Optimality gap (%)	CPU time (hrs)
BARON	29,054	21%	12*
SCIP	29,892	92%	12*
Bilevel Decomposition	**28,991**	**9%****	**4***

* Exceeded maximum CPU time
** Estimated gap
For the Bilevel Decomposition Algorithm, the master problem (MILP) was solved using CPLEX and the subproblem (nonconvex MINLP) was solved using BARON

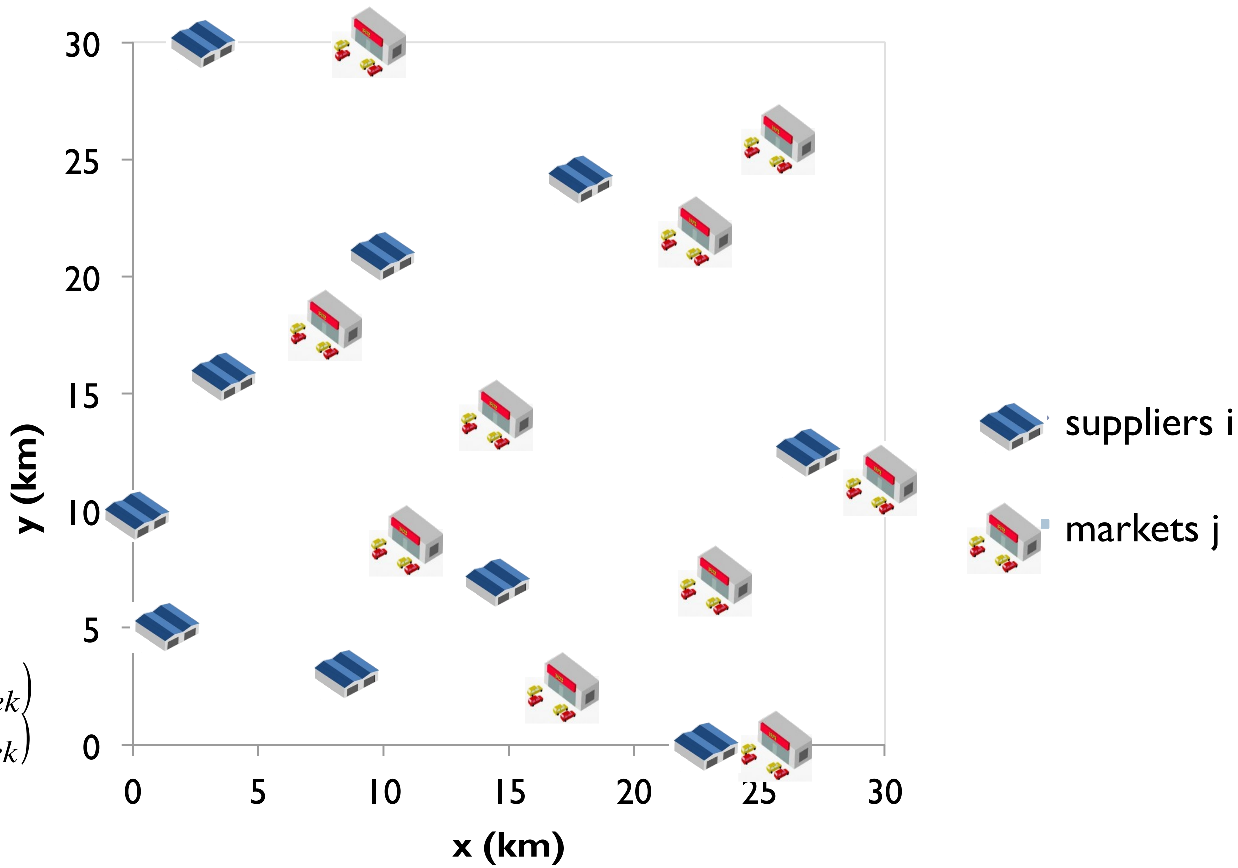
Large-scale problems: Example 2

Problem

- 10 suppliers
- Different availability of raw material for each supplier

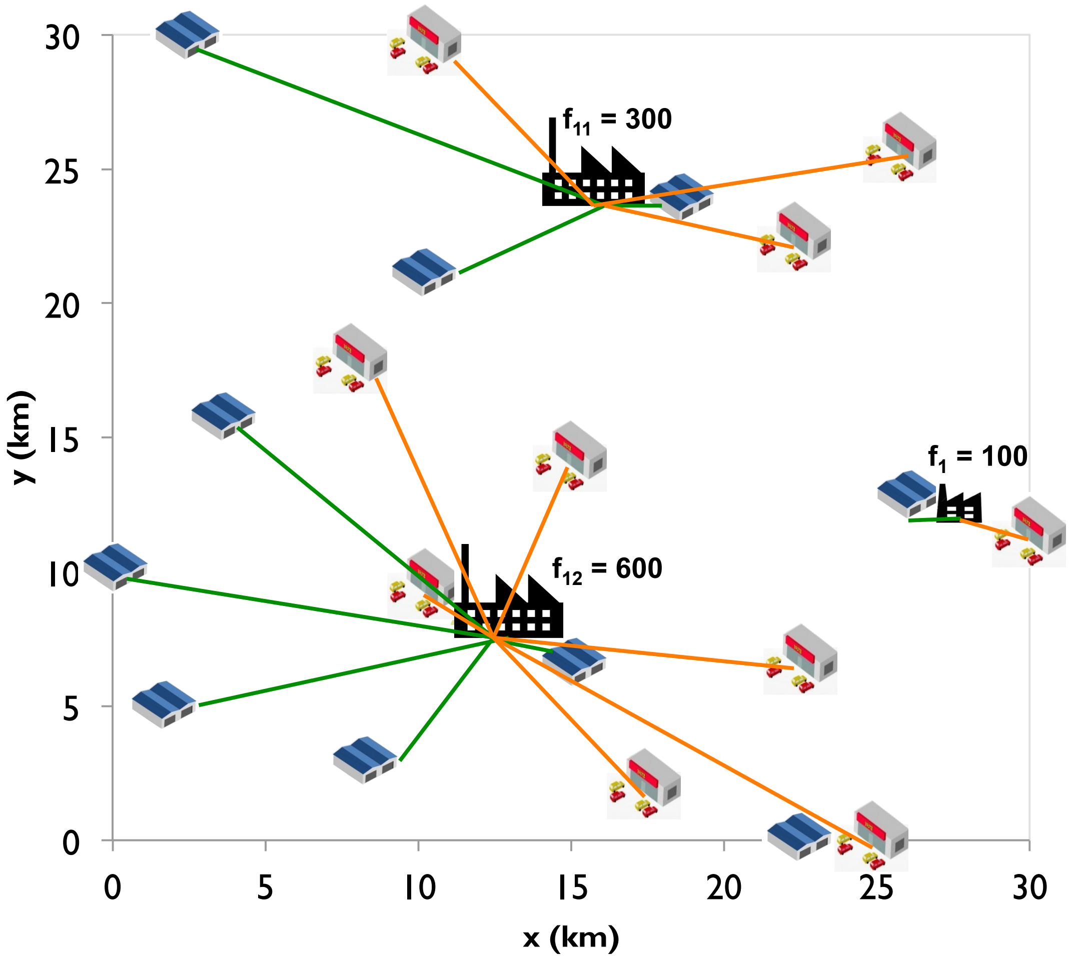
- 10 markets
- Demand of 100 $^{tons}\!/_{week}$ per market

- 12 potential facilities $(cv_k = 90\%)$
  - 10 distributed $\left(mc_m = 100 \, ^{tons}\!/_{week}\right)$
  - 2 centralized $\left(mc_n = 1000 \, ^{tons}\!/_{week}\right)$



Large-scale problems: Example 2

Optimal network found: 2 centralized + 1 distributed facilities



$24,984,000 /week

Large-scale problems: Example 2

Computational results

Global Optimization:

METHOD	Cost $\left(\$10^3/_{week}\right)$	Optimality gap (%)	CPU time (hrs)
BARON	24,990	0.6%	12*
SCIP	25,181	7.6%	12*
Bilevel Decomposition	**24,984**	**0.2%****	**4***

* Exceeded maximum CPU time
** Estimated gap
For the Bilevel Decomposition Algorithm, the master problem (MILP) was solved using CPLEX and the subproblem (nonconvex MINLP) was solved using BARON

Conclusions

Nonconvex GDP reformulated as an MINLP

Commercial global solvers can solve small problems fairly easy

Computationally expensive to solve large-scale problems

- Bilevel decomposition algorithm
  - Although at this point it cannot rigorously solve the large-scale problems to optimality, provides superior results
  - Potential to be improved

Future work

Develop new cuts to tighten the relaxation

- Improve the performance of both the solvers and the algorithm

Rethink master problem formulation

- So as it can be solved to optimality faster

Apply formulation to different problem structures

- Investigate how the network configuration is affected by changes in the parameters
- Explore which conditions favor distributed and/or centralized manufacturing networks.

Apply the model to biomass and electric power systems supply chain