Models and Computational Strategies for Multistage Stochastic Programming under Endogenous and Exogenous Uncertainties

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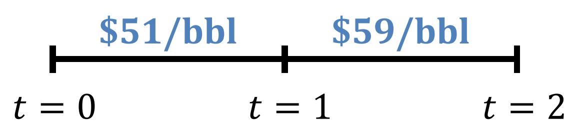
Types of Uncertainty

- Classified by the way the uncertainty is resolved: **exogenous** or **endogenous** (Jonsbråten, 1998)

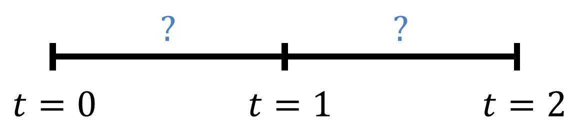
- **Exogenous uncertainty**
  - Parameter values **revealed independently of operation decisions**
  - Realizations occur automatically in each time period
  - **Market uncertainty** (e.g., crude-oil prices)



- **Endogenous uncertainty**
  - Realizations are **affected by operation decisions**
  - Two distinct types: **Type 1** and **Type 2** (Goel and Grossmann, 2006)

Types of Uncertainty

- **Type-2 endogenous uncertainty (Goel and Grossmann, 2006)**
  - Decisions affect the **timing of realizations**
  - A parameter is Type-2 endogenous if its **true value cannot be determined until a decision is made**
  - These are technical parameters that represent **intrinsic properties of a given source** (e.g., size of an oilfield)
  - Not associated with a particular time period

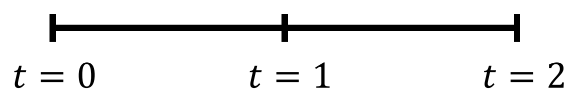


  - We will be focusing on **exogenous** and **Type-2 endogenous uncertainties**

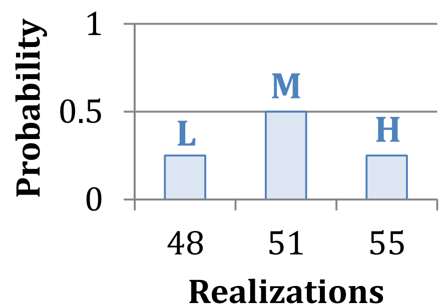
Stochastic Programming

- **Stochastic programming** is a scenario-based framework for optimization under uncertainty (Birge and Louveaux, 2011)

- Two assumptions:
  - Time horizon is represented by a set of **discrete time points**

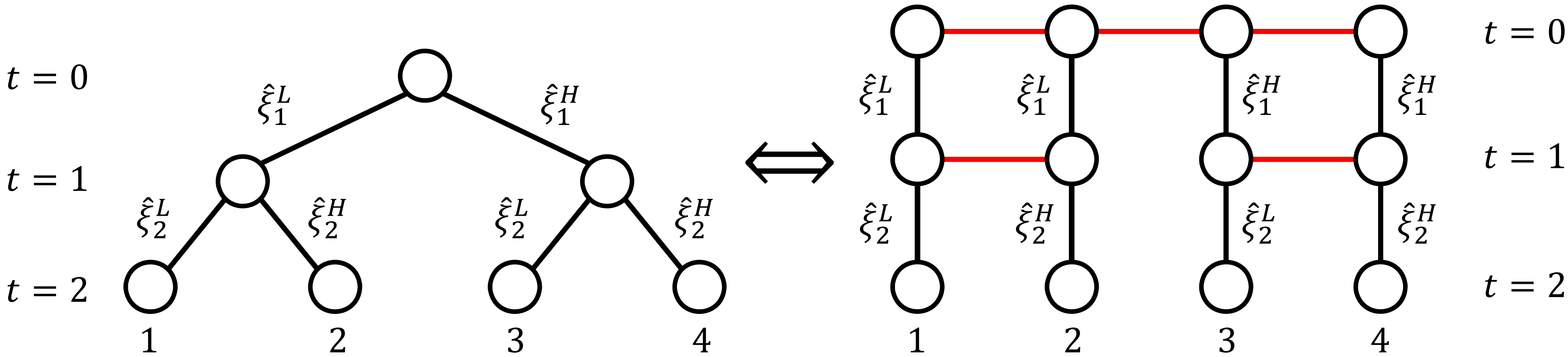


  - Uncertain parameters are described by a **discretized probability distribution**



- This allows us to model stochastic processes with **scenario trees**

MSSP under Exogenous Uncertainty

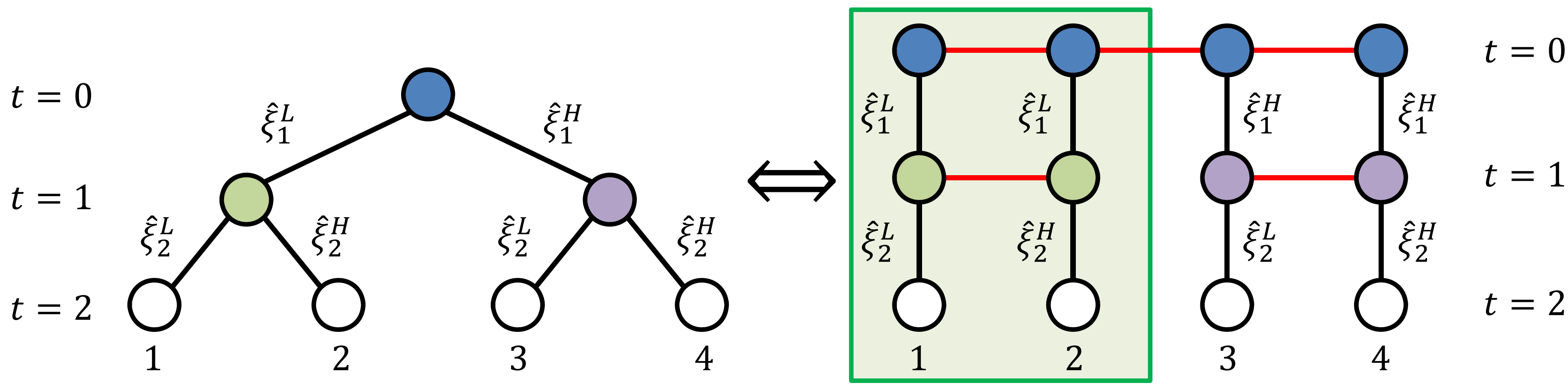


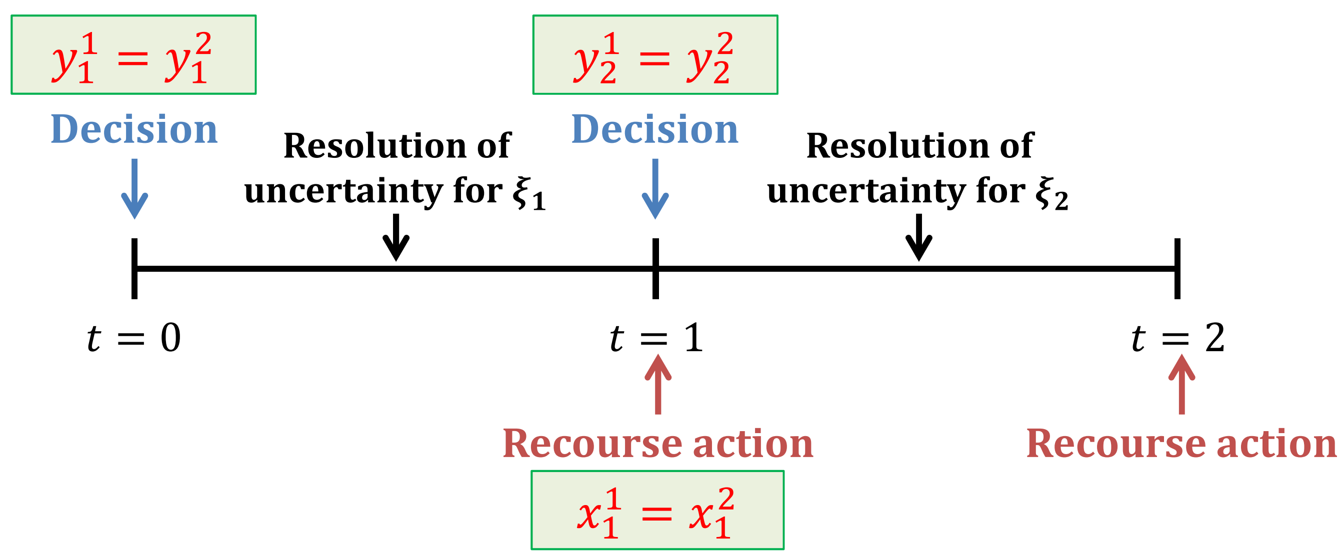
Standard Scenario Tree

Alternative Scenario Tree (Ruszczyński, 1997)

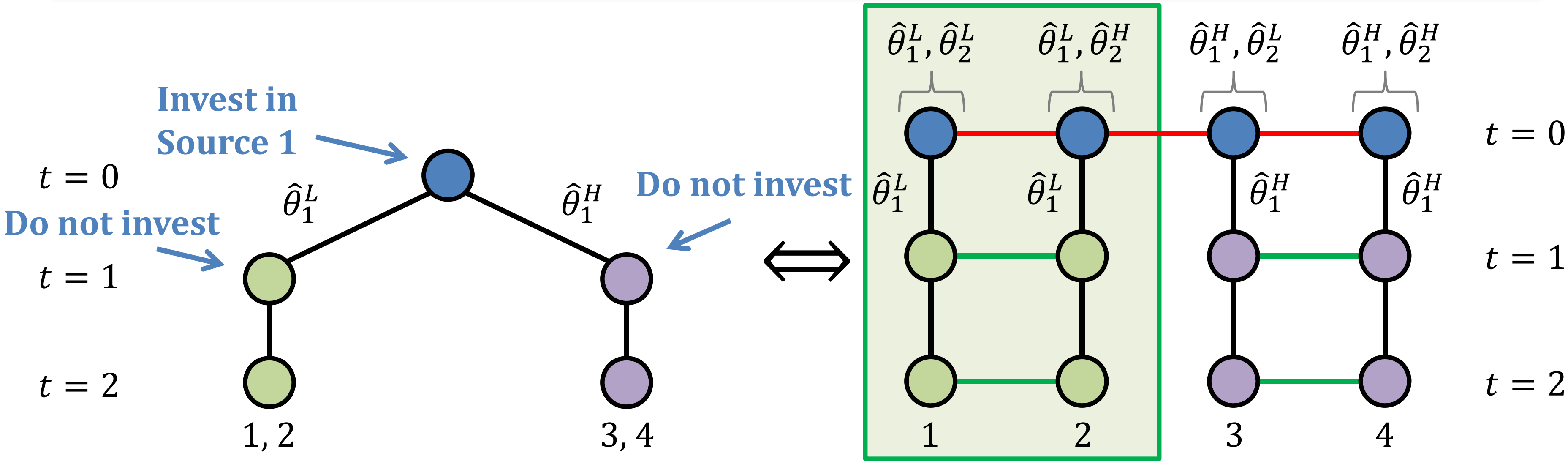
- Alternative form of tree gives each scenario a **unique set of nodes**
- Scenarios with the same information at time t are said to be **indistinguishable** at that time
- We must **make the same decisions in indistinguishable scenarios**
  - Enforced by **non-anticipativity constraints**

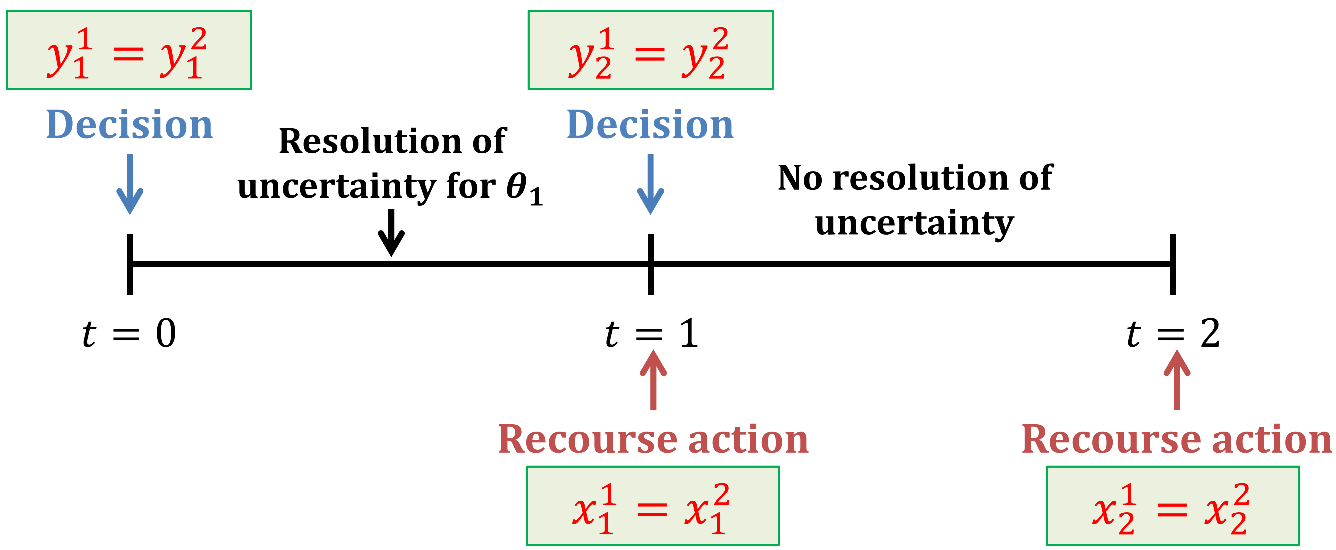
MSSP under Exogenous Uncertainty



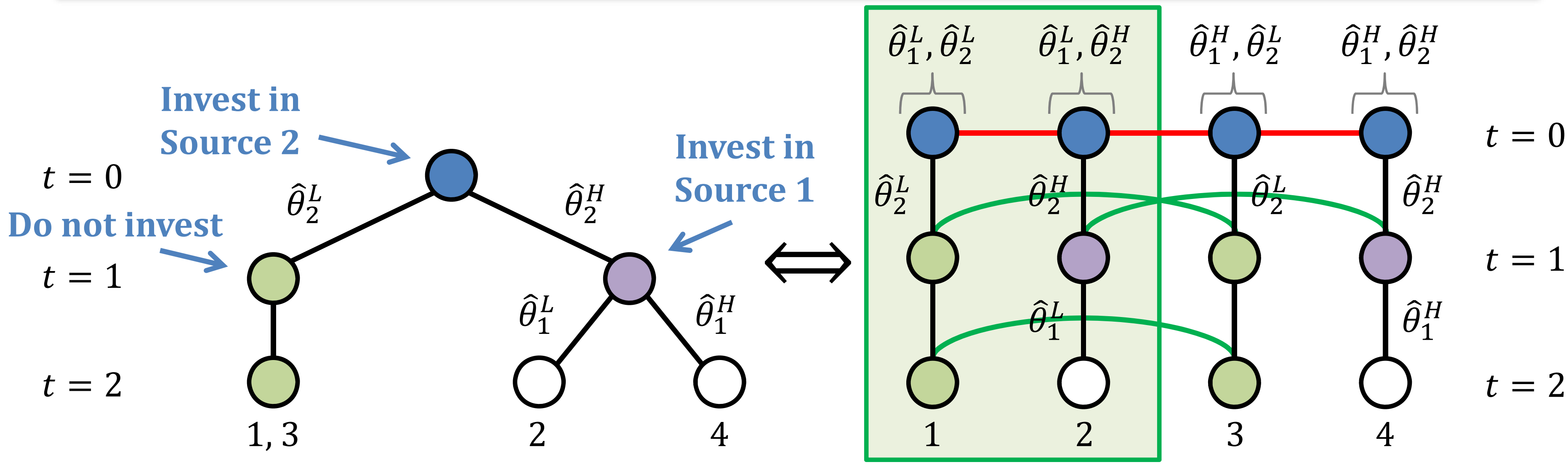


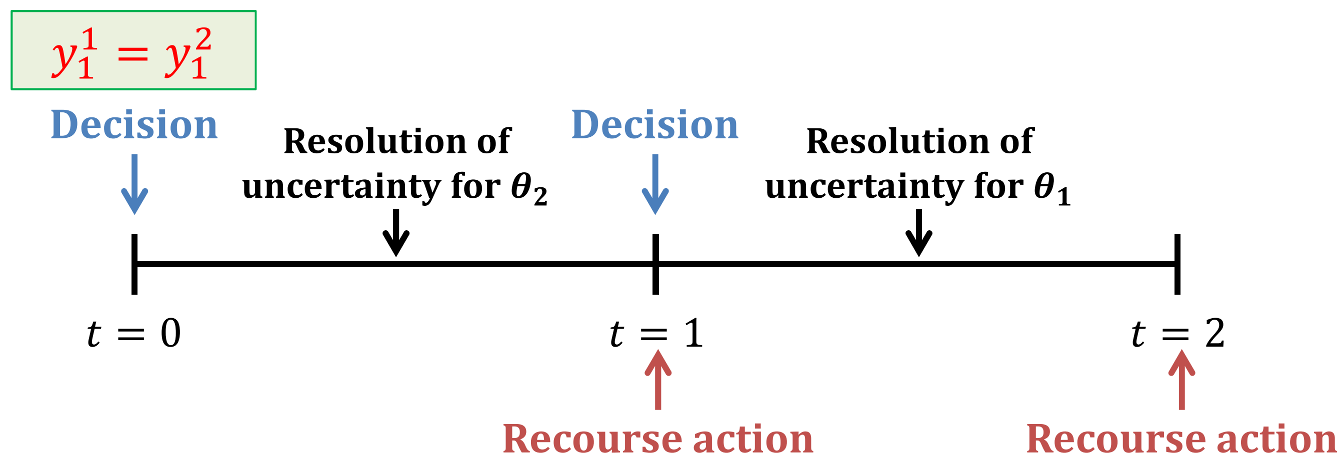
MSSP under Endogenous Uncertainty





MSSP under Endogenous Uncertainty

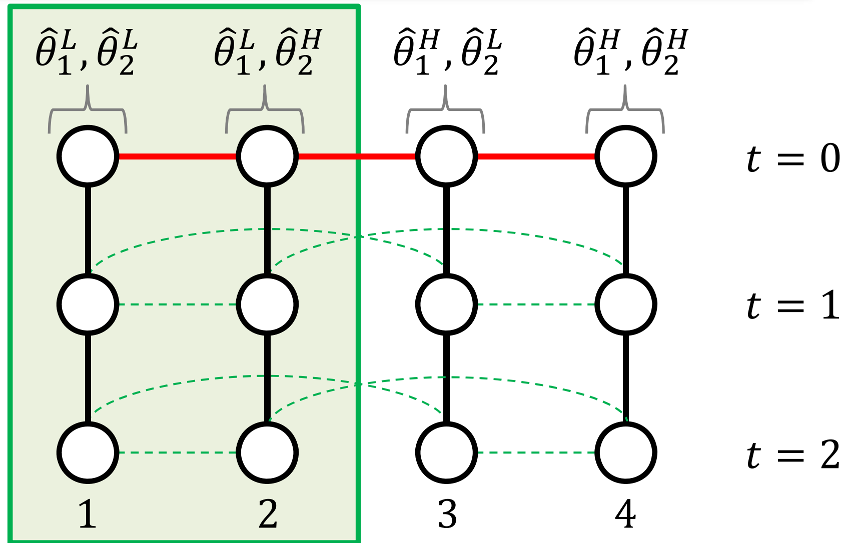


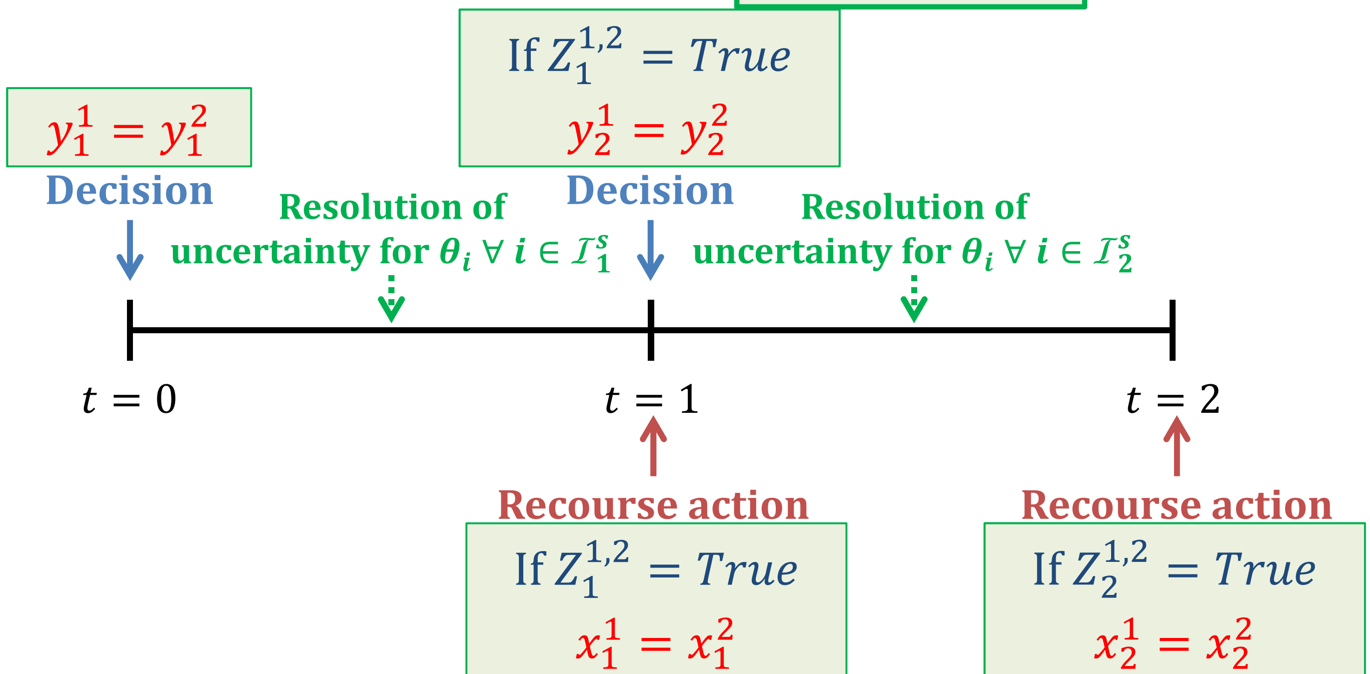


MSSP under Endogenous Uncertainty

$$\begin{bmatrix} Z_t^{s,s'} \\ x_t^s = x_t^{s'} \\ y_{t+1}^s = y_{t+1}^{s'} \end{bmatrix} \vee \left[\neg Z_t^{s,s'} \right]$$

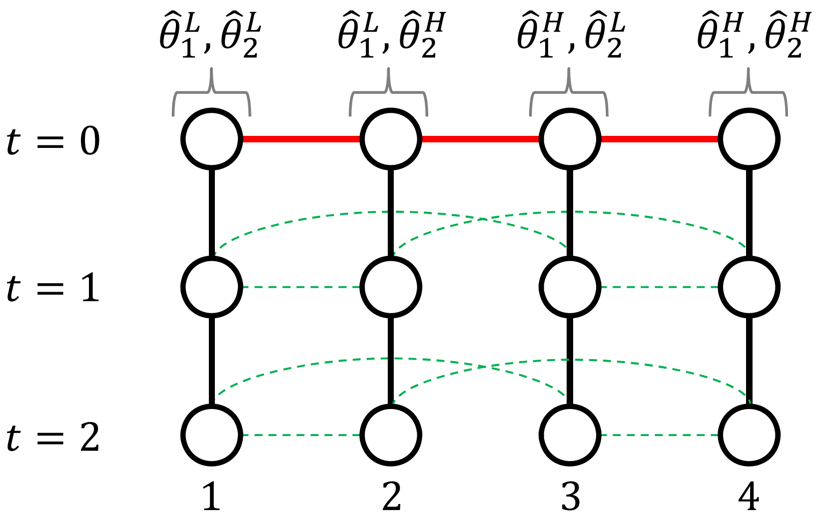
$Z_t^{s,s'} = True$ **if (s, s') indistinguishable**





Description of Endogenous & Exogenous Uncertainty

Endogenous

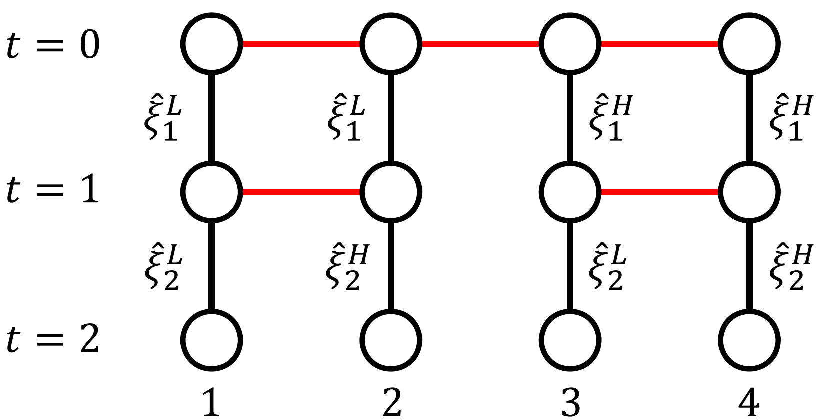


$$\mathcal{R}_N := \times_{i \in \mathcal{I}} \times_{h \in \mathcal{H}_i} \Theta_{i,h}$$
$$= \left\{ \left(\hat{\theta}_{1,1}^1, \dots, \hat{\theta}_{I,H_I}^1\right), \dots, \left(\hat{\theta}_{1,1}^{|\Theta_{1,1}|}, \dots, \hat{\theta}_{I,H_I}^{|\Theta_{I,H_I}|}\right) \right\}$$

$$|\mathcal{R}_N| = \prod_{i \in \mathcal{I}} \prod_{h \in \mathcal{H}_i} |\Theta_{i,h}|$$

$$\mathcal{S}_N = \{1, 2, \dots, |\mathcal{R}_N|\}$$

Exogenous



$$\mathcal{R}_X := \times_{t \in \mathcal{T}} \times_{j \in \mathcal{J}} \Xi_{j,t}$$
$$= \left\{ \left(\hat{\xi}_{1,1}^1, \dots, \hat{\xi}_{J,T}^1\right), \dots, \left(\hat{\xi}_{1,1}^{|\Xi_{1,1}|}, \dots, \hat{\xi}_{J,T}^{|\Xi_{j,t}|}\right) \right\}$$

$$|\mathcal{R}_X| = \prod_{t \in \mathcal{T}} \prod_{j \in \mathcal{J}} |\Xi_{j,t}|$$

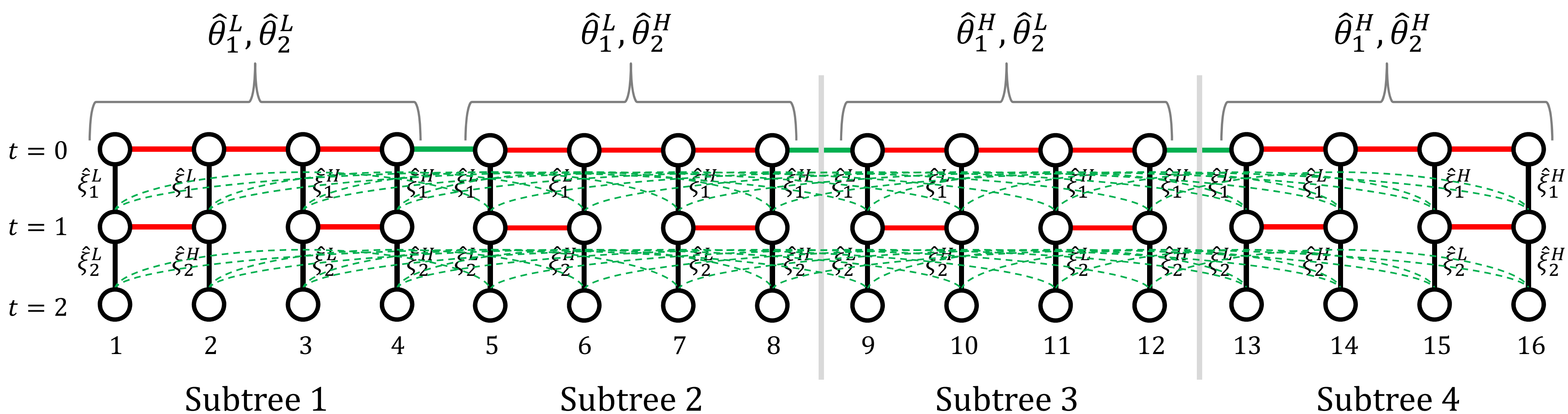
$$\mathcal{S}_X = \{1, 2, \dots, |\mathcal{R}_X|\}$$

Description of Endogenous & Exogenous Uncertainty

Endogenous and Exogenous

$$\mathcal{R} := \mathcal{R}_N \times \mathcal{R}_X = \left\{ \begin{array}{c} \left(\hat{\theta}^1_{1,1},\dots,\hat{\theta}^1_{I,H_I},\hat{\xi}^1_{1,1},\dots,\hat{\xi}^1_{J,T}\right),\dots,\left(\hat{\theta}^1_{1,1},\dots,\hat{\theta}^1_{I,H_I},\hat{\xi}^{|\Xi_{1,1}|}_{1,1},\dots,\hat{\xi}^{|\Xi_{J,T}|}_{J,T}\right),\dots, \\ \left(\hat{\theta}^{|\Theta_{1,1}|}_{1,1},\dots,\hat{\theta}^{|\Theta_{I,H_I}|}_{I,H_I},\hat{\xi}^1_{1,1},\dots,\hat{\xi}^1_{J,T}\right),\dots,\left(\hat{\theta}^{|\Theta_{1,1}|}_{1,1},\dots,\hat{\theta}^{|\Theta_{I,H_I}|}_{I,H_I},\hat{\xi}^{|\Xi_{1,1}|}_{1,1},\dots,\hat{\xi}^{|\Xi_{J,T}|}_{J,T}\right) \end{array} \right\}$$

$$\mathcal{S} = \{1, 2, \dots, |\mathcal{R}|\}$$



Complete **'composite'** scenario tree

MSSP Formulation: Endogenous & Exogenous

$$\min_{b,y,x} \phi = \sum_{s \in \mathcal{S}} p^s \sum_{t \in \mathcal{T}} \left({}^y c_t^s y_t^s + {}^x c_t^s x_t^s + {}^w c_t^s w_t^s + \sum_{i \in \mathcal{I}} {}^b c_{i,t}^s b_{i,t}^s \right)$$

Objective function (minimize expected cost)

$$\text{s.t.} \quad \sum_{\tau=1}^{t} \left({}^y A_{\tau,t}^s y_\tau^s + {}^x A_{\tau,t}^s x_\tau^s + {}^w A_{\tau,t}^s w_\tau^s + \sum_{i \in \mathcal{I}} {}^b A_{i,\tau,t}^s b_{i,\tau}^s \right) \le a_t^s \qquad \forall\, t \in \mathcal{T},\ s \in \mathcal{S}$$

Decision-governing & period-linking constraints

$$b_{i,1}^s = b_{i,1}^{s'} \qquad \forall\, (s,s') \in \mathcal{SP}_F,\ i \in \mathcal{I}$$
$$y_1^s = y_1^{s'} \qquad \forall\, (s,s') \in \mathcal{SP}_F$$

First-period NACs

$$x_t^s = x_t^{s'} \qquad \forall\, (t,s,s') \in \mathcal{SP}_X$$
$$b_{i,t+1}^s = b_{i,t+1}^{s'} \qquad \forall\, (t,s,s') \in \mathcal{SP}_X,\ i \in \mathcal{I}$$
$$y_{t+1}^s = y_{t+1}^{s'} \qquad \forall\, (t,s,s') \in \mathcal{SP}_X$$

Exogenous NACs

$$x_t^s = x_t^{s'} \qquad \forall\, (t,s,s') \in \mathcal{SP}_N,\ t \in \mathcal{T}_E$$
$$b_{i,t+1}^s = b_{i,t+1}^{s'} \qquad \forall\, (t,s,s') \in \mathcal{SP}_N,\ t \in \mathcal{T}_E,\ i \in \mathcal{I}$$
$$y_{t+1}^s = y_{t+1}^{s'} \qquad \forall\, (t,s,s') \in \mathcal{SP}_N,\ t \in \mathcal{T}_E$$

Fixed endogenous NACs

$$-x_t^{UB}\left(1 - z_t^{s,s'}\right) \le x_t^s - x_t^{s'} \le x_t^{UB}\left(1 - z_t^{s,s'}\right) \qquad \forall\, (t,s,s') \in \mathcal{SP}_N,\ t \in \mathcal{T}_C$$
$$-\left(1 - z_t^{s,s'}\right) \le b_{i,t+1}^s - b_{i,t+1}^{s'} \le \left(1 - z_t^{s,s'}\right) \qquad \forall\, (t,s,s') \in \mathcal{SP}_N,\ t \in \mathcal{T}_C,\ t < T,\ i \in \mathcal{I}$$
$$-y_{t+1}^{UB}\left(1 - z_t^{s,s'}\right) \le y_{t+1}^s - y_{t+1}^{s'} \le y_{t+1}^{UB}\left(1 - z_t^{s,s'}\right) \qquad \forall\, (t,s,s') \in \mathcal{SP}_N,\ t \in \mathcal{T}_C,\ t < T$$

Conditional endogenous NACs

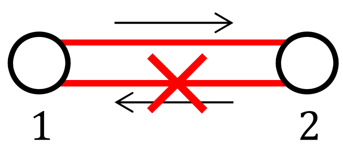
$$Z_t^{s,s'} \Leftrightarrow F\left(b_{i,1}^s, b_{i,2}^s, \dots, b_{i,t}^s\right) \qquad \forall\, (t,s,s') \in \mathcal{SP}_N,\ t \in \mathcal{T}_C,\ \{i\} = \mathcal{D}^{s,s'}$$

Endogenous indistinguishability constraints

$$b_{i,t}^s \in \{0,1\},\ y_t^s \in \mathcal{Y}_t^s,\ x_t^s \in \mathcal{X}_t^s,\ w_t^s \in \mathcal{W}_t^s \qquad \forall\, i \in \mathcal{I},\ t \in \mathcal{T},\ s \in \mathcal{S}$$
$$Z_t^{s,s'} \in \{True, False\},\ z_t^{s,s'} \in \{0,1\} \qquad \forall\, (t,s,s') \in \mathcal{SP}_N,\ t \in \mathcal{T}_C$$

Bounds & integrality restrictions

Eliminating Redundant NACs

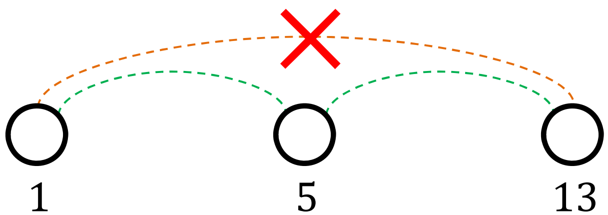


Symmetry

Property 1

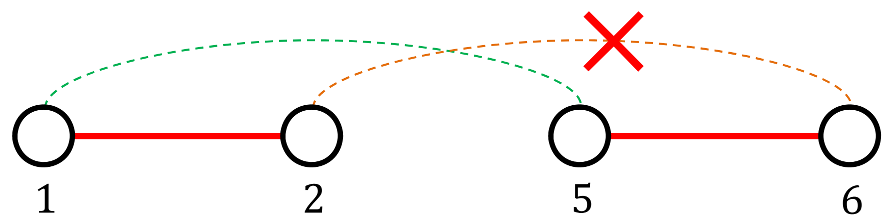
Adjacency

Properties 2a & 2b



Transitivity

Properties 3 & 4



Grouping

Property 5

- Even after eliminating redundant constraints, **models are typically still too large** to be solved directly

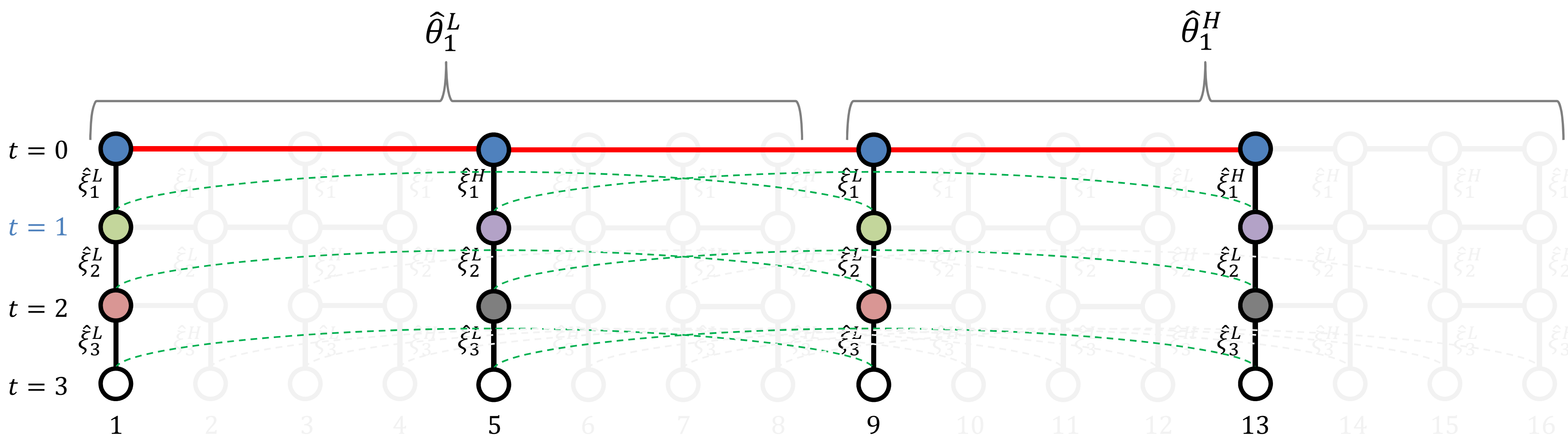
- We consider **special solution methods**:
  - Sequential scenario decomposition (SSD) heuristic
  - Lagrangean decomposition (LD)

Solution Method 1: SSD Heuristic

- Basic idea behind **sequential scenario decomposition (SSD)**:
  - Select a **subset of scenarios** in each time period
  - Solve an **MILP subproblem** involving only those scenarios for the full time horizon
  - **Take binary decisions** from the subproblem solution
  - **Fix these binary decisions** in scenarios of the original problem to satisfy first-period and exogenous NACs
  - Proceed to next time period and **repeat**
  - **Solve the resulting model** to obtain a feasible solution to the original problem

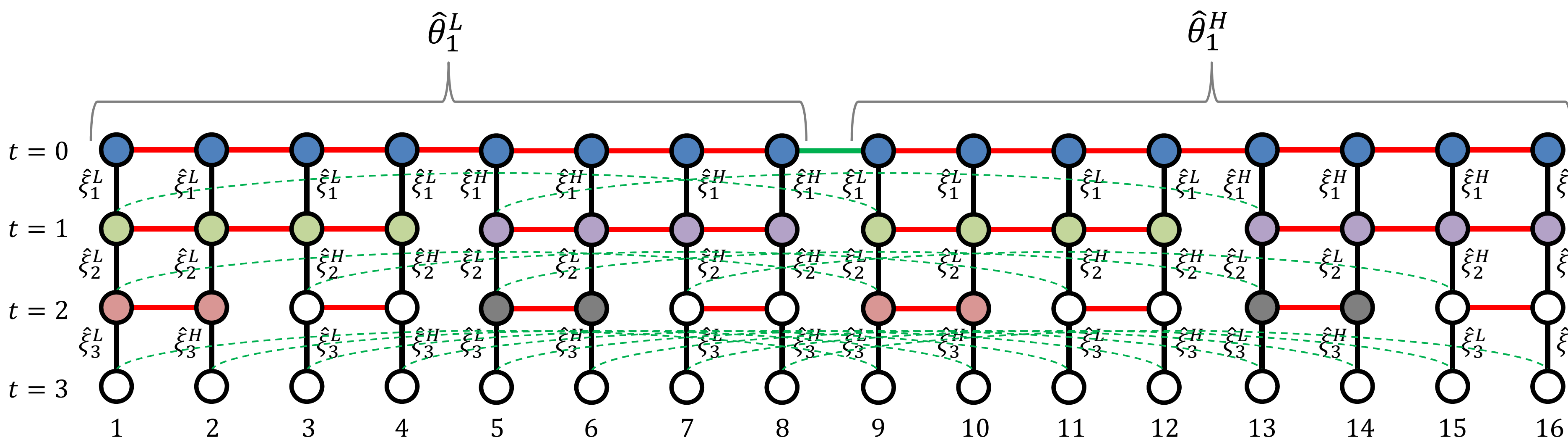
Solution Method 1: SSD Heuristic

- **Solve an MILP subproblem** that consists of only the selected scenarios $\{1, 5, 9, 13\}$
- This subproblem includes first-period NACs and endogenous NACs, but **no exogenous NACs**



Solution Method 1: SSD Heuristic

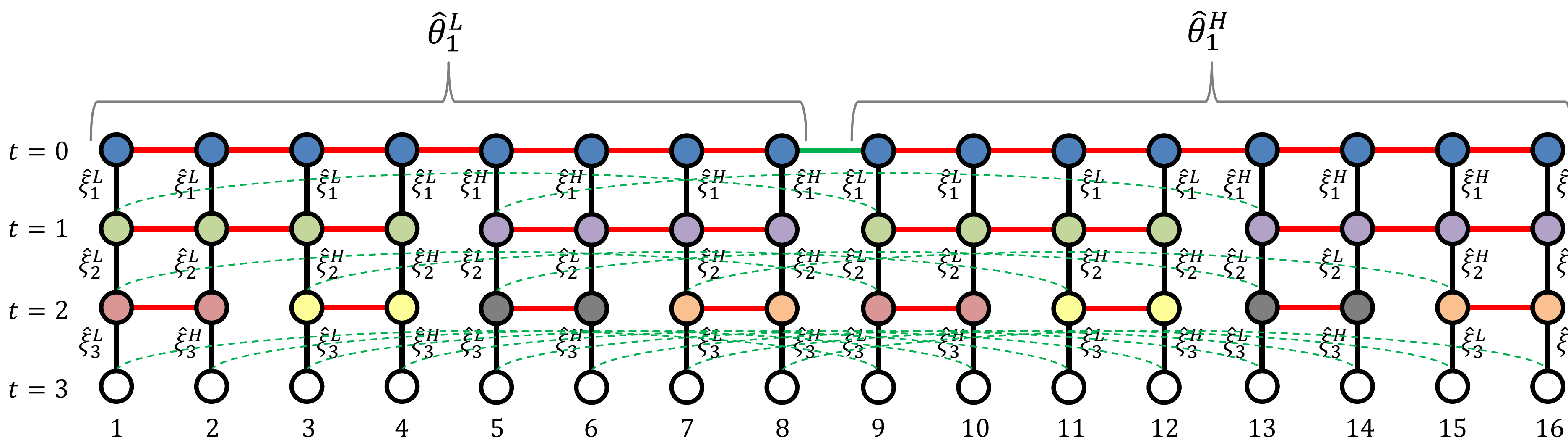
- **Extract binary investment decisions from solution, and fix in corresponding scenario groups** to satisfy first-period and exogenous NACs



Solution Method 1: SSD Heuristic

- After repeating procedure for each subproblem, **all binary variables are fixed**. Thus, the scenario tree is fixed

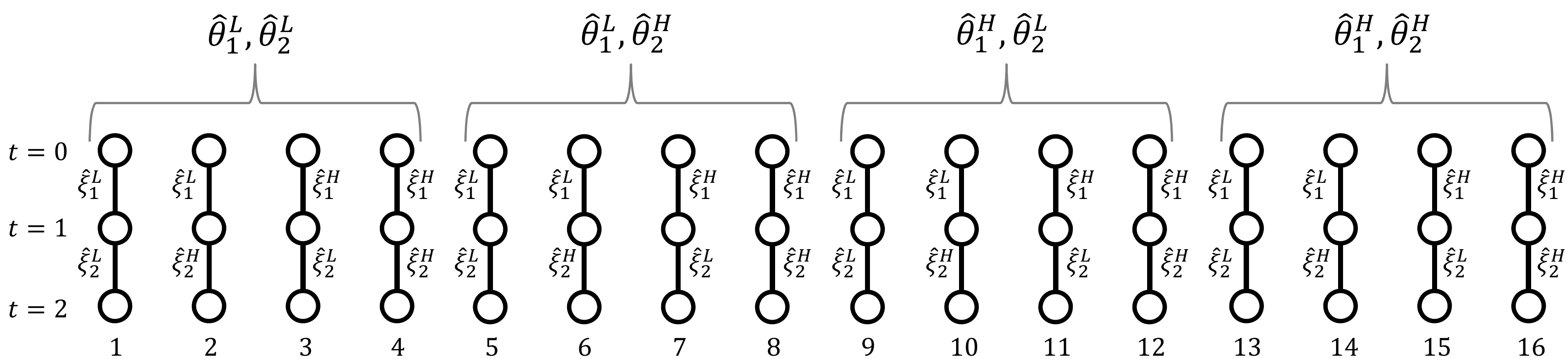
- The solution of the resulting problem is a **feasible (but not necessarily optimal) solution** to the original MILP



Solution Method 2: Lagrangean Decomposition

- If non-anticipativity constraints are removed, the scenario tree **decomposes into independent scenarios**

- Set up **Lagrangean dual problem**:
  - Relax **conditional endogenous** NACs
  - Dualize **first-period** and **exogenous** NACs



Solution Method 2: Lagrangean Decomposition

$$\min_{y} \phi = \sum_{s \in \mathcal{S}} p^s \sum_{t \in \mathcal{T}} {}^{y}c_t^s y_t^s$$

$$\text{s.t.} \quad \sum_{\tau=1}^{t} {}^{y}A_{\tau,t}^s y_\tau^s \le a_t^s \qquad \forall\, t \in \mathcal{T},\ s \in \mathcal{S}$$

$$y_1^s = y_1^{s'} \qquad \forall\, (s,s') \in \mathcal{SP}_F$$

$$y_{t+1}^s = y_{t+1}^{s'} \qquad \forall\, (t,s,s') \in \mathcal{SP}_X$$

$$y_{t+1}^s = y_{t+1}^{s'} \qquad \forall\, (t,s,s') \in \mathcal{SP}_N,\ t \in \mathcal{T}_E$$

$$-y_{t+1}^{UB}\left(1 - z_t^{s,s'}\right) \le y_{t+1}^s - y_{t+1}^{s'} \le y_{t+1}^{UB}\left(1 - z_t^{s,s'}\right) \qquad \forall\, (t,s,s') \in \mathcal{SP}_N,\ t \in \mathcal{T}_C,\ t < T$$

$$Z_t^{s,s'} \Leftrightarrow F\left(y_1^s, y_2^s, \dots, y_t^s\right) \qquad \forall\, (t,s,s') \in \mathcal{SP}_N,\ t \in \mathcal{T}_C$$

$$y_t^s \in \mathcal{Y}_t^s \qquad \forall\, t \in \mathcal{T},\ s \in \mathcal{S}$$

$$Z_t^{s,s'} \in \{True, False\},\ z_t^{s,s'} \in \{0,1\} \qquad \forall\, (t,s,s') \in \mathcal{SP}_N,\ t \in \mathcal{T}_C$$

Solution Method 2: Lagrangean Decomposition

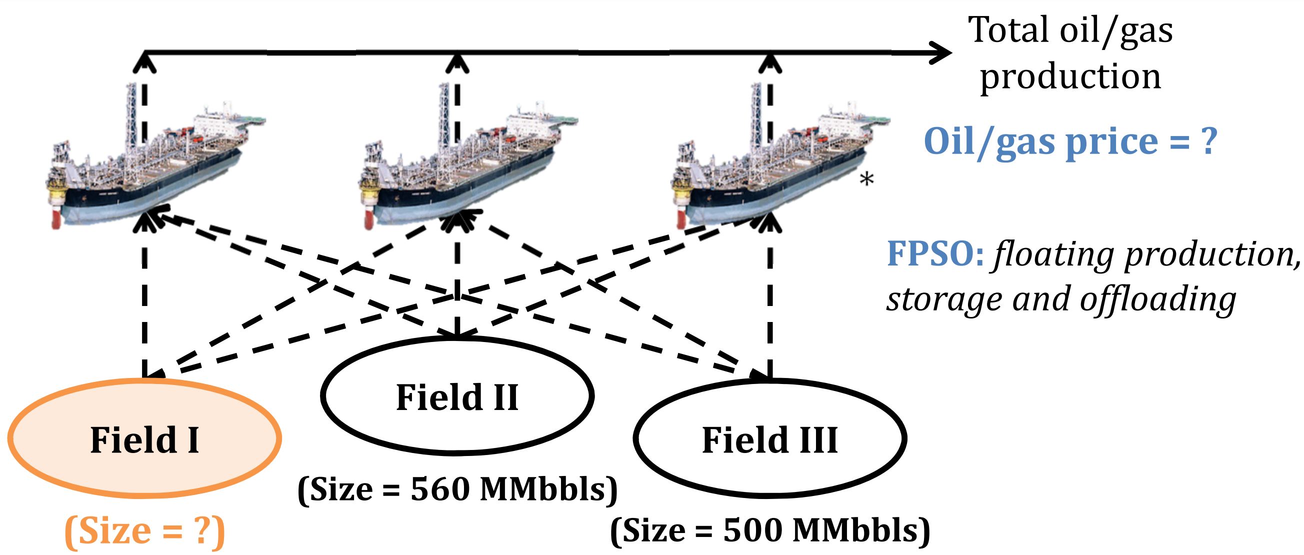
$$\min_{y} \phi_{LD} = \sum_{s \in \mathcal{S}} p^s \sum_{t \in \mathcal{T}} \left({}^{y}c_t^s y_t^s + y_1^s \left(\sum_{(s,s') \in \mathcal{SP}_F} {}^{F}\lambda_1^{s,s'} - \sum_{(s',s) \in \mathcal{SP}_F} {}^{F}\lambda_1^{s',s} \right) + y_{t+1}^s \left(\sum_{(t,s,s') \in \mathcal{SP}_X} {}^{X}\lambda_t^{s,s'} - \sum_{(t,s',s) \in \mathcal{SP}_X} {}^{X}\lambda_t^{s',s} \right) + y_{t+1}^s \left(\sum_{\substack{(t,s,s') \in \mathcal{SP}_N \\ t \in \mathcal{T}_E}} {}^{N}\lambda_t^{s,s'} - \sum_{\substack{(t,s',s) \in \mathcal{SP}_N \\ t \in \mathcal{T}_E}} {}^{N}\lambda_t^{s',s} \right) \right)$$

$$\text{s.t.} \quad \sum_{\tau=1}^{t} {}^{y}A_{\tau,t}^s y_\tau^s \le a_t^s \qquad \forall\, t \in \mathcal{T},\ s \in \mathcal{S}$$

$$y_t^s \in \mathcal{Y}_t^s \qquad \forall\, t \in \mathcal{T},\ s \in \mathcal{S}$$

- ✓ **Lagrangean relaxation**
- ✓ **Decompose problem by scenario**
  - ❑ Solve scenario subproblems in parallel
  - ❑ Update multipliers using subgradient method

Example: Oilfield Development Planning



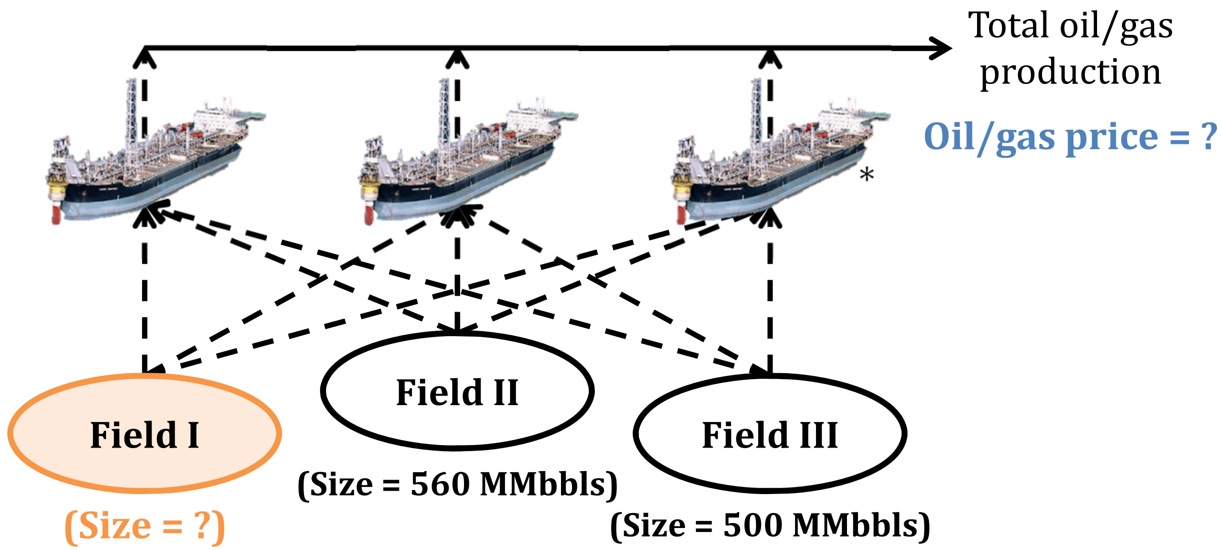
- Planning over **5 years**
- Determine **optimal investment and operations decisions**
- **Here-and-now decisions:** FPSO installations and expansions, field-FPSO connections (9 possible), well-drilling schedule (30 potential wells)
- **Recourse decisions:** Oil production rate

Objective: **Maximize total expected NPV**

Gupta and Grossmann, 2014a

* www.rigzone.com

Example: Oilfield Development Planning



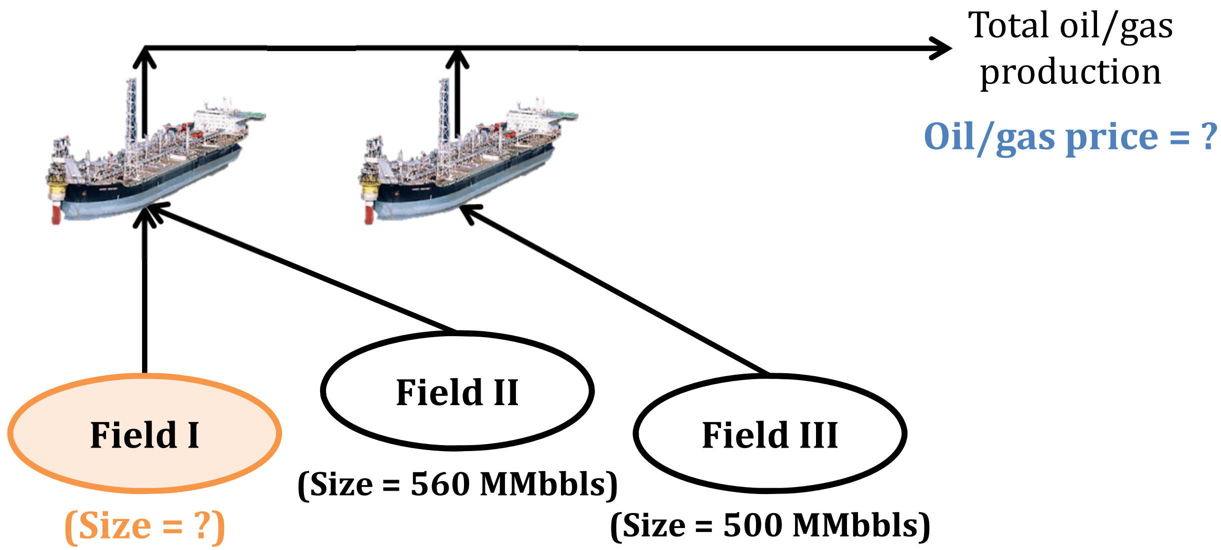
(2 realizations for size)$^{(1 \text{ uncertain size})}$ ⟹ **2 endogenous combinations**

(2 realizations for price)$^{(5 \text{ time periods})}$ ⟹ **32 exogenous combinations**

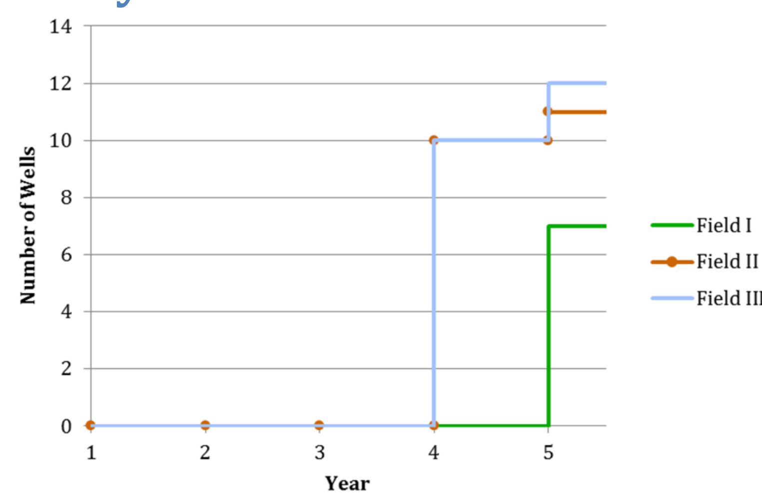
(**2 endogenous combinations**) × (**32 exogenous combinations**) ⟹ **64 scenarios**

* www.rigzone.com

Example: Oilfield Development Planning



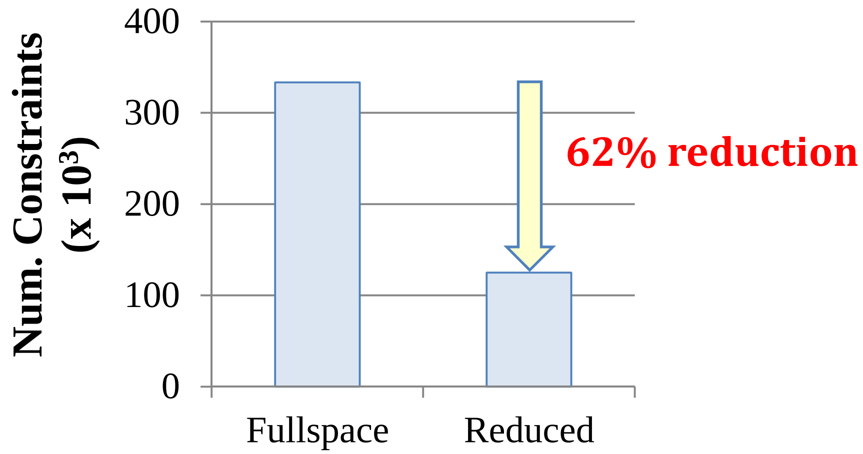
- **Begin installing all infrastructure in first year**
- **Drilling cannot start until year 4** due to lead time for FPSO installation
- **Drill fields with known size first**



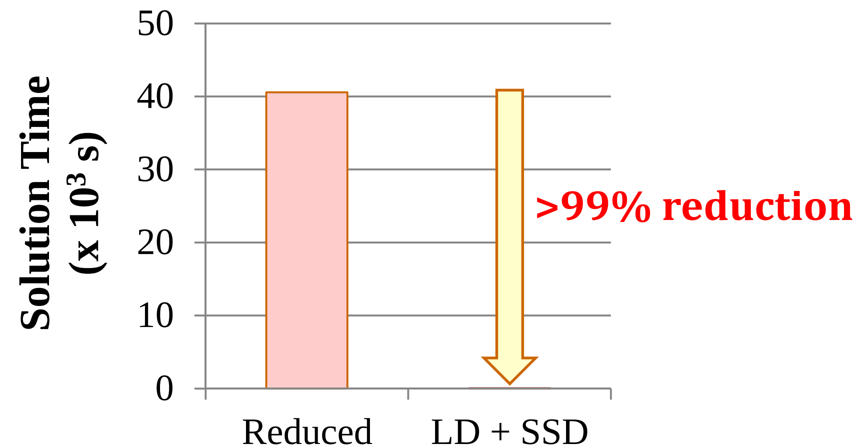
Example: Oilfield Development Planning

Problem Type	Total Expected NPV ($B)		Optimality Gap	Number of Constraints	Continuous Variables	0, 1 Variables	Solution Time (s)
	Lower Bound	Upper Bound					
Fullspace	-	-	-	333,249	70,465	7,360	-
Reduced Model	6.968	10.495	50.61%	124,980	70,465	7,000	40,562
SSD	7.166	-	0.20%	28,687[1]	17,329[1]	1,224[1]	41
LD	-	7.180		3,776	2,203	218	14

[1] Largest subproblem



Comparison of Instance Sizes



Comparison of Solution Methods

Solved in GAMS 24.3.3 with CPLEX 12.6.0.1 on a machine with an Intel Core i7 CPU @ 2.93 GHz and 12 GB RAM

Conclusions

- Multistage stochastic optimization problems often contain **both endogenous and exogenous** uncertain parameters

- We have presented a **'composite' scenario tree** that captures endogenous and exogenous realizations

- We have proposed a **reduced model** for **MSSP under both types of uncertainty** by eliminating redundant NACs based on the structure of the underlying scenario tree

- To demonstrate practical solution methods for this class of problems, we have applied **Lagrangean decomposition** and a novel **sequential scenario decomposition heuristic** to an example problem
  - Both alternative methods are considerably faster than solving the reduced model directly

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