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Global Optimization Approach to the Multiperiod Blending Problem

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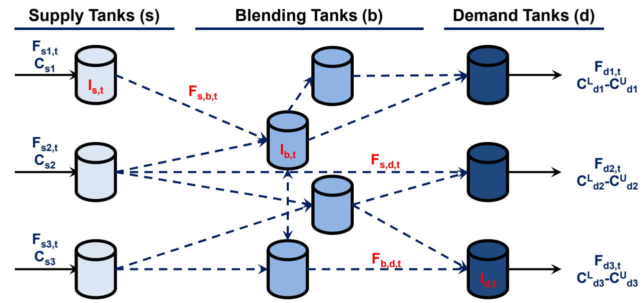
Multiperiod blending problem is defined over supply, blending and demand tanks



Given:

- Topology, initial conditions and flow profit/costs
- Supply tank flow and qualities
- Demand tank flow and quality limits

Determine

- Flows between which tanks in which time periods
- Inventories/qualities for tanks in each period
- **Maximum total profit of blending operation**

Assumptions:

- Supply qualities are constant
- No simultaneous input/output to blending tanks
- Perfect mixing

MINLP formulation contains bilinear terms

$$Max\ \sum_{t\epsilon T}[\sum_{n\epsilon S\cup B}\sum_{d\epsilon D}\beta_d F_{ndt} - \sum_{s\epsilon S}\sum_{n\epsilon B\cup D}\beta_s F_{snt} - \sum_{nn'\epsilon N}(\alpha_{nn'}y_{nn't} + \beta_{nn'}F_{nn't})]$$

Profit of products - Cost of raw materials - Network flow cots (fixed and variable)

Subject to:

$F_{nn't} \le F^U_{nn'}y_{nn't}$	$\forall\ nn'\epsilon N, t\epsilon T$	*If $y_{nn't} = 0$ then there is no flow, otherwise enforce flow bounds*
$F_{nn't} \ge F^L_{nn'}y_{nn't}$	$\forall\ nn'\epsilon N, t\epsilon T$	
$C_{qbt-1} \le C^U_{qd} + M(1 - y_{qbt})$	$\forall\ q\epsilon Q, d\epsilon D, b\epsilon B, t\epsilon T$	*If $y_{qbt} = 0$ then no restriction in qualities, otherwise enforce demand quality specifications*
$C_{qbt-1} \ge C^L_{qd} - M(1 - y_{qbt})$	$\forall\ q\epsilon Q, d\epsilon D, b\epsilon B, t\epsilon T$	
$C^{IN}_{qs} \le C^U_{qd} + M(1 - y_{sbt})$	$\forall\ q\epsilon Q, d\epsilon D, s\epsilon S, t\epsilon T$	
$C^{IN}_{qs} \ge C^L_{qd} - M(1 - y_{sbt})$	$\forall\ q\epsilon Q, d\epsilon D, s\epsilon S, t\epsilon T$	
$I_{st} = I_{st-1} + F^{IN}_{st} - \sum_{n\epsilon B\cup D} F_{snt}$	$\forall\ s\epsilon S, t\epsilon T$	*Total mass balance in tanks*
$I_{bt} = I_{bt-1} + \sum_{n\epsilon S\cup B} F_{nbt} - \sum_{n\epsilon B\cup D} F_{bnt}$	$\forall\ b\epsilon B, t\epsilon T$	
$I_{dt} = I_{dt-1} + \sum_{n\epsilon S\cup B} F_{ndt} - F^{OUT}_{dt}$	$\forall\ d\epsilon D, t\epsilon T$	

$$I_{bt}C_{qbt} = I_{bt-1}C_{qbt-1} + \sum_{s\epsilon S} F_{sbt}\,C^{IN}_{qs} + \sum_{b'\epsilon B} F_{b'bt}C_{qb't-1} - \sum_{n\epsilon B\cup D} F_{bnt}C_{qbt-1} \quad \forall\ q\epsilon Q, b\epsilon B, t\epsilon T$$

Mass balance in blending tanks: Only non-linear constraints

$y_{nbt} + y_{bn't} \le 1$	$\forall\ b\epsilon B, n\epsilon S\cup B, n'\epsilon B\cup D, t\epsilon T$	*No simultaneous flow in and flow out in blending tanks*
$I^L_n \le I_{nt} \le I^U_n$	$\forall\ n\epsilon TA, t\epsilon T$	
$y_{nn't}\epsilon\{0,1\}$	$\forall\ nn'\epsilon N, t\epsilon T$	
$F_{nn't} \ge 0;\ I_{nt} \ge 0;\ 0 \le C_{qbt} \le 1$	$\forall nn'\epsilon N, q\epsilon Q, b\epsilon B, t\epsilon T$	

Discretization technique transforms bilinear terms into mixed-integer linear constraints

Select one of the two variables to discretize

$$v * u = w$$

k selects the powers of B

Any integer Base can be used

$$v = \sum_{k=p}^{P}\sum_{j=0}^{B-1} B^k * j * z_{jk}$$

j selects over the digits

$$\sum_{j=0}^{B-1} z_{jk} = 1$$

Only one z for each power of B is selected

Example.
1.53 can be represented as:

- $10^0\cdot\mathbf{1} + 10^{-1}\cdot\mathbf{5} + 10^{-2}\cdot\mathbf{3}$
- $z_{1,0} = 1$, $z_{5,-1} = 1$, and $z_{3,-2} = 1$. Other $z_{j,k} = 0$

$$w = \sum_{k=p}^{P}\sum_{j=0}^{B-1} B^k * j * \hat{u}_{jk}$$

$$u = \sum_{j=0}^{B-1} \hat{u}_{jk} \qquad \forall\ k$$

$$u^L * z_{jk} \le \hat{u}_{jk} \le u^U * z_{jk}$$

Discretization provides upper bound. However, a slack variable can be introduced to obtain lower bound

Improving the formulation: *"Better parameters and MILP cuts"*

"Optimal" big-M improves solution performance

Big-M parameter in constraints

$$C_{qbt-1} \le C_{qd}^{U} + M_{qbt}^{1}(1 - y_{qbt})$$
$$C_{qbt-1} \ge C_{qd}^{L} - M_{qbt}^{2}(1 - y_{qbt})$$
$$C_{qs}^{IN} \le C_{qd}^{U} + M_{sdt}^{3}(1 - y_{sdt})$$
$$C_{qs}^{IN} \ge C_{qd}^{L} - M_{sdt}^{4}(1 - y_{sdt})$$

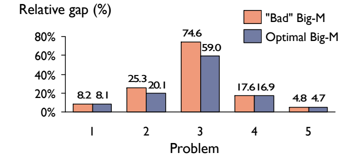
Valid "Bad Big-M"

$$M_{qbt}^{1} = M_{qbt}^{2} = M_{sdt}^{3} = M_{sdt}^{4} = 1$$

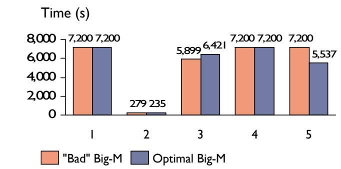
Valid "Optimal Big-M"

$$M_{qbt}^{1} = M_{sdt}^{3} = 1 - C_{qd}^{U}$$
$$M_{qbt}^{2} = M_{sdt}^{4} = C_{qd}^{U}$$

Improvement in relaxation gap



Improvement in solution time



Cuts can be generated to improve solution times of the discretized problem (MILP)

Cuts using summation of mass balances in blending tanks

- **1a** Summation over qualities
- **1b** Summation over blending tanks
- **1c** Summation over time periods
- **1d** Summation over qualities and blending tanks
- **1e** Summation over blending tanks and time periods
- **1f** Summation over qualities and time periods
- **1g** Summation over qualities, blending tanks and time periods

McCormick

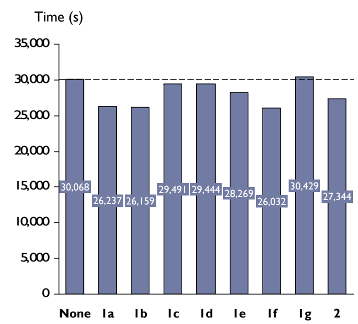
- **2** McCormick envelopes for the bilinear terms

Cuts improve solution time

But they do not seem to work well when combined

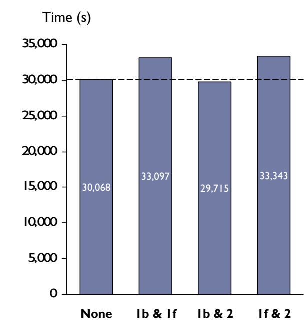
MILP cuts

Accumulated time to solve 5 problems



Best cuts combined

Accumulated time to solve 5 problems



Improving discretization technique: *"Base and variable selection"*

Two main decisions in the discretization technique

Variable to discretize

Bilinear terms are $F * C$ and $I * C$

Discretizing quality
- Good lower and upper bound
- Only one discretization for both bilinear terms

Discretize flow and inventory
- Same number of 0-1 variables, independent of the number of qualities in the problem

Base for discretization

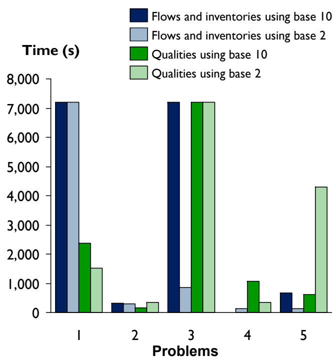
We can use other bases for more/fewer binary variables
- 1.53 in base 10: $10^{0}\cdot\mathbf{1} + 10^{-1}\cdot\mathbf{5} + 10^{-2}\cdot\mathbf{3}$
- 1.53 in base 5: $10^{-2}\cdot(5^{3}\cdot\mathbf{1}+5^{2}\cdot\mathbf{1}+5^{1}\cdot\mathbf{0}+5^{0}\cdot\mathbf{3})$
- Base 10 requires 30 binary variables
- Base 5 requires 20 binary variables

Using base 2 provides fewest binaries

Using base 10 is more intuitive

For 2 qualities, it is not clear which discretization variable and base selection is the best

Solution times (7,200s limit)
2 quality instances



Notes

In problem 4 using Flows and inventory with base 10 did not find a solution

There is no clear winner
- Flow and inventory with base 2 is best in problems 3, 4 and 5
- Quality base 2 is best in problem 1, but worst in 5
- Quality base 10 is best in problem 2

For 6 qualities, discretizing flow and inventory and using base 2 is the best alternative

Optimality gap after 2 hours
6 quality instances

Method	Problem 1	2	3	4	5
Flow & Inv. Base 10	NA	23.5%	36.0%	0.10%	NA
Flow & Inv. Base 2	**7.6%**	**13.4%**	**0.16%**	**0.10%**	**5.30%**
Quality Base 10	NA	NA	NA	NA	NA
Quality Base 2	NA	NA	NA	NA	NA

NA = No solution found within limit time

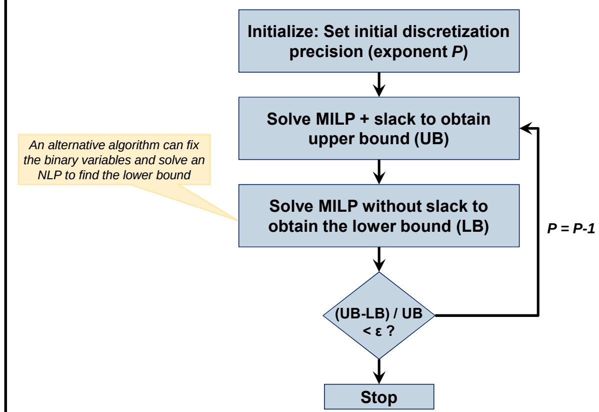
Notes

No solution found in any of the problems when discretizing quality

Problem 4 was solved to specified gap when discretizing flow and inventory
- ~6,900 sec. to optimality with base 10
- ~1,300 sec. to optimality with base 2

Improving the algorithm: *"Consolidate all improvements"*

Discretization technique can be incorporated in an algorithm to find lower and upper bounds



Two algorithms were used to compare the impact of the improvements

Original algorithm

Uses naïve Big-M parameter in the problem formulation

Discretize quality using base 10

Always uses the best MILP solution found as upper bound
- Even when the MILP algorithm does not achieve optimality within the time limit
- This might lead to sub optimality, since the upper bound can cut the global optimum

Improved algorithm

Uses "optimal Big-M"

Discretize flow and inventory using base 2

Includes constraint 1f in the MILP

Uses problem UB in the MILP subproblems as cutoff in the branch and bound

Uses the MILP solution found as UB only when the MILP reached optimality
- If it does not reach optimality, it uses the best UB from the MILP solver
- Improved algorithm guarantees global optimality

New algorithm provides higher quality solutions

Qualities	Instance	Best solution found: Original	Best solution found: Improved	Time (s): Original	Time (s): Improved	(UB – LB) / UB[1]: Original	(UB – LB) / UB[1]: Improved
2	1	**45.27**	45.13	>10800	>10800	**0.1%**	2.7%
	2	13.53	**13.53**	566	**219**	0.0%	0.1%
	3	9.15	**9.23**	>10800	>10800	0.8%	**0.2%**
	4	20.04	20.04	1,913	**774**	0.0%	0.1%
	5	53.93	**53.96**	830	**280**	0.1%	0.1%
4	1	NA	**45.23**	>10800	>10800	NA	**3.0%**
	2	13.53	13.53	4,329	**320**	0.0%	0.1%
	3	9.15	**9.23**	**7,265**	>10800	-0.6%	**0.2%**
	4	NA	**20.04**	>10800	>10800	NA	**0.0%**
	5	50.04	**50.98**	>10800	>10800	-0.1%	**3.0%**
6	1	NA	**42.30**	>10800	>10800	NA	**6.0%**
	2	9.32	**11.68**	>10800	**7,308**	22.7%	**0.0%**
	3	7.10	**9.23**	>10800	>10800	19.2%	**0.2%**
	4	NA	**20.04**	7,301	**6,020**	NA	**0.0%**
	5	44.71	**51.11**	>10800	>10800	11.7%	**3.0%**
8	1	NA	**39.31**	>10800	>10800	NA	**15.3%**
	2	NA	**11.22**	>10800	>10800	NA	**10.8%**
	3	7.62	**9.23**	>10800	>10800	-33.1%	**0.2%**
	4	NA	**19.65**	>10800	**2,038**	NA	**0.1%**
	5	NA	**46.20**	>10800	>10800	NA	**13.1%**

NA = No integer solution found

1. Negative GAP when original solver uses a sub optimal solution of the MILP relaxation as upper bound