A new Decomposition Algorithm for Multistage Stochastic Programs with Endogenous Uncertainties

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Multistage Stochastic Model (Endogenous Uncertainty)

MSSP model (MILP) has a decomposable structure

$$\min \quad z = \sum_{s \in S} p^s \sum_{t \in T} c_t x_t^s$$

$$s.t. \quad \sum_{\tau \le t} A_\tau^s x_\tau^s \le a_t^s \qquad \forall t, s$$

Scenario Constraints

$$x_t^s = x_t^{s'} \quad \forall t \in T_I, \forall (s,s') \in P_3$$

Initial NACs

$$B_t^s x_t^s + C_t^s z_t^{s,s'} \le d_t^s \qquad \forall t \in T_C, \forall (s,s') \in P_3$$

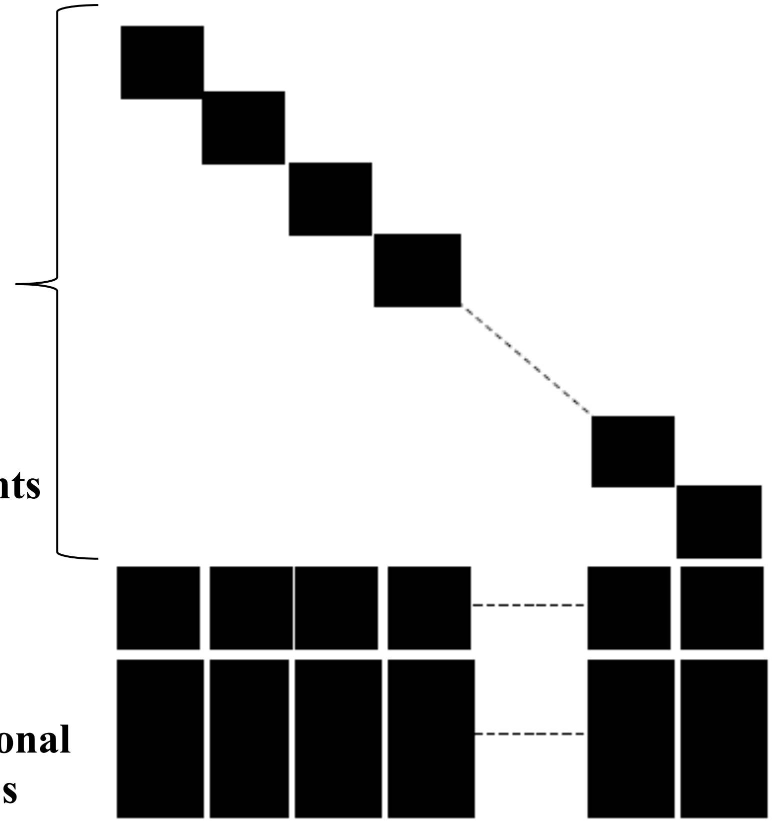
$$-M(1 - z_t^{s,s'}) \le x_t^s - x_t^{s'} \quad \forall t \in T_C, \forall (s,s') \in P_3$$

$$M(1 - z_t^{s,s'}) \ge x_t^s - x_t^{s'} \quad \forall t \in T_C, \forall (s,s') \in P_3$$

Conditional NACs

$$x_{jt}^s \in \{0,1\} \qquad \forall t, s, \forall j \in J'$$

$$x_{jt}^s \in R \qquad \forall t, s, \forall j \in J \setminus J'$$



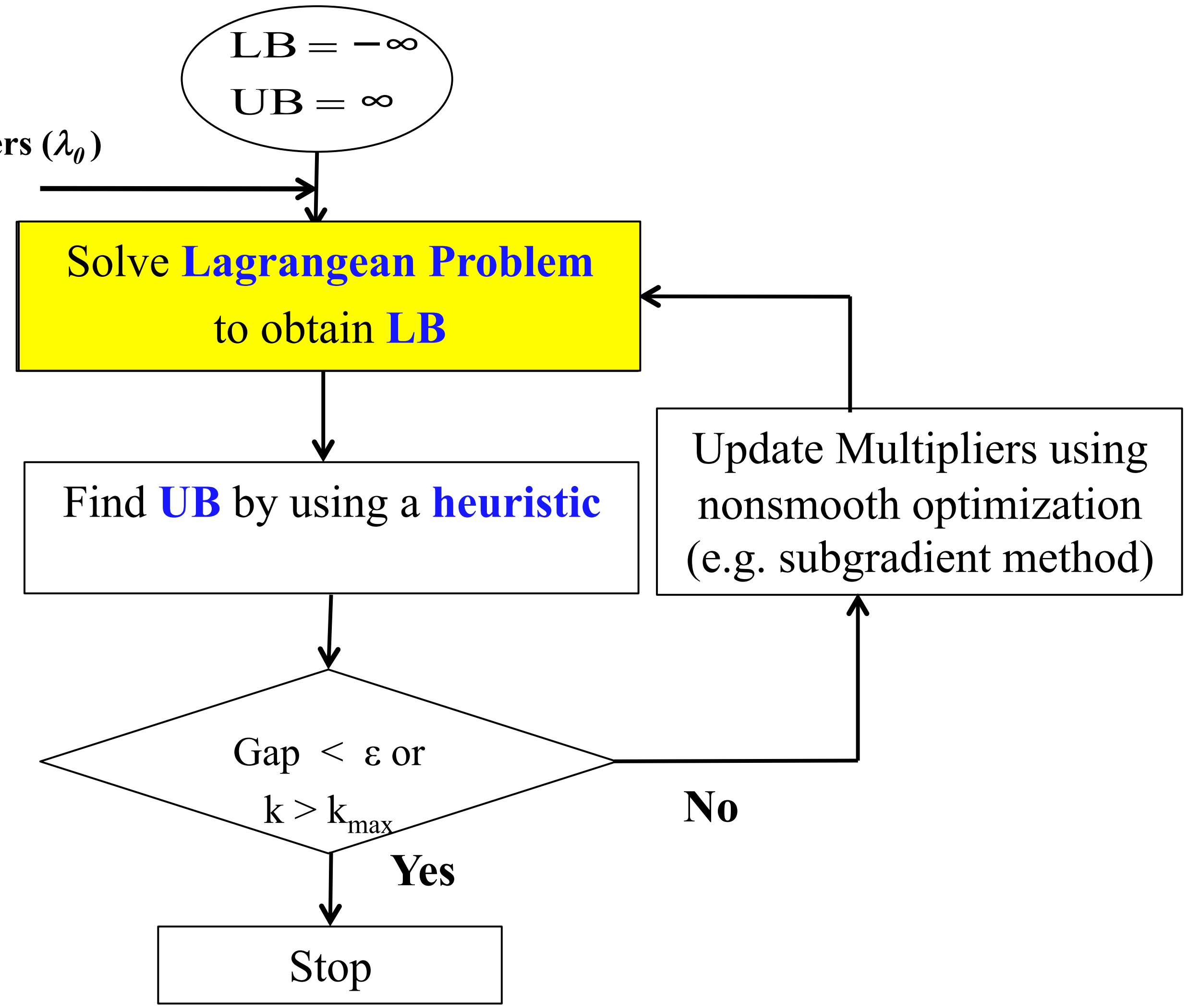
Non-anticipativity constraints link the scenarios (complicating constraints)

Lagrangean Decomposition

Lagrangean Decomposition Algorithm

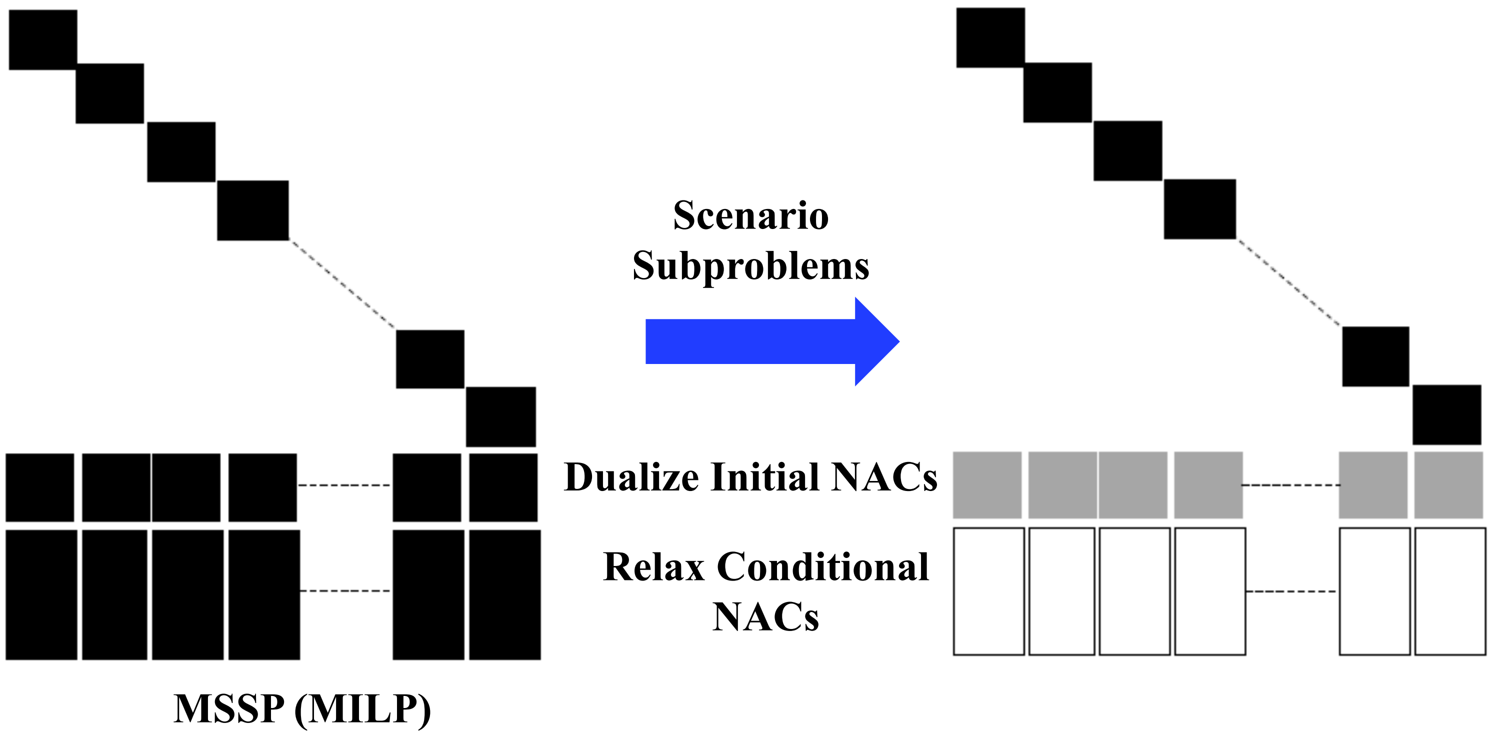
Lagrangean problem depends on how we decompose the fullspace model

Initial Multipliers (λ_0)
k=0



Duality gap may exist due to the presence of integer variables

Standard Lagrangean Decomposition Approach



Drawbacks:

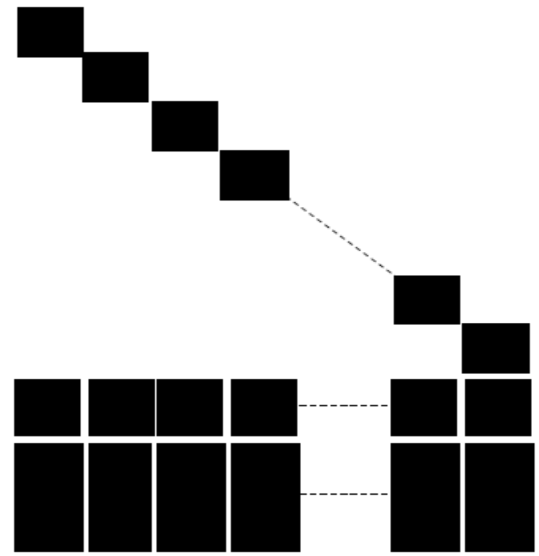
- ➢**Dual bound** at root node can be **weak** since **conditional NACs are ignored**
- ➢**Total nodes and the number of iterations at each node in B&B tree can be very large**
- ➢**Good heuristic is needed** to generate a **feasible solution**
- ➢**Efficient branching rules** in the tree search are not trivial to identify

Gupta and Grossmann (2011), Tahan et al. (2009), Goel and Grossmann (2006)

Goal and Basic Idea

Goal

Propose a new efficient decomposition algorithm for multistage stochastic programs under endogenous uncertainty and compare it with the standard decomposition approach



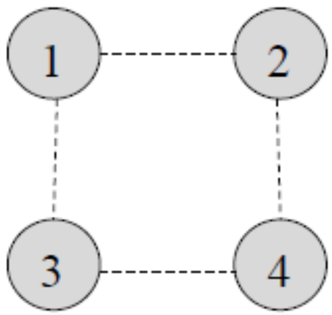
Basic Idea

The main idea relies on decomposing the MSSP (MILP) model into scenario groups instead individual scenario

Keep a subset of NACs as explicit constraints while relaxing/dualizing the rest of NACs → **Partial Decomposition**

Illustration: Standard vs. Proposed Decomposition

Standard Decomposition Approach (scenario based)



Dualize initial NAC and relax conditional NACs

2 Uncertain Parameters

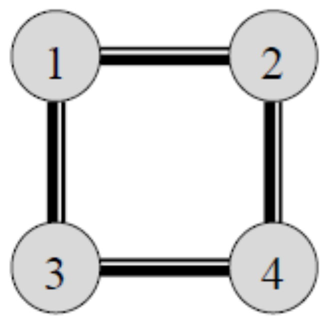
4 Scenarios

4 (s,s’) pairs for NACs

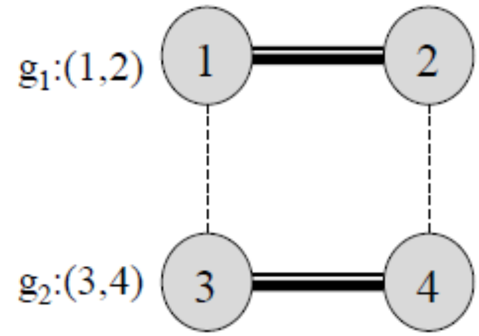
P_3:{(1,2),(1,3),(2,4),(3,4)}

MSSP Model

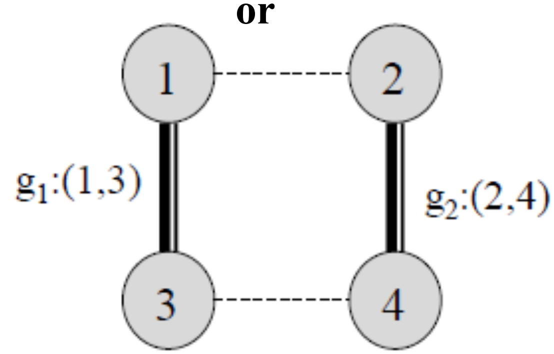
(Node represents scenario and line represent linking NACs)



Proposed Decomposition Approach (scenario group based)



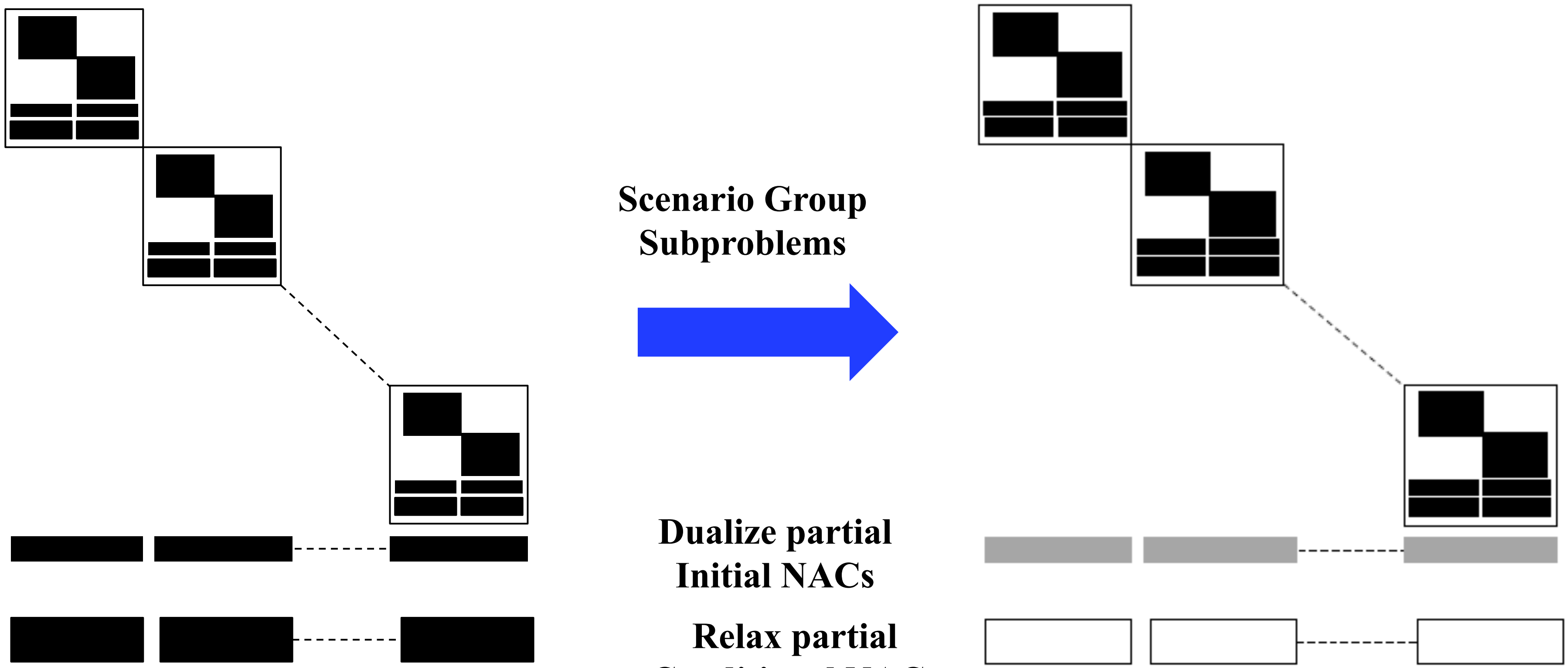
or



Construct the scenario group sets:
For **each endogenous uncertain** parameter take those scenarios in a group which **differs only in that particular uncertain parameter**

Select a scenario group set that provides the:
(a) **tightest initial bound** or
(b) **largest variations** in the objective values

Proposed Decomposition Approach

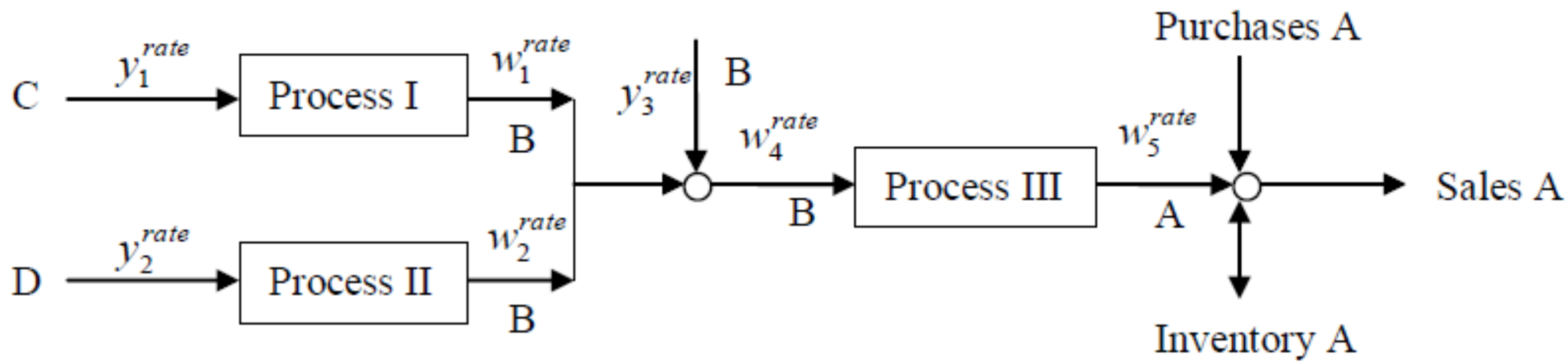


Reformulated MSSP (MILP) in scenario group form

Lagrangean Decomposition based on partial relaxation of the Conditional NACs

> ***Proposition:*** ***The dual bound obtained from the proposed decomposition approach at root node is at-least as tight as the one obtained from the standard approach***

Example: Process Network Planning under Uncertain Yield



- **3 Process, 10 years planning horizon**
- **Process III yield = 0.7**
- **Endogenous uncertainty: Process I yield: {0.69,0.81}, Process II yield: {0.65,0.85}**
- **4 Scenarios with equal probability of 0.25**
- **Determine optimal investment and operating decisions minimizing total expected cost**

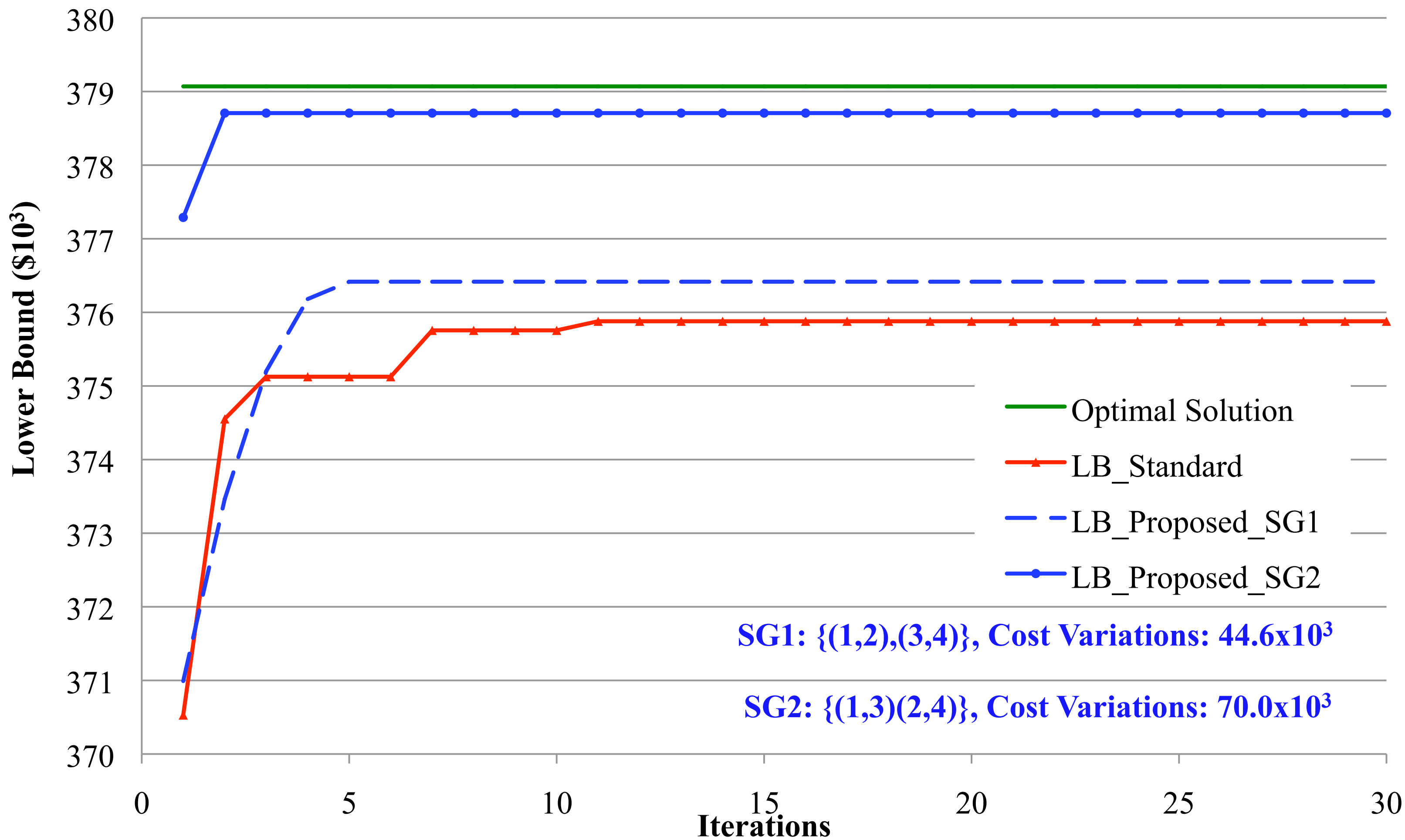
Model Statistics

Problem Type	Number of Constraints	Continuous Variables	Binary Variables
Reduced Model (MLR)	1,869	845	120
Individual Scenario	192	202	30

Optimal Solution

$379.07x10^3

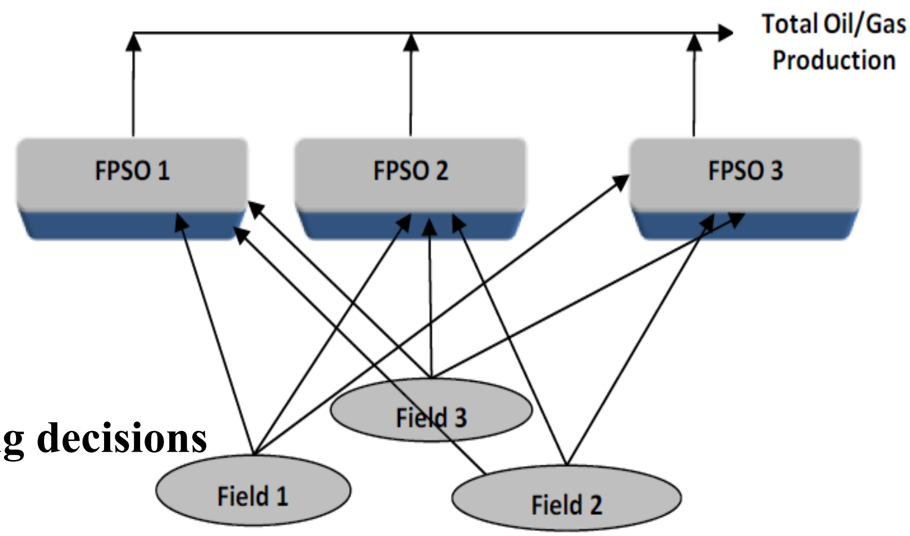
Process Network Example: Comparison of the Decomposition Approaches



Proposed approach with SG2 takes only 2 iterations and 0.94s to reach within 1% of gap compared to 30 iterations and 8.89s with 1.33% gap in the standard approach

Example: Oilfield Planning under Uncertain Field parameters

- 3 Oilfields, 3 potential FPSOs,
- 9 possible connections, 30 wells
- 10 years planning horizon
- Endogenous uncertainty
 (Field 1 and Field 2 parameters)
- 4 Scenarios with equal probability of 0.25
- Determine optimal investment and operating decisions
- Maximizing total expected NPV



Oil deliverability $Q^d = \alpha_o \cdot g(fc)$

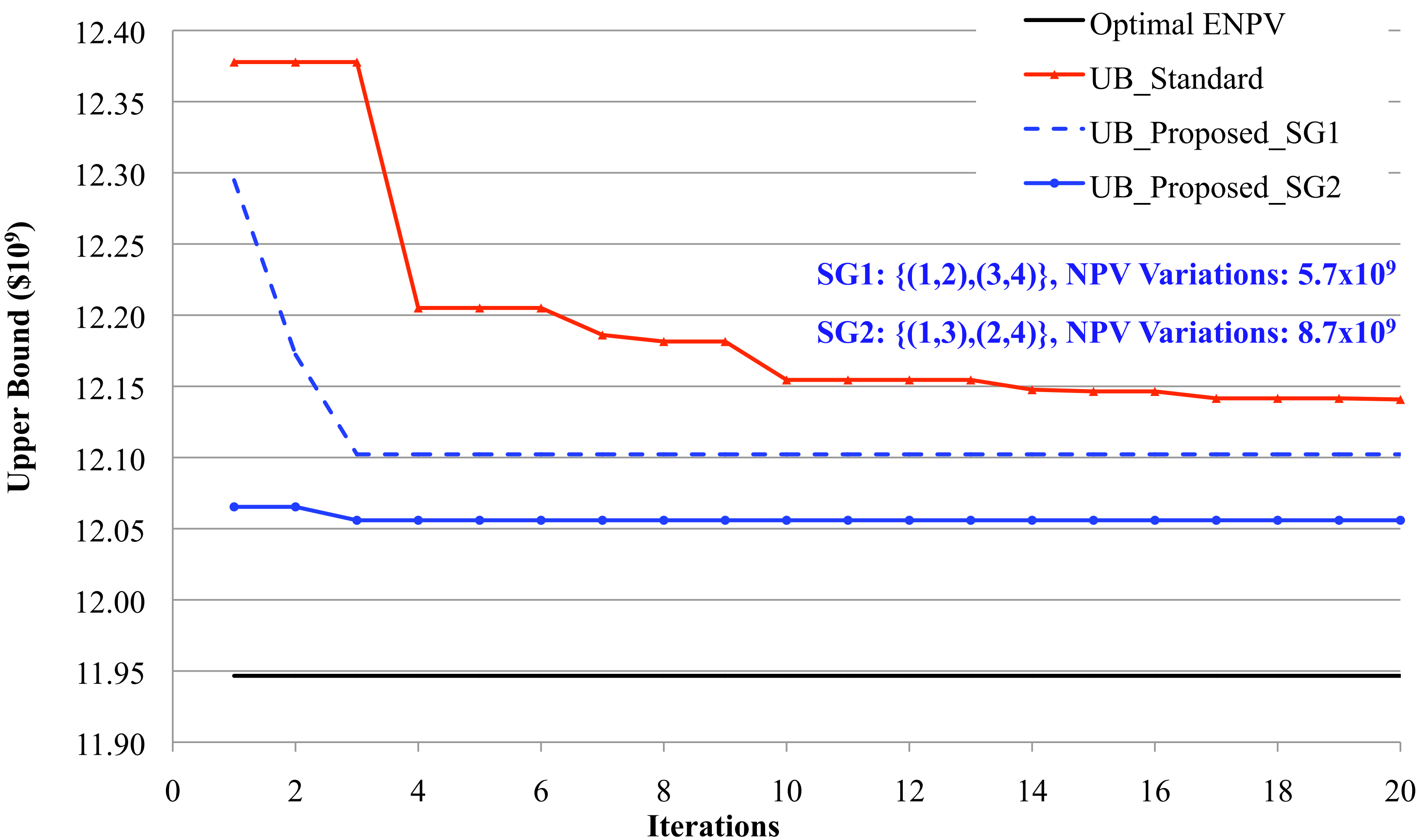
water-oil-ratio $wor = \alpha_w \cdot g(fc)$

gas-oil-ratio $gor = \alpha_g \cdot g(fc)$

Uncertainty in Field sizes and parameters $\alpha_o, \alpha_w, \alpha_g$

Assumption: Uncertain parameters for a field are correlated

Oilfield Example: Comparison of the Decomposition Approaches



Proposed approach with SG2 takes only 1 iteration and 84s to reach within 1% of gap compared to 20 iterations and 438s with 1.66% gap in the standard approach

Conclusions

- A new decomposition algorithm is proposed for multistage stochastic programs under endogenous uncertainties based on scenario group partitions
- The results on process network and oilfield planning problems show that the dual bound obtained in few iterations at the root node using proposed method is much stronger than the standard decomposition approach
- Potential reduction in the total nodes required in the duality based branch and bound search and improvement in the quality of the feasible solution