

Decomposition method for the Multiperiod Blending Problem

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Motivation and goals

Motivation

Multiperiod blending problem is a general model for many applications, and it is difficult to solve

- Gasoline and crude oil blending are some of the applications
- The model contains mixed-integer variables and bilinear constraints

Goals

Generate “good solutions” fast

- Guaranteeing global optimality is not a priority
- Solutions must be feasible

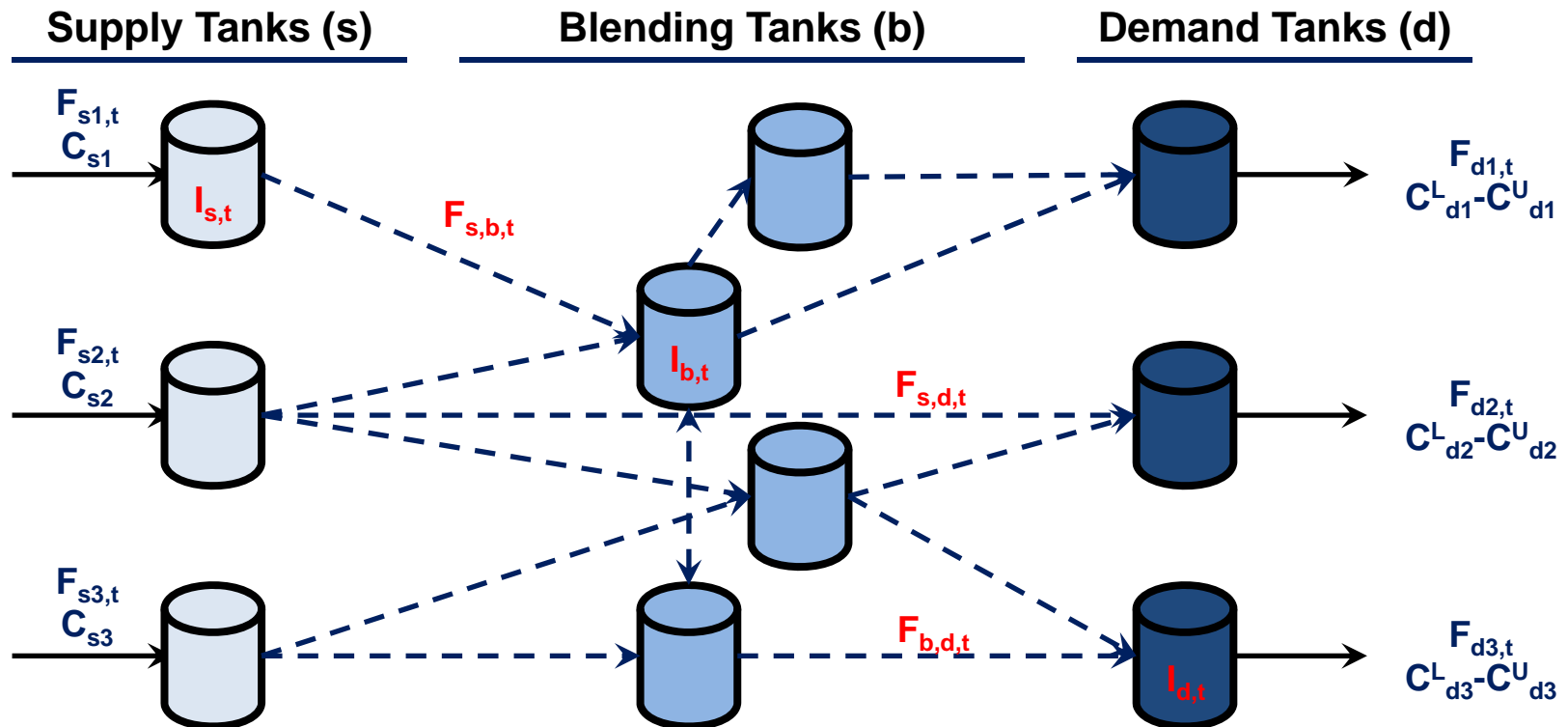
Developments

Generated new problem formulation with stronger MILP relaxation for the MINLP

Decompose the problem to simplify search for feasible solutions

- Solving smaller MINLPs with fewer 0-1 variables and bilinear terms
- “Guided” by an MILP relaxation of the problem

Multiperiod blending problem is defined over supply, blending and demand tanks



Given:

- Topology, initial conditions and flow profit/costs
- Supply tank flow and concentration
- Demand tank flow and concentration limits

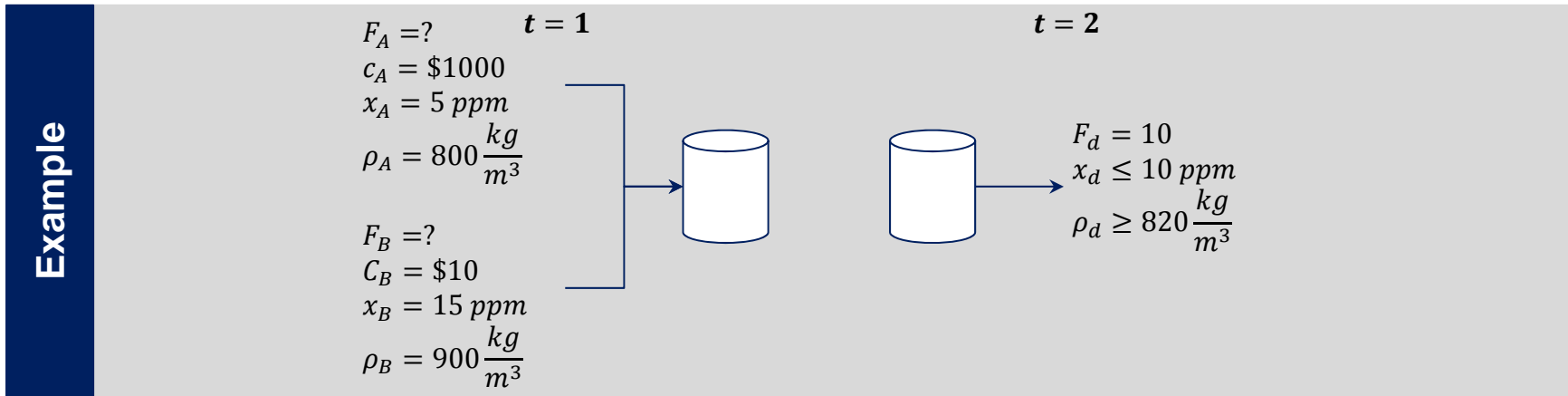
Determine

- Flows between which tanks in which time periods
- Inventories/concentrations for tanks in each period
- **Maximum total profit of blending operation**

Assumptions:

- Supply concentrations are constant
- **No simultaneous input/output to blending tanks**
- Perfect mixing

Individual flow and inventory constraints for sources instead of specs.



Total flow + fractions

$$\min c_A F_A + c_B F_B$$

s.t.

$$I_1 = F_A + F_B$$

$$I_1 x_{I1} = F_A x_A + F_B x_B$$

$$I_1 \rho_{I1} = F_A \rho_A + F_B \rho_B$$

Bilinear terms

$$x_d \leq 10$$

$$\rho_d \geq 820$$

$$I_2 = I_1 - F_d$$

$$x_{I2} = x_{I1}$$

$$\rho_{I2} = \rho_{I1}$$

Individual flows (specs.)

$$\min c_A F_A + c_B F_B$$

s.t.

$$I_1 = F_A + F_B$$

$$I_{x1} = F_A x_A + F_B x_B$$

$$I_{\rho1} = F_A \rho_A + F_B \rho_B$$

Inventory of specs. (@t = 1)

$$\xi I_1 = F_d \quad 0 \leq \xi \leq 1$$

$$\xi I_{x1} = F_{dx} \quad F_{dx} \leq 10 F_d$$

$$\xi I_{\rho1} = F_{d\rho} \quad F_{d\rho} \geq 820 F_d$$

$$I_2 = I_1 - F_d$$

$$I_{x2} = I_{x1} - F_{dx}$$

$$I_{\rho2} = I_{\rho1} - F_{d\rho}$$

Source base model

$$\min c_A F_A + c_B F_B$$

s.t.

$$I_1 = F_A + F_B$$

$$I_{A1} = F_A$$

$$I_{B1} = F_B$$

Inventory of sources (@t = 1)

Transformation of spec. constraints to sources

$$\xi I_1 = F_d \quad 0 \leq \xi \leq 1$$

$$\xi I_{A1} = F_{dA} \quad 5 F_{dA} + 15 F_{dB} \leq 10 F_d$$

$$\xi I_{B1} = F_{dB} \quad 800 F_{dA} + 9000 F_{dB} \geq 820 F_d$$

$$I_2 = I_1 - F_d$$

$$I_{A2} = I_{A1} - F_{dA}$$

$$I_{B2} = I_{B1} - F_{dB}$$

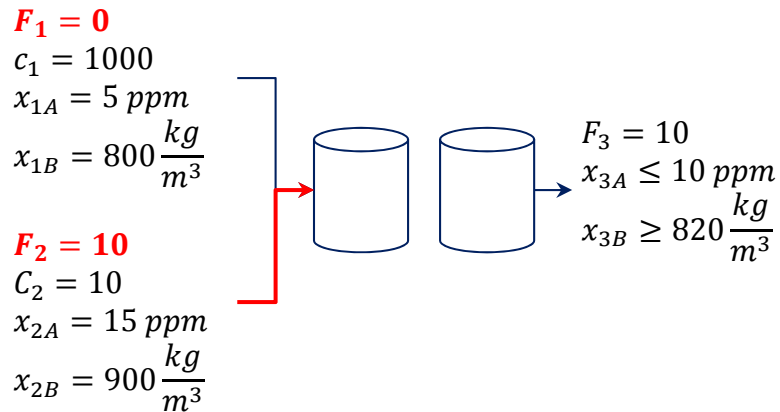
Redundant constraints

$$F_d = F_{dA} + F_{dB}$$

$$I_2 = I_{A2} + I_{B2}$$

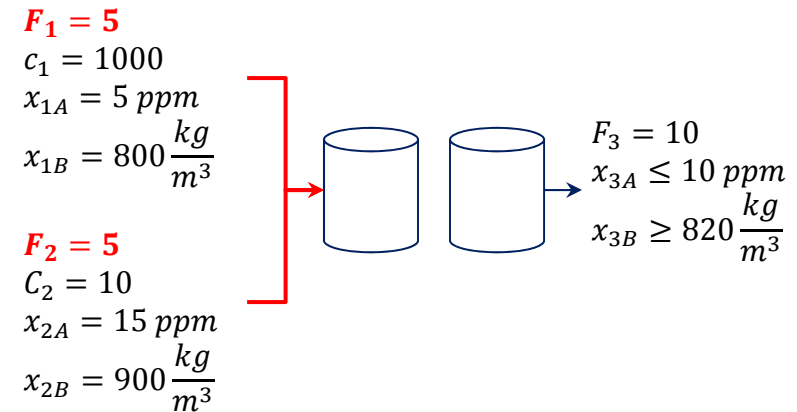
Source based model provides stronger linear relaxation

Total flow + fractions and Individual flows (specs.)



Solution of linear relaxation is infeasible for original problem

Source based model



Solution of linear relaxation is optimal for original problem!

Using redundant constraints helps to improve the linear relaxations of bilinear terms

Formulation with redundant constraints¹

$$I_{bt} = I_{bt-1} + \sum_{n \in IN} F_{nbt} - \sum_{n \in OUT} F_{bnt}$$

Flow times concentration formulation

$$I_{bt} C_{qbt} = I_{bt-1} C_{qbt-1} + \sum_{n \in IN} F_{nbt} C_{qnt-1}$$

$$C_{qbt} = C_{qbt-1}$$

$$\hat{I}_{sbt} = \hat{I}_{sbt-1} + \sum_{n \in IN} \hat{F}_{snbt} \quad \forall s$$

$$\hat{I}_{sbt} = \hat{I}_{sbt-1} - \sum_{n \in OUT} \hat{F}_{sbnt} \quad \forall s$$

$$F_{nbt} = \sum_s \hat{F}_{snbt}$$

$$I_{bt} = \sum_s \hat{I}_{sbt}$$

Relate total flows/inventories of sources with individual ones

Include individual flow and inventory for sources (not specs)

Results Problems solved²

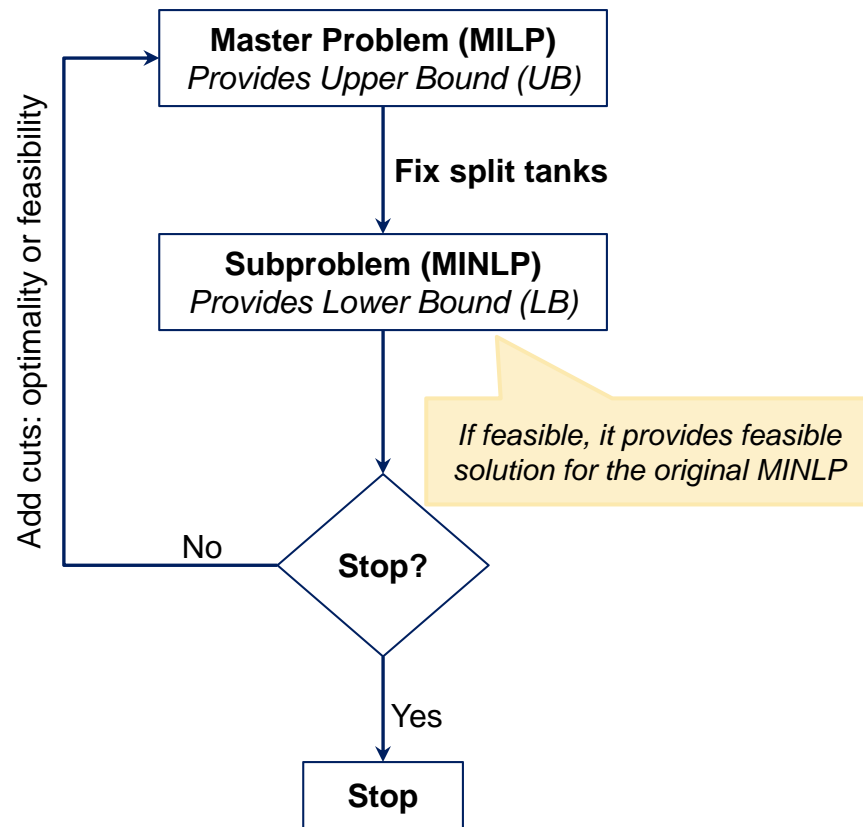
| | w/o RC | w RC |
|-----------------|--------|------|
| Antigone | 29% | 31% |
| Baron | 21% | 29% |
| SCIP | 31% | 42% |

1. Similar to the "pq" formulation in pooling problems. M. Tawarmalani and N.V. Sahinidis. Convexification and global optimization in continuous and mixed-integer nonlinear programming: theory, algorithms, software, and applications, chapter 9, pages 281–310. Kluwer Academic Publishers, 2002.

2. For 48 test cases with 8 blending tanks. Instances have 6 and 8 time periods. Qualities vary from 1 to 10 Antigone 1.1., Baron 14.0.3., SCIP 3.1

Decomposition algorithm seeks to find feasible solutions in short periods of time

Algorithm



Decisions in the algorithm

Stop criteria

- UB and LB gap
- Solution time

Master MILP (MINLP relaxation)

- Remove nonlinear constraints
- McCormick
- Piecewise McCormick
- Multiparametric disaggregation

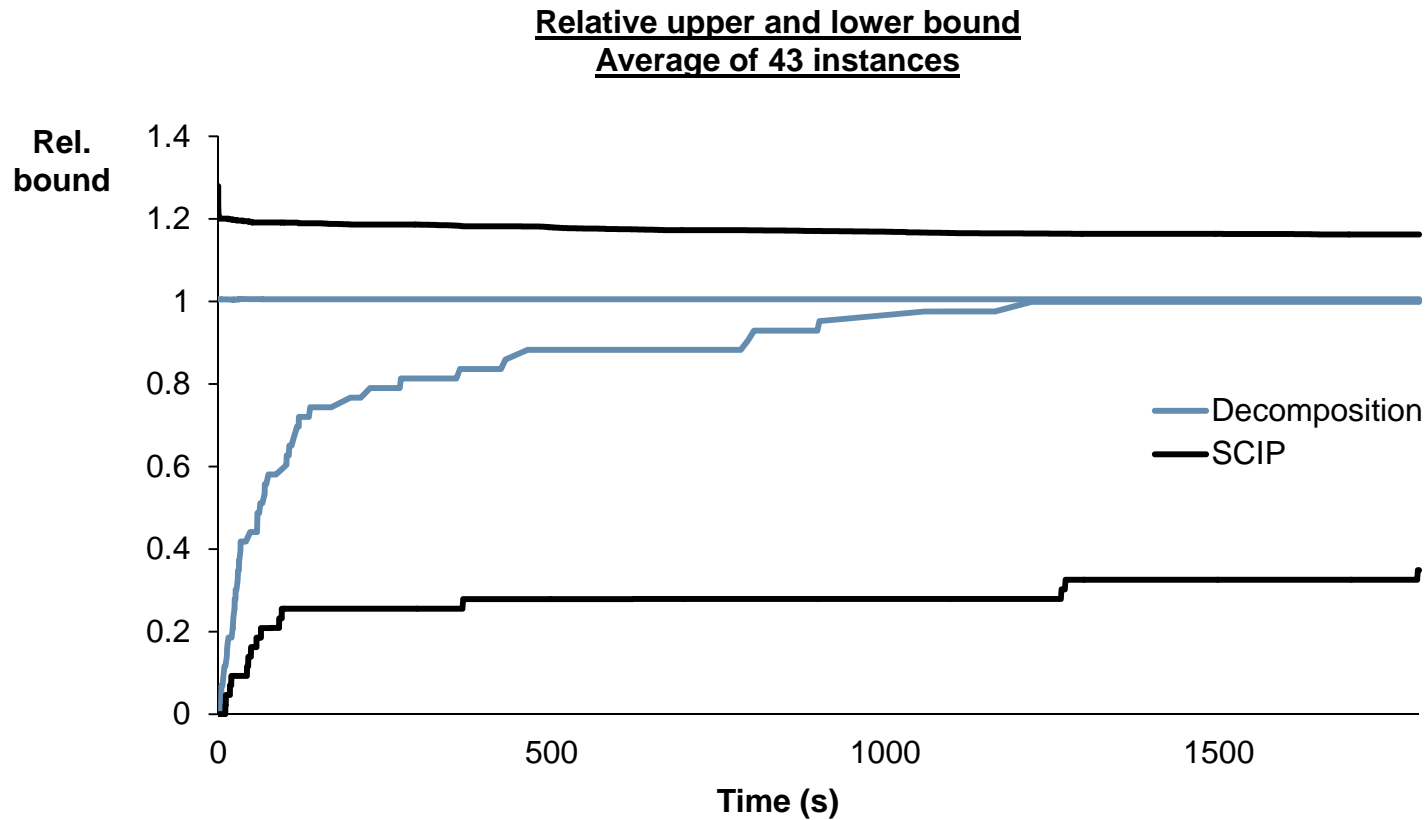
Solving MINLP

- Convex MINLP solver provides feasible solution, but optimality cut is not valid
- Global solution through commercial solver
- Global solution through specialized technique

Other considerations

- “no-good” are included in the subproblem, eliminating regions already evaluated in previous subproblems

Results show that the algorithm predicts strong bounds to solve the multiperiod blending problem



Note: In a 2.93 GHz Processor, Intel® Core™ i7. 4GB of RAM. SCIP 3.1.

First iteration provides good upper bound and feasible solutions in several instances

| Problem (instances) | MINLP | | | | MILP first iteration | | MINLP first iteration (subproblem) | | | |
|------------------------|--------------|----------|-------|--------|----------------------|------------------|------------------------------------|------------------|------------------|-------|
| | 0-1 vars. | Bilinear | Vars. | Const. | Avg. time (s) | Avg. Norm. UB | Avg. 0-1 | Avg. Bilinear | Avg. time (s) | Feas. |
| A (6) | 240 | 128 | 552 | 984 | 5.2 | 1.001 | 56 | 40 | 7.2 | 5 |
| B (6) | 240 | 256 | 600 | 1176 | 14.9 | 1.007 | 59 | 83 | 67.4 | 4 |
| C (6) | 240 | 640 | 772 | 1752 | 22.5 | 1.007 | 57 | 200 | 68.3 | 5 |
| D (6) | 240 | 1280 | 984 | 2712 | 22.8 | 1.037 | 58 | 405 | 68.8 | 2 |
| E (6) | 320 | 176 | 736 | 1312 | 15.6 | 1.001 | 69 | 45 | 38.3 | 6 |
| F (6) | 320 | 352 | 800 | 1568 | 11.0 | 1.007 | 72 | 91 | 157.2 | 2 |
| G (6) | 320 | 889 | 992 | 2336 | 24.3 | 1.004 | 75 | 235 | 207.8 | 2 |
| H (6) | 320 | 1760 | 1312 | 3616 | 41.3 | 1.011 | 76 | 470 | 177.3 | 0 |

Test problems still small for industrial problem size

Industrial application problem size comparison

| | <u>Current examples</u> | <u>Gasoline blending</u> | <u>Crude blending</u> |
|-----------------------|-------------------------|--------------------------|-----------------------|
| # of tanks | 8 | 4 ✓ | ~45 ✗ |
| # of qualities | 1 - 10 | >10 ✓ | >10 ✓ |
| Time periods | 6 - 8 | ~20 ✗ | ~20 ✗ |

Conclusions

We presented a strong formulation for the multiperiod blending problem

- Including redundant constraints for the total mass balance of sources

Decomposition algorithm developed to solve the problem shows promising results

- The algorithm iteratively solves an MILP and a smaller MINLP
- It generally finds good feasible solutions fast, while providing a good upper bound

Future work includes improvements in the algorithm and formulation

- Symmetry breaking constraints for the problem
- Develop parallelization of the algorithm