Rescheduling Bulk Gas Production and Distribution

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AIR PRODUCTS

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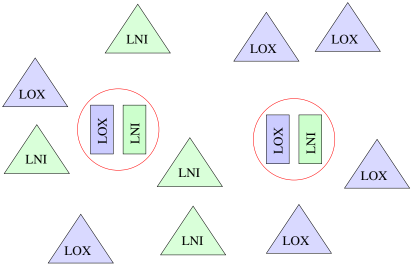
Liquid Bulk Gas Production-Distribution

- Sites $\mathcal{S}$
- Products $\mathcal{P} = \{\text{LOX}, \text{LNI}\}$
- Customers $\mathcal{C}$



Planning Problem

How should one set production levels at the sites $s \in \mathcal{S}$ and sourcing decisions (amount delivered from $s \in \mathcal{S}$ to $c \in \mathcal{C}$) in order to meet customer demand at minimum cost?

Bulk Gas Wrinkles

Production

- Most sites operate in two modes:
  - Regular Mode
  - Extended Mode (Costs more than regular)
- Physics of Production
  - Maximum total production: (LOX + LNI)
  - Individual production limit. (Fraction of total)

Competitor Arrangements

- Enter contractual "take-or-pay" arrangements with competitors.
- Allowed to remove (equal) fixed amount of product from each other's sites
  - $\mathcal{Q} \subseteq \mathcal{S}$: Set of *'Pick up'* locations
  - $\mathcal{R} \subseteq \mathcal{S}$: Set of *'Take out'* locations

A Simple Planning Model

Objective

- Production Cost + Distribution Cost

$$\min \sum_{p\in\mathcal{P}}\sum_{s\in\mathcal{S}}(\alpha_{ps}x_{ps} + \beta_{ps}e_{ps}) + \sum_{p\in\mathcal{P}}\sum_{s\in\mathcal{S}}\sum_{c\in\mathcal{C}}(d_{sc}y_{psc})$$

- Variables
  - x_{ps}: Regular production amount of $p \in \mathcal{P}$ at $s \in \mathcal{S}$
  - e_{ps}: Extended production amount of $p \in \mathcal{P}$ at $s \in \mathcal{S}$
- Parameters
  - α_{ps}: Regular mode per unit production cost of $p \in \mathcal{P}$ at $s \in \mathcal{S}$
  - β_{ps}: Extended mode per unit production cost of $p \in \mathcal{P}$ at $s \in \mathcal{S}$
  - d_{sc} per-unit delivery cost from $s \in \mathcal{S}$ to $c \in \mathcal{C}$

Constraints

Maximum Production Level

$$\sum_{p \in \mathcal{P}} x_{ps} \leq M_s, \qquad \sum_{p \in \mathcal{P}} e_{ps} \leq N_s \quad \forall s \in \mathcal{S}$$

$$x_{ps} \leq \Lambda_p M_s, \qquad e_{ps} \leq \Lambda_p N_s \quad \forall p \in \mathcal{P},\ \forall s \in \mathcal{S}$$

- Parameters
  - M_s, N_s: Regular mode maximum total production at $s \in \mathcal{S}$
  - N_s: Extended mode maximum total production at $s \in \mathcal{S}$
  - Λ_P: Maximum "air-fraction" of $p \in \mathcal{P}$
- This is a fairly crude (approximate) model of production

Constraints

Contract Amount Limit

$$\sum_{q \in \mathcal{Q}} x_{pq} \leq \Phi_p \qquad \forall p \in P$$

Customer Demand

$$\sum_{s \in \mathcal{S}} y_{psc} \geq B_{pc}, \quad \forall p \in \mathcal{P}, \; \forall c \in \mathcal{C}$$

- Variables
  - y_{psc}: Amount of $p \in \mathcal{P}$ shipped from $s \in \mathcal{S}$ to $c \in \mathcal{C}$
- Parameters
  - Φ_p: Contract amount for $p \in \mathcal{P}$
  - B_{pc}: Customer $c \in \mathcal{C}$ demand for $p \in \mathcal{P}$

Constraints

Inventory Balance

$$x_{ps} + e_{ps} - \sum_{c \in \mathcal{C}} y_{psc} - z_{ps} = \Delta I_{ps}, \quad \forall p \in \mathcal{P},\ \forall s \in \mathcal{S}$$

Resource: Driver Hours and Truck Hours

$$\begin{aligned} \sum_{p \in \mathcal{P}} \sum_{c \in \mathcal{C}} d_{sc} y_{psc} &\leq D_s, \quad \forall s \in \mathcal{S} \\ \sum_{c \in \mathcal{C}} d_{sc} y_{psc} &\leq K_{ps}, \quad \forall p \in \mathcal{P},\ \forall s \in \mathcal{S} \end{aligned}$$

- Variables
  - ΔI_{ps}: Change in inventory of $p \in \mathcal{P}$ at $s \in \mathcal{S}$
- Parameters
  - z_{ps}: Amount of $p \in \mathcal{P}$ competitor removes from $s \in \mathcal{S}$
  - D_s: Available driver hours at $s \in \mathcal{S}$
  - K_{ps}: Available truck hours of $p \in \mathcal{P}$ at $s \in \mathcal{S}$

Using the Production-Distribution Model

- Model is used to set monthly production levels and customer sourcing decisions
- Sometimes, during the course of the month, things get "out of skew"
  - A customers is about to be run out
  - Plants don't have enough product to meet short-term customer demand

What Happens in Practice

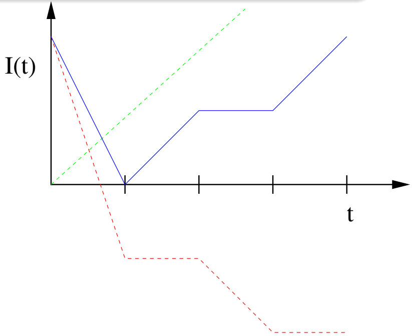
- Daily planners (attempt) to do a manual adjustment to the monthly schedule in order to meet customer demand
- Sometimes, the planning model will be re-run given the current (changed) input conditions.

Why?

One Possible Explanation

Known variation in plant supply and customer demand during the course of the month

- Customer Usage
- Aggregate/Prorated Delivery Volume
- True Customer Inventory



What's the Cure!?

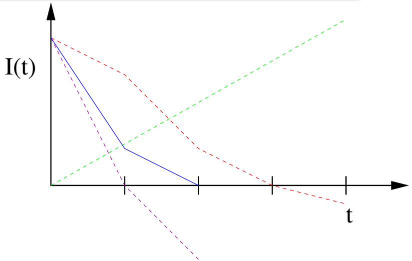
The rescheduling burden can be alleviated by solving the model at a finer time aggregation

Why?

Another Possible Explanation

Unknown (random) variation in components of the model

- A plant just went down for an unscheduled maintenance.
- A customer used three times as much as forecast



What's the Cure?

- In that case, a planning model that directly deals with the inherent uncertainty might be warranted
  - Stochastic Programming
  - Robust Optimization

What to Do?

- In either case, building and implementing a new model within APCI is not to be taken lightly.

Project Scope

- The company is interested in understanding why this rescheduling must often take place

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Step #1

- Build and experiment with a multi-period version of the planning model.
- Will allow us to experiment with instances solved at variety of time grains
- Would need a multi-period model in order to create a stochastic program anyway

A Multi-Period Planning Model

- $\mathcal{T} = \{1, 2, \ldots, T\}$: Set of time periods
- Essentially add a "time index" to all the variables and parameters
- Make inventory a variable that can be carried from one period to the next, e.g

$$x_{pst} + e_{pst} - \sum_{c \in \mathcal{C}} y_{psct} - z_{pst} + I_{pr,t-1} - I_{prt} = 0 \quad \forall p \in \mathcal{P},\ \forall r \in \mathcal{R},\ \forall t \in$$

- Also add inventory cost to objective

$$\sum_{p \in \mathcal{P}} \sum_{s \in \mathcal{S}} \sum_{t \in \mathcal{T}} \gamma_{ps} I_{pst}$$

Creating the Model

- Model built and created in Mosel modeling language
- Mosel is convenient
  - Air Products uses Mosel
  - If we wish to build a stochastic programming model later on, Mosel has new modules for building stochastic programs
- Model reads instance data from (properly formatted) text files

The Facts

1. Models need data
2. Data is hard to come by
3. $\Rightarrow$ We will create our own data.

- Instance Generation-Simulation code being created in C++
- Data is random, but reasonable
- **Sites**
  - Daily Production Rate
  - with random outages
- **Customers**:
  - Normally Distributed
  - On-Off
  - Call-in

Class Structure

- `Site`
  - Location
  - DailyMaxProduction
  - SiteProductInfo (NumTrucks, Initial Inventories, etc.)
- `Customer`
  - Location
  - Product
  - (Abstract) `DemandDistribution` Class

Class Structure

- `InstanceFamily`
  - NumDays
  - Sites
  - Customers
- `Instance`
  - Something that can be solved!
  - `create(InstanceFamily &if, vector<int> &daysPerPeriod)`

- All classes have a `makeRandom()` method that will instantiate itself with random, reasonable, data.
- When data is available, we can extended classes to instantiate themselves by reading from a file

Sample Instance Creation

```
int
main(int argc, char *argv[])
{
  InstanceFamily testInstance;

  // 3 sites, 12 customers, 10 days
  testInstance.makeRandom(3, 12, 10);

  Instance instance;
  vector<int> daysPerPeriod(4);
  daysPerPeriod[0] = 2;
  daysPerPeriod[1] = 2;
  daysPerPeriod[2] = 3;
  daysPerPeriod[3] = 3;

  instance.create(testInstance, daysPerPeriod);
  instance.writeMosel('EWO_AP.dat');
}
```

Experiments

Experiment #1 — Can We?

- Build large multi-period models
- See if state-of-the-art commercial solvers (XPRESS/CPLEX) as well as open-source solvers (Clp, GLPK) can solve instances in reasonable computing time

Experiment #2 — What do we gain?

1. Solve same `InstanceFamily` with large and small time buckets
2. Simulate customer inventories
   - Deliveries made in equal (daily) increments
   - Customer demand varies daily

Experiments

Metrics

1. Total number of customer outages
2. Average customer outage amount
3. Others?

Other Experiments

1. Measure cost and benefit of solving for a “robust” solution in which customer demand is slightly exceeded
2. How to handle “competitor relationships”
   - As a Parameter?
   - As a Variable?
   - Random Variable!

Results

Hopefully Some Preliminary Results by 3/15!