Global Optimization of Optimal Power Flow

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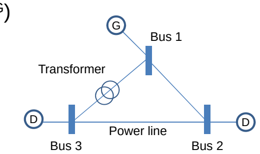
Outline

- Steady state problem (AC-OPF)
  - Problem Formulation.
  - Branch and Bound based global optimization algorithm using Semidefinite programming (SDP) relaxations.
  - Results
- Dynamic problem (M-OPF)
  - Problem Formulation.
  - Results.
- Conclusions & Future Work.

AC-OPF definition

- Given: Topology of the power grid: generators, loads, power lines.
- Objective:
  - Find operating points for generators, (P^G,Q^G)
  - Minimizing cost of generation – f(P^G)
- Satisfy constraints on
  - Power demands (P^D, Q^D)
  - Bus voltages (V)
  - Line, thermal & generation limits.



AC-OPF formulation

- Nonconvex Quadratically Constrained Quadratic Program (QCQP):

Define $\mathbf{x} = [\mathrm{Real}(V)^T,\ \mathrm{Imag}(V)^T]$

$$
\begin{aligned}
&\min\ c_0 + c_1\mathbf{P}^G + c_2(\mathbf{P}^G)^2 \\
&\text{s.t. } (P_i^G + \mathbf{j}Q_i^G) - (P_i^D + \mathbf{j}Q_i^D) = \sum_{i\sim j}(P_{ij} + \mathbf{j}Q_{ij}),\ i \in \mathcal{N} \\
&\quad \boxed{P_{ij} + \mathbf{j}Q_{ij} = \mathbf{x}^T Y_{ij}\mathbf{x},}\ (i,j) \in \mathcal{L} \\
&\quad \boxed{(V_i^{\min})^2 \le \mathbf{x}^T M_i \mathbf{x}} \le (V_i^{\max})^2,\ i \in \mathcal{N} \\
&\quad (P_i^{\min} + \mathbf{j}Q_i^{\min}) \le (P_i^G + \mathbf{j}Q_i^G) \le (P_i^{\max} + \mathbf{j}Q_i^{\max}),\ i \in \mathcal{N}^{\mathcal{G}} \\
&\quad P_{ij}^2 + Q_{ij}^2 \le (S_{ij}^{\max})^2,\ P_{ij} \le P_{ij}^{\max}\ (i,j) \in \mathcal{L}
\end{aligned}
$$

Nonconvex constraints

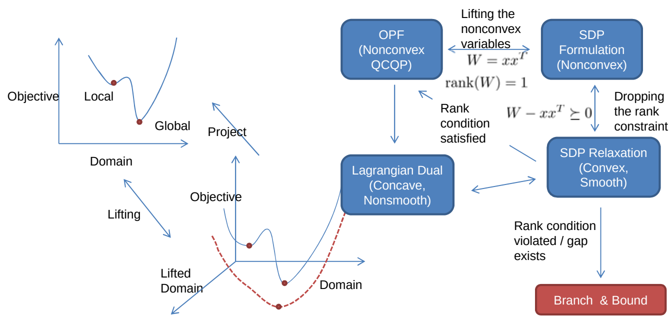
Power balance

Power flows

Physical limits

- FERC study: Lot of local solutions exist (~ 20% improvement can be achieved)

SDP Relaxation of OPF



- Semidefinite programming formulation: Lift the product terms $x_i x_j$ to an element W_{ij} of an n x n matrix **W**
  - Turn out to be very strong in a lot of instances.
  - Harder to solve but a price worth paying to get global optimality!
- For QCQP's, optimal objective fn. value: SDP = Lagrangian Dual <= QCQP

Test Cases

- Most IEEE benchmark instances satisfied sufficient rank condition and show low gap at root node.

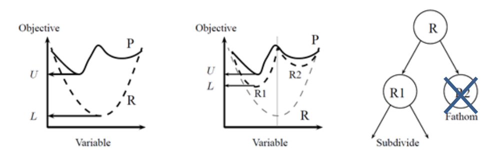
Case	SDP Branch and Bound					BARON (LP)		
	Root gap %	SDP rank	Close gap %	Time (sec)	Nodes	Root gap %	Close gap %	Time (sec)
3 bus	0	1	< 0.1	2.44	1	10.52	< 0.1	0.55
6 bus	0	1	< 0.1	2.15	1	68.71	12.05	300
9 bus	0	1	< 0.1	2.27	1	75.29	57.62	300
14 bus	0	1	< 0.1	2.42	1	100.00	100.00	300
30 bus	0	1	< 0.1	3.38	1	100.00	100.00	300
39 bus	0.005	1	< 0.1	4.89	1	96.52	34.57	300
57 bus	0.001	1	< 0.1	6.94	1	100.00	100.00	300
118 bus	0.005	1	< 0.1	22.6	1	100.00	100.00	300

- SDP relaxations tighter than LP relaxations used by BARON

- Modified IEEE instances (demands, line flow limits)

Case	SDP Branch and Bound				
	Root gap %	SDP Rank	Close gap %	Time (sec)	Nodes
3 bus	0.39	3	< 0.1	12.03	19
6 bus	0.38	3	< 0.1	6.08	9
9 bus	0.36	5	< 0.1	45.66	59
14 bus	0.16	3	< 0.1	19.16	21
30 bus	0.19	7	< 0.1	24.50	19
39 bus	0.74	9	0.59	300	163
57 bus	2.31	3	0.32	300	275
118 bus	0.17	5	<0.1	138.85	9

Branch & Bound Algorithm

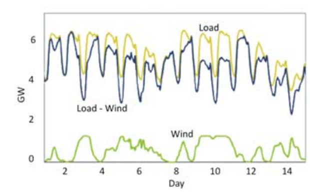


(a) Lower and Upper Bounding (b) Domain Subdivision (c) Search tree

1. Upper bounding: Local NLP solver (IPOPT)
2. Lower bounding: SDP relaxation (SeDuMi).
3. Branching rules: Rectangular/Radial partitioning.
4. Bounds tightening: Strengthening constraints.

Multi-Period OPF – 'Smart Grid'

- Transmission short falls due to
  - Fluctuating demands.
  - Integrating renewables into the grid e.g., Wind: Hard to forecast.
  - Base load power plants cannot change their output frequently.
- Integrating storage into the grid.
- M-OPF to decide optimal storage policy and generation dispatch.



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Multi-Period OPF Formulation

- Nonconvex QCQP:

Define $\mathbf{x}(t) = [\text{Real}(V)^T,\ \text{Imag}(V)^T]$

$$\min \sum_{t \in T} c_0 + c_1 \mathbf{P}^G(t) + c_2(\mathbf{P}^G(t))^2$$

Storage term

$$\text{s.t.}\ (P_i^G(t) + \mathbf{j}Q_i^G(t)) - (P_i^D(t) + \mathbf{j}Q_i^D(t)) \boxed{- R_i(t)} = \sum_{i \sim j} (P_{ij}(t) + \mathbf{j}Q_{ij}(t)),\ i \in \mathcal{N}$$

Power balance

$$\boxed{P_{ij}(t) + \mathbf{j}Q_{ij}(t) = \mathbf{x}(t)^T Y_{ij} \mathbf{x}(t),}\ (i,j) \in \mathcal{L}$$

$$\boxed{(V_i^{\min})^2 \leq \mathbf{x}(t)^T M_i \mathbf{x}(t)} \leq (V_i^{\max})^2,\ i \in \mathcal{N}$$

Power flows

$$(P_i^{\min} + \mathbf{j}Q_i^{\min}) \leq (P_i^G(t) + \mathbf{j}Q_i^G(t)) \leq (P_i^{\max} + \mathbf{j}Q_i^{\max}),\ i \in \mathcal{N}^{\mathcal{G}}$$

$$P_{ij}^2(t) + Q_{ij}^2(t) \leq (S_{ij}^{\max})^2,\ P_{ij}(t) \leq P_{ij}^{\max}\ (i,j) \in \mathcal{L}$$

Physical limits

$$\boxed{\begin{aligned} \Delta P_i^{\min} \leq P_i^G(t+1) - P_i^G(t) \leq \Delta P_i^{\max}, \\ \Delta Q_i^{\min} \leq Q_i^G(t+1) - Q_i^G(t) \leq \Delta Q_i^{\max}, \end{aligned}} \begin{aligned} & i \in \mathcal{N}^{\mathcal{G}} \\ & i \in \mathcal{N}^{\mathcal{G}} \end{aligned}$$

$$\boxed{B_i(t) = B_i(t-1) + \eta R_i(t),}\ B^{\min} \leq B_i(t) \leq B^{\max}, R^{\min} \leq R_i(t) \leq R^{\max},\ i \in \mathcal{N}^{\mathcal{S}}$$

Storage & ramp constraints

Nonconvex

Time coupled

- SDP based Branch & Bound algorithm for multi-period OPF problems.

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Conclusions & Future Work

Conclusions:

- Globally optimal solutions to nonconvex OPF & multi-period OPF problems.
- Branch and Bound algorithm uses SDP relaxations to provide strong lower bounds.
- Effect of storage & ramp constraints in offsetting demand variations, integrating renewable energy sources, smoothing out generation profiles.

Future Work:

- Addressing large scale problems through efficient decomposition techniques or distributed computations.
- Speed up of branch and bound algorithm by employing effective heuristics.

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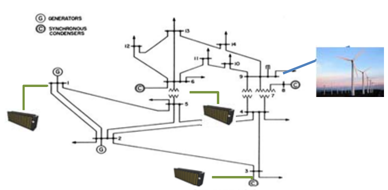
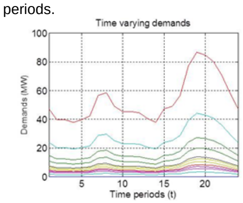
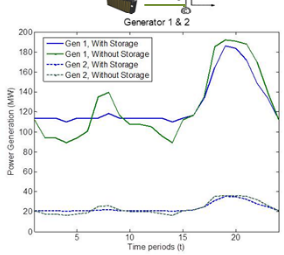
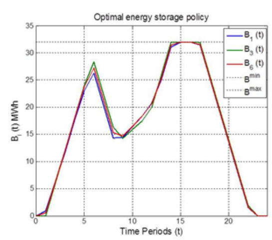
Case Study 1: IEEE14 example

- 14 buses, 2 generators, 1 wind farm, 3 storage units, 24 time periods.









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