CP Based Tool for Tightening the MILP Relaxation of a Crude-Oil Operations Scheduling MINLP

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Enterprise-Wide Optimization, 11 March 2009

TOTAL

Carnegie Mellon

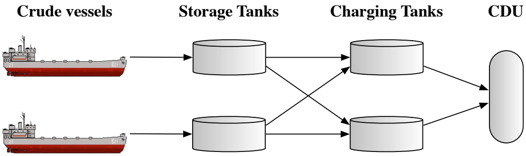
Introduction

Goals

- Optimize the schedule of operations for the crude-oil unloading and blending problem using a continuous-time mathematical formulation
- Reduce the computation expense by breaking the symmetries in the MINLP model
- Use CP inference techniques to improve performance
- Use CP bound contraction to tighten the MILP relaxation of the MINLP

Crude-oil operations scheduling problem

- Scheduling horizon $[0, H]$
- 4 types of resources:
  - Crude-oil marine vessels
  - Storage tanks
  - Charging tanks
  - Crude Distillation Units (CDUs)
- 3 types of operations:
  - Unloading: Vessel unloading to storage tanks
  - Transfer: Transfer from storage tanks to charging tanks
  - Distillation: Distillation of charging tanks

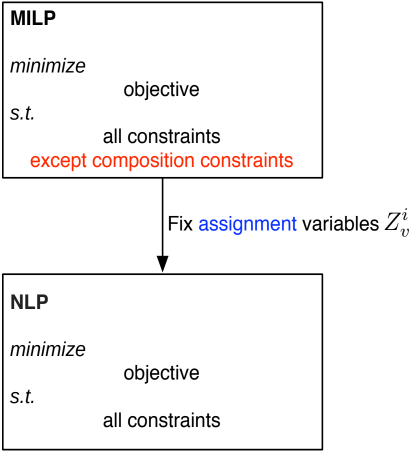


Solution method

Decomposition steps

The MINLP model is solved using the following MILP - NLP decomposition procedure :

1. **Solve** the MILP relaxation
2. **Check** feasibility: the solution may not satisfy the nonlinear composition constraints
3. If infeasible, **Fix** assignment variables
4. **Solve** the resulting NLP (with nonlinear composition constraints)



Profit maximization vs Cost minimization

- Original linear objective function:

$$\max \sum_{i \in T} \sum_{r \in R_D} \sum_{v \in I_r} \sum_{c \in C} G_c \cdot V_{ivc}$$

- New nonlinear objective function:

$$\begin{aligned} Z = {} & SWITCHINGCOST \sum_{r \in CDU} \left(\sum_{i,v \in I_r} Z_{iv} - 1 \right) \\ & + UNLOADINGCOST \sum_{i,v \in UNLOAD} D_{iv} \\ & + SEAWAITINGCOST \sum_{i,v \in UNLOAD} W_{iv} \\ & + FIXEDSTORAGECOST \\ & + \sum_{i,r_1,r_2,v \in O_{r_1} \cap I_{r_2}} \left(\dot{C}_{r_2} - \dot{C}_{r_1} \right) \left(H - S_{iv} - \frac{D_{iv}}{2} \right) V_{iv} \end{aligned}$$

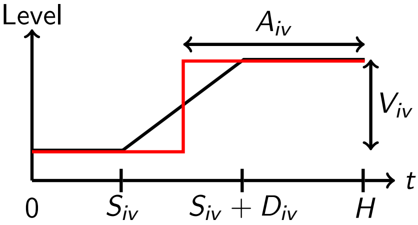
- Lee et al., 2006 (discrete-time, linear objective)
- Jia et al., 2003 (continuous-time, linear approximation objective)
- Karuppiah et al., 2008 (continuous-time, exact nonlinear objective, global optimization)

Storage cost variation

- Variation of storage cost:

$$\Delta COST_{iv} = \sum_{i,r_1,r_2,v \in O_{r_1} \cap I_{r_2}} \left(\dot{C}_{r_2} - \dot{C}_{r_1}\right)\left(H - S_{iv} - \frac{D_{iv}}{2}\right) V_{iv}$$
$$= \sum_{i,r_1,r_2,v \in O_{r_1} \cap I_{r_2}} \left(\dot{C}_{r_2} - \dot{C}_{r_1}\right) A_{iv} V_{iv}$$

- where:
  - $\dot{C}_r$ is the storage cost rate
  - $A_{iv} = H - S_{iv} - \frac{D_{iv}}{2}$ is the time remaining after the middle of operation time interval
  - V_{iv} is the volume transferred



Reformulation and linear relaxation

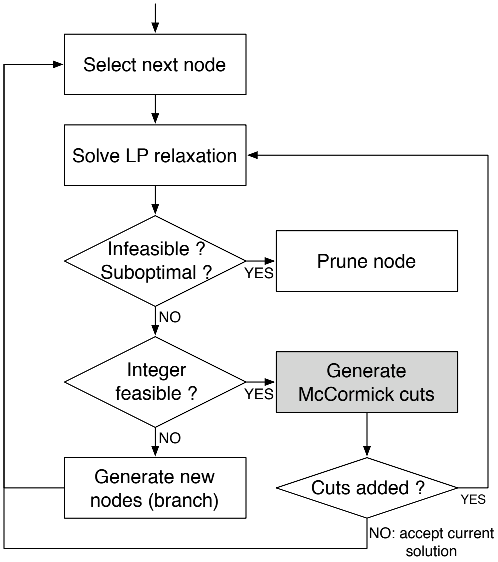
- Reformulation of storage costs:

$$\Delta COST = \sum_{i,r_1,r_2,v \in O_{r_1} \cap I_{r_2}} \left(\dot{C}_{r_2} - \dot{C}_{r_1} \right) A_{iv} V_{iv}$$
$$= \sum_{i,r_1,r_2,v \in O_{r_1} \cap I_{r_2}} \left(\dot{C}_{r_2} - \dot{C}_{r_1} \right) X_{iv}$$

- where $X_{iv} = A_{iv} V_{iv}$
- Linear relaxation using McCormick under- and overestimators (McCormick, 1976):

$$X_{iv} \geq A_{iv}^{L} V_{iv} + A_{iv} V_{iv}^{L} - A_{iv}^{L} V_{iv}^{L}$$
$$X_{iv} \geq A_{iv}^{U} V_{iv} + A_{iv} V_{iv}^{U} - A_{iv}^{U} V_{iv}^{U}$$
$$X_{iv} \leq A_{iv}^{L} V_{iv} + A_{iv} V_{iv}^{U} - A_{iv}^{L} V_{iv}^{U}$$
$$X_{iv} \leq A_{iv}^{U} V_{iv} + A_{iv} V_{iv}^{L} - A_{iv}^{U} V_{iv}^{L}$$

Branch & Cut with McCormick cuts



- McCormick cuts generation implementing in Ilog Cplex 11 using a Lazy Constraint Callback
- Cut generation procedure executed only at integer feasible nodes to reduce computational expense

CP based McCormick cuts

- Let p be a discrete solution of the MINLP defined by the discrete values taken by binary variables Z_{iv}
- p is defined by the corresponding sequence of operations $(v_1^p, \ldots, v_n^p)$
- Let $(CP)^p$ be a CP model defined by:

$$(CP)^p \begin{cases} \text{MINLP constraints} & \\ & Z_{iv} = 1 \quad \forall (i,v), v = v_i^p \\ & Z_{iv} = 0 \quad \forall (i,v), v \neq v_i^p \end{cases}$$

- CP constraint propagation can be used to obtain tight bounds for discrete solution p:

$$(CP)^p \xrightarrow[\text{Ilog Solver}]{\text{constraint propagation}} \left(\left(A_{iv}^L\right)^p, \left(A_{iv}^U\right)^p, \left(V_{iv}^L\right)^p, \left(V_{iv}^U\right)^p \right)$$

CP based McCormick cuts

- Consider an MILP node for which the LP relaxation is integer feasible
- Let p be the corresponding discrete solution
- The following "basic McCormick cuts" are valid for p:

$$X_{iv} \geq \left(A_{iv}^L\right)^p V_{iv} + A_{iv}\left(V_{iv}^L\right)^p - \left(A_{iv}^L\right)^p\left(V_{iv}^L\right)^p$$
$$X_{iv} \geq \left(A_{iv}^U\right)^p V_{iv} + A_{iv}\left(V_{iv}^U\right)^p - \left(A_{iv}^U\right)^p\left(V_{iv}^U\right)^p$$
$$X_{iv} \leq \left(A_{iv}^L\right)^p V_{iv} + A_{iv}\left(V_{iv}^U\right)^p - \left(A_{iv}^L\right)^p\left(V_{iv}^U\right)^p$$
$$X_{iv} \leq \left(A_{iv}^U\right)^p V_{iv} + A_{iv}\left(V_{iv}^L\right)^p - \left(A_{iv}^U\right)^p\left(V_{iv}^L\right)^p$$

- However, if not all binary variables Z_{iv} are fixed, they are not valid for the current node

CP based McCormick cuts

- Instead with use following "big-M McCormick cuts":

$$X_{iv} \geq \left(A_{iv}^{L}\right)^{p} V_{iv} + A_{iv}\left(V_{iv}^{L}\right)^{p} - \left(A_{iv}^{L}\right)^{p}\left(V_{iv}^{L}\right)^{p} - M_1 \cdot (n - \sum_{i} Z_{iv_i^p})$$

$$X_{iv} \geq \left(A_{iv}^{U}\right)^{p} V_{iv} + A_{iv}\left(V_{iv}^{U}\right)^{p} - \left(A_{iv}^{U}\right)^{p}\left(V_{iv}^{U}\right)^{p} - M_2 \cdot (n - \sum_{i} Z_{iv_i^p})$$

$$X_{iv} \leq \left(A_{iv}^{L}\right)^{p} V_{iv} + A_{iv}\left(V_{iv}^{U}\right)^{p} - \left(A_{iv}^{L}\right)^{p}\left(V_{iv}^{U}\right)^{p} + M_3 \cdot (n - \sum_{i} Z_{iv_i^p})$$

$$X_{iv} \leq \left(A_{iv}^{U}\right)^{p} V_{iv} + A_{iv}\left(V_{iv}^{L}\right)^{p} - \left(A_{iv}^{U}\right)^{p}\left(V_{iv}^{L}\right)^{p} + M_4 \cdot (n - \sum_{i} Z_{iv_i^p})$$

where:

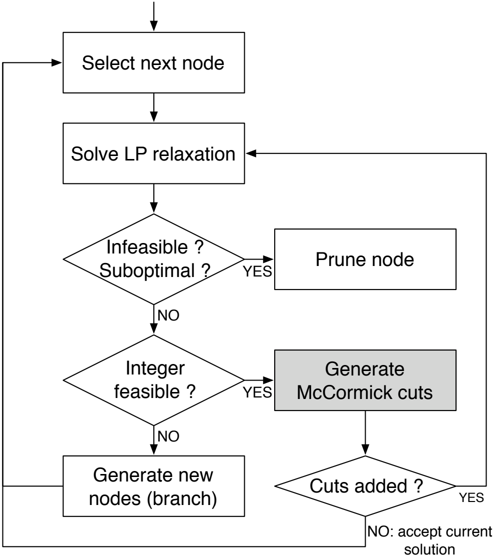
$$M_1 = \left(A_{iv}^{L}\right)^{p} V_{iv}^{U} + A_{iv}^{U}\left(V_{iv}^{L}\right)^{p} - \left(A_{iv}^{L}\right)^{p}\left(V_{iv}^{L}\right)^{p}$$

$$M_2 = \left(A_{iv}^{U}\right)^{p} V_{iv}^{U} + A_{iv}^{U}\left(V_{iv}^{U}\right)^{p} - \left(A_{iv}^{U}\right)^{p}\left(V_{iv}^{U}\right)^{p}$$

$$M_3 = A_{iv}^{U} V_{iv}^{U} - \left\{\left(A_{iv}^{L}\right)^{p} V_{iv}^{L} + A_{iv}^{L}\left(V_{iv}^{U}\right)^{p} - \left(A_{iv}^{L}\right)^{p}\left(V_{iv}^{U}\right)^{p}\right\}$$

$$M_4 = A_{iv}^{U} V_{iv}^{U} - \left\{\left(A_{iv}^{U}\right)^{p} V_{iv}^{L} + A_{iv}^{L}\left(V_{iv}^{L}\right)^{p} - \left(A_{iv}^{U}\right)^{p}\left(V_{iv}^{L}\right)^{p}\right\}$$

Branch & Cut with McCormick cuts



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Computational results

- 4 problems from Lee et al. (1996) solved with 2 approaches
  - BasicRelaxation: McCormick constraints added at the root node only (modeling stage)
  - ExtendedRelaxation: McCormick cuts at each integer feasible nodes
- MILP: Ilog Cplex ; CP: Ilog Solver ; NLP: CONOPT 3
- Optimality gap is reduced (3.48% vs 14.83% average gap)
- CPU time is increased by 9.5%

Pb	MILP Solution	MILP CPU	MILP CP	MILP Nodes	NLP Solution	Gap
BasicRelaxation						
1	199.1	9s	-	22	222.3	11.7%
2	297.8	215s	-	55	362.9	21.9%
3	254.6	224s	-	73	287.6	13.0%
4	331.8	600s	-	20	374.0	12.7%
ExtendedRelaxation						
1	213.7	11s	1s	20	222.3	4.0%
2	343.1	246s	19s	57	351.2	2.4%
3	269.2	337s	16s	95	287.6	6.8%
4	371.3	554s	18s	19	374.0	0.7%

Table: Results obtained with BasicRelaxation and ExtendedRelaxation algorithms

Computational results

- For problem 1, at the node where the optimal solution is found, 2 rounds of McCormick cuts are added:
  - First round increase objective value from 199.1 to 212.4
  - Second round increase objective value from 212.4 to 213.7
  - No cuts are add during the third round

- Comparison with generic MINLP solvers for problems 1 and 2

	Problem 1			Problem 2		
Algorithm	Solution	CPU	Gap	Solution	CPU	Gap
BasicRelaxation	222.3	9s	11.7%	362.9	215s	21.9%
ExtendedRelaxation	222.3	11s	4.0%	351.2	246s	2.4%
DICOPT	233.5	14s	-	351.2	1235s	-
sBB	Local Infeas.	14s	-	Local Infeas.	697s	-
Bonmin-OA	222.3	27s	-	No solution	+3600s	-
AlphaECP	222.3	260s	-	358.0	+3600s	-
BARON	222.3	+3600s	4.1%	No solution	+3600s	-

Table: Results obtained with several algorithms on problems 1 and 2

Conclusion and future work

Conclusion

- Continuous-time MINLP formulation for the crude-oil operations problem (Mouret et al., 2009)
- CP based MILP relaxation tightening tool for the minimization of total costs
  - Reduced optimality gap
  - Same or better NLP feasible solution obtained

Future work

- Improve CP model for better bound tightening
- Use optimality-based reduction techniques (Sahinidis, 2003) to derive a cut valid for the optimal solution (but not for all feasible solutions):

$$x^* = x^U(\text{multiplier } \lambda^*) \Rightarrow x \geq x^U - \frac{U - L^*}{\lambda^*} \text{ (for minimization problem)}$$

- Extend the MILP search with spatial branch & bound for complete global optimization