A Novel Global Optimization Approach to the Multiperiod Blending Problem

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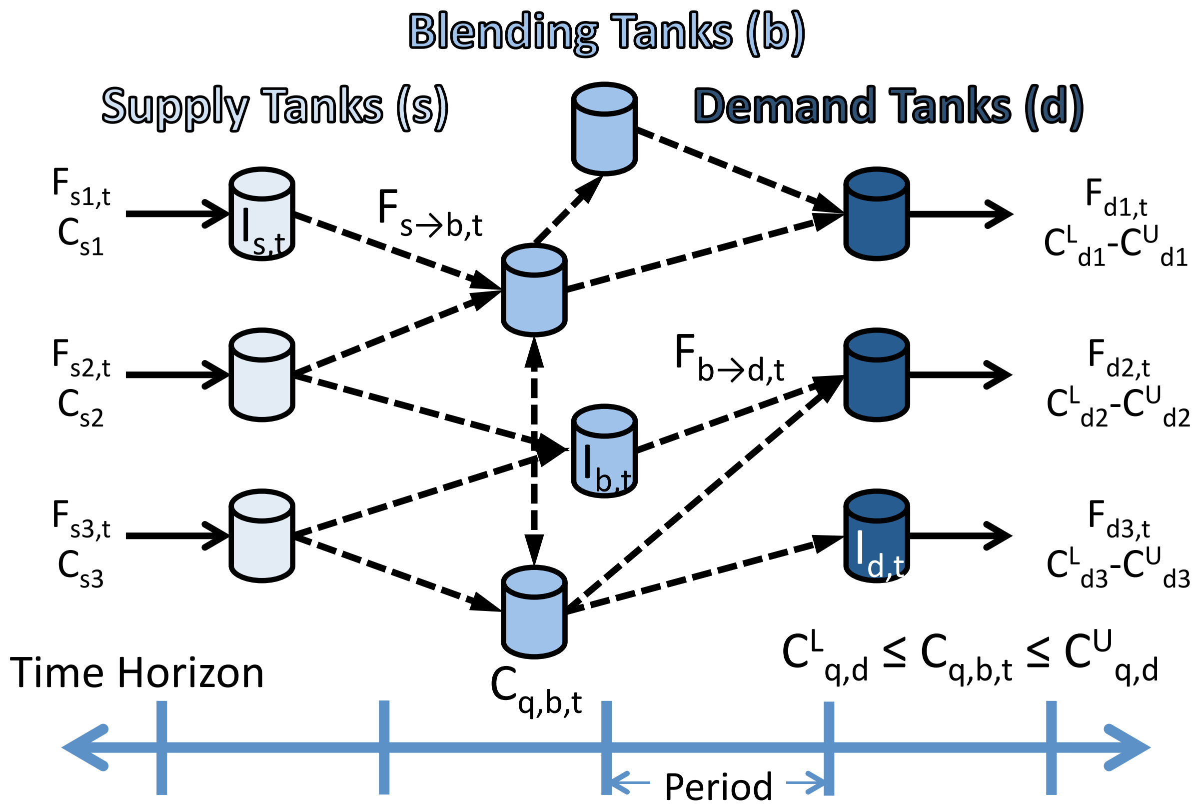
Problem Definition

Given Information

- **Time Horizon (Periods)**
- **Network Topology**
- **Supply Tanks**
  - Amount entering
  - Concentrations
- **Demand Tanks**
  - Amount withdrawn
  - Concentration limits
- **Initial Conditions**
  - Inventories
  - Concentrations
- **Economic Costs**
  - Network flow costs
  - Raw material costs
  - Profit for meeting demand



Determine

- Flows between which tanks in which time periods
- Inventories and concentrations for all tanks in each time period
- **Minimize total cost** of blending operation

Problem Definition

Assumptions

- Supply concentrations are **constant**
- No **simultaneous input/output** to **blending tanks**
  - Avoids dynamic concentration changes
- **Perfect mixing**

Time Periods

- Generally **hours** to **days**
- Time periods are **coupled** by inventories
  - Requires **simultaneous optimization** over all periods
  - **Example:** Storage for excess demand in a later time period

Mathematical Model

Objective: Minimize cost

- Mass Balances
  - Overall Flows
  - Individual Components (Blending)
- Flow/Inventory Bounds
- Operational Constraints
- Demand Specifications

} Process Constraints

Variables

- Flows, Concentrations, and Inventories **(Continuous)**
- Existence/Nonexistence of Streams **(Binary)**

Complicating **nonconvex bilinearities F·C and I·C** appear in the individual component mass balances

Requires **global optimization** techniques

Resulting model is an **MINLP**

GloMIQO Results

GloMIQO							
Tanks	6	8	8				
Time Periods	3	3	3				
Wall Time (s)	1771	>7200	>7200	>7200	>7200	>7200	0 (Fail)
LB	13.359	47.246	7.179	13.830	54.147	9.226	22.718
UB	13.372	48.171	12.653	13.974	54.285	9.262	22.741

GloMIQO still cannot close the gap in less than 2 hours

GloMIQO closes the gap faster than BARON

… but need a new approach!

Radix-Based Discretization

Discretize concentration C using a **disjunction** for levels of precision *p* to *P*

$$\text{Let } C = \sum_{k=p}^{P} \lambda_k$$

$$\bigvee_{j=0}^{9} \left[\lambda_k = 10^k \cdot j\right] \quad \forall\, k \in \{p, \ldots, P\}$$

Convex hull for this disjunction

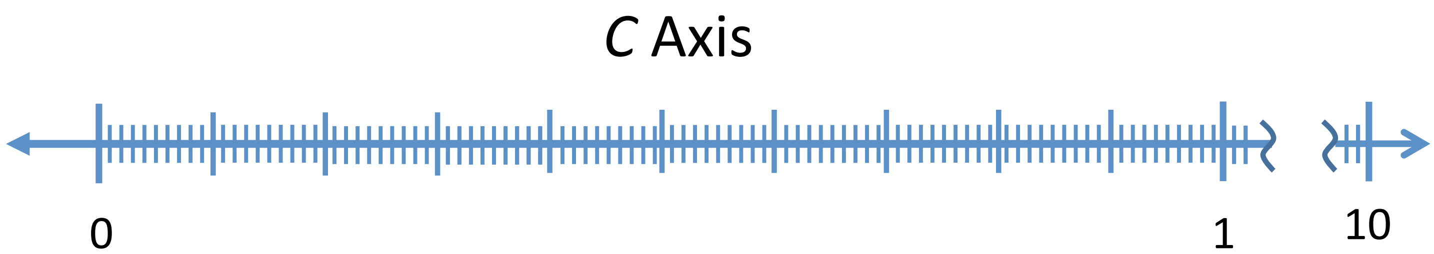
$$\lambda_k = \sum_{j=0}^{9} \lambda_{j,k} \quad \forall\, k \in \{p, \ldots, P\}$$

$$\lambda_{j,k} = 10^k \cdot j \cdot z_{j,k} \quad \forall\, k \in \{p, \ldots, P\}, j \in \{0, \ldots, 9\}$$

$$\sum_{j=0}^{9} z_{j,k} = 1 \quad \forall\, k \in \{p, \ldots, P\}$$

$$z_{j,k} \in \{0, 1\} \quad \forall\, k \in \{p, \ldots, P\}, j \in \{0, \ldots, 9\}$$

Radix-Based Discretization



p (Smallest Power)	0	-1	-2
P (Largest Power)	0	0	0
Range	0-9	0-9.9	0-9.99
Increment	1	0.1	0.01
Significant Digits	1	2	3
Binary Variables	10	20	30

Linear increase in binary variables for **order of magnitude** increase in precision

The **range of the variable** is **discretized** over a region determined by p and P. This region must encapsulate the **full range** of the variable being discretized.

Radix-Based Discretization

Given that $C = \sum_{k=p}^{P} \sum_{j=0}^{9} 10^k \cdot j \cdot z_{j,k}$ the bilinear product $u = F \cdot C$ can be replaced by this set of constraints using exact linearization:

$$u = \sum_{k=p}^{P} \sum_{j=0}^{9} 10^k \cdot j \cdot \hat{F}_{j,k}$$

$$\hat{F}_{j,k} \leq F^U \cdot z_{j,k} \qquad \forall\ k \in \{p, ..., P\}, j \in \{0, ..., 9\}$$

$$\sum_{j=0}^{9} \hat{F}_{j,k} = F \qquad \forall\ k \in \{p, ..., P\}$$

$$\sum_{j=0}^{9} z_{j,k} = 1 \qquad \forall\ k \in \{p, ..., P\}$$

$$z_{j,k} \in \{0, 1\} \qquad \forall\ k \in \{p, ..., P\}, j \in \{0, ..., 9\}$$

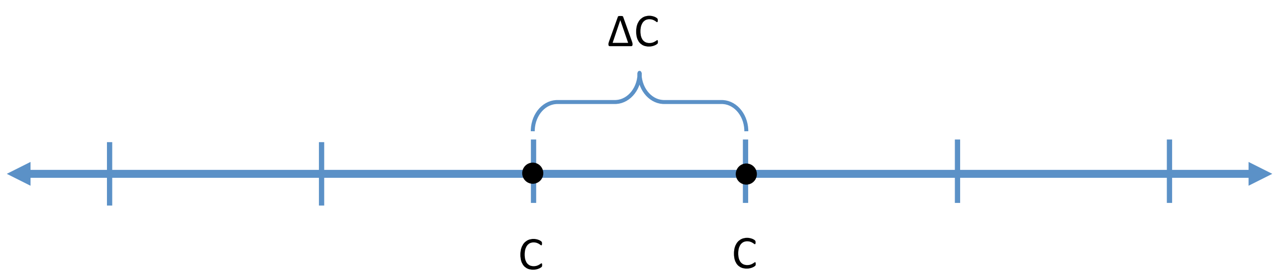
Teles, Castro, & Matos (2011)

Discretization **replaces bilinearity** with **mixed-integer linear constraints**

The **MINLP** model can be reformulated as an **MILP approximation**

Lower Bounding

- RBD yields an **upper bound**
- What about a **lower bound**?



- We introduce ΔC as a **slack variable**
  - ΔC ranges between 0 and 10^p (the size of the “gaps”)
  - **Relaxed RBD constraints** can be added to “fill the gap” and **relax the original problem**

Kolodziej, Castro and Grossmann (2012)

Algorithm 1

- Solve the MILP discretized problem for upper bound
- Solve the MILP relaxed problem for lower bound
- If gap < ε, stop. Otherwise, let $p = p - 1$ and repeat

Algorithm 2

- Solve the MILP relaxed problem for upper bound
- Fix the process binary variables
- Solve the resulting NLP with an NLP solver for a loewer bound

Algorithm 2

- Computational Results

Tanks	6	8	8	8	8	8	8
Time Periods	3	3	3	3	4	4	4

Remarks

- **Using a base 2 discretization yields the smallest problem sizes.**

  Performance is slightly better in most cases.

- **Discretizing over flows instead of concentrations slower performance.**

  Trend might be reversed for larger number of properties