# *COLUMN GENERATION HEURISTICS FOR SPLIT PICKUP AND DELIVERY VEHICLE ROUTING PROBLEM FOR INTERNATIONAL CRUDE OIL TRANSPORTATION*

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March 12, 2013

# Outline

1. Introduction
2. Ship scheduling problem for international crude oil transportation
3. Problem modeling and formulation
4. Solution approach: Column generation heuristics
5. Computational experiments
6. Conclusion and future works

# Introduction

- Oil is one of the most consumed energy resource in Japan.
- Japan has to import crude oil from other countries.

Table: Value of imports of crude oil in 2011

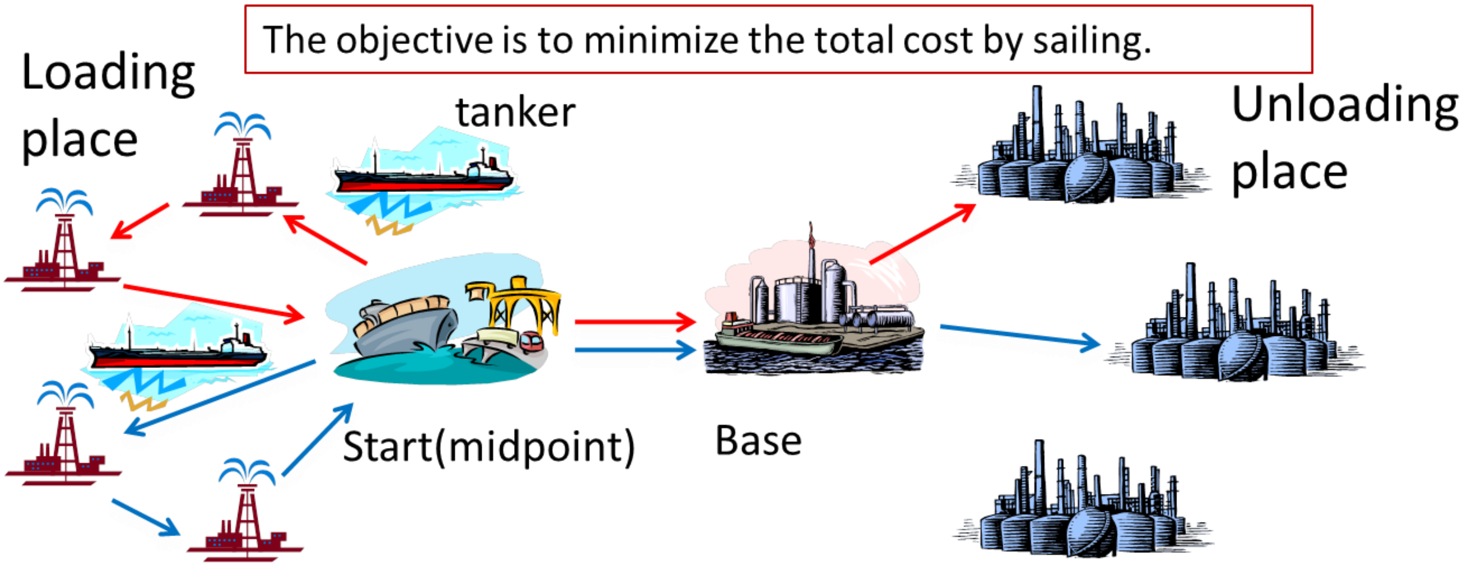
| Rank | Country | The value of imports (US $ billions) |
|---|---|---|
| 1 | United States | 461.53 |
| 2 | China | 235.75 |
| 3 | Japan | 185.01 |
| 4 | India | 137.34 |

(Reference: http://ecodb.net/ranking/imf_tmgo.html)

Pickup and delivery transportation problem is significant for global logistics.

# Introduction

- Pickup and delivery crude oil transportation scheduling problem.



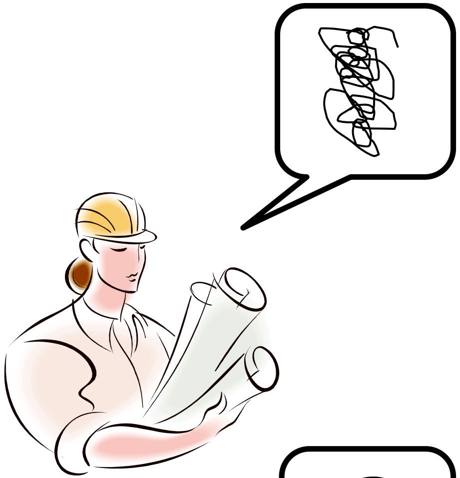

The objective is to minimize the total cost during pickup and delivery.

# Introduction

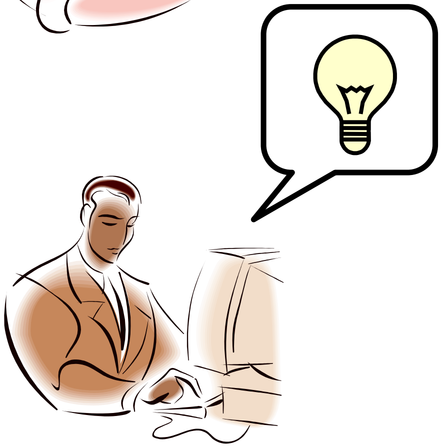
- Before……human's decision

  (calculation by hand or experience)

  → When the scale is too large,
  decision-making becomes difficult.



Our purpose is to decide faster and plan better schedules.



Global logistics problem → The importance will increase

# Previous works on VRP with split deliveries

- Exact algorithms → takes much computing time
- Heuristic algorithm (Saving method, Passen et al, 2011)
  → Optimality cannot be ensured.

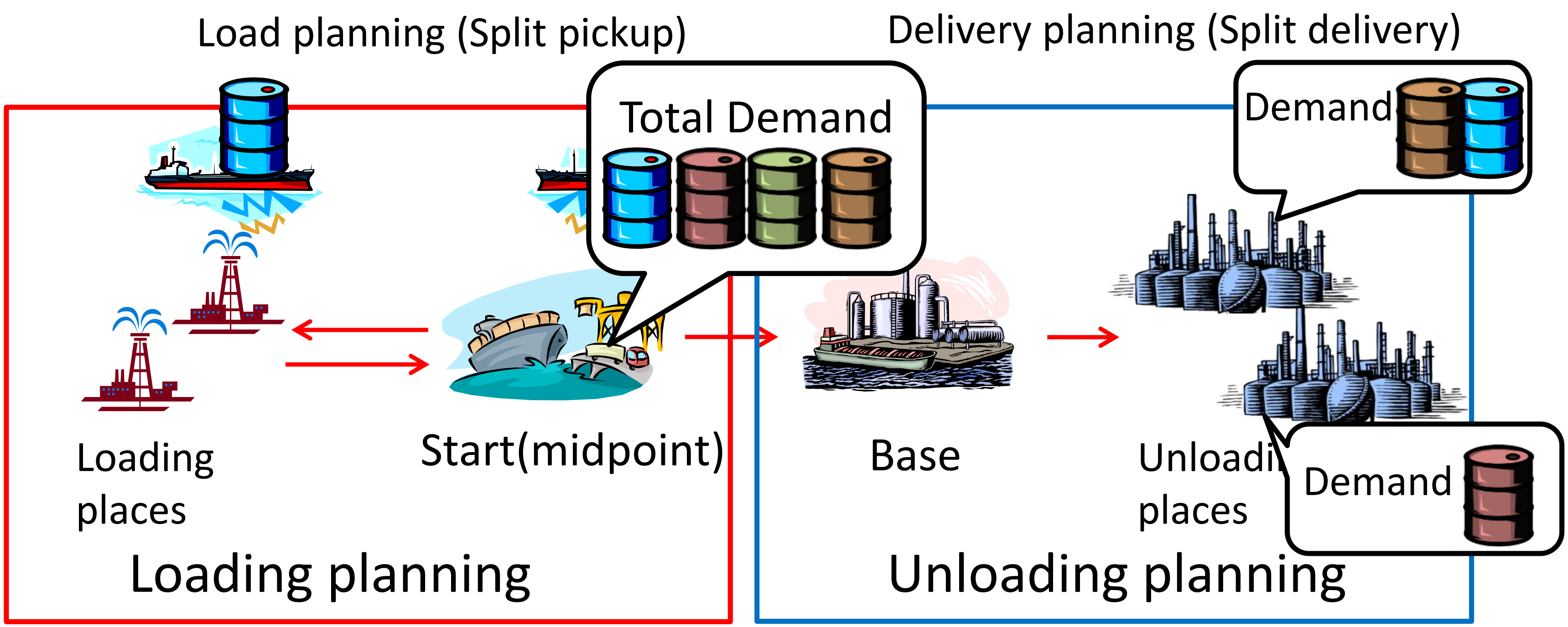
- Column generation
  → optimal for continuous relaxation of Dantzig-Wolfe reformulation
  - Column generation for split delivery VRP (Jin et al, 2007)
  - Branch and price and cut (Brønmo et al, 2010) (Hennig et al, 2012)

  However, practical constraints for crude oil transportation is not considered.

- Objective of this work

  We propose a column generation heuristic for real case study of international crude oil transportation problems.

# Problem description




- Objective function: To minimize
  - the total distances
  - the cost imposed by visiting loading places

# Problem description

- The number of available tankers is fixed. (Cannot increase)
- Capacity of tankers are different for each one.
- The limits of loading volume and unloading volume in each loading place and unloading place are different for each place, respectively.
- The number of visiting time is limited.
- The assignments of loading places and unloading places for each oil are different. (Oil to unloading places is one-to-many)
- Etc.

# Input data and decision variable

Given

Distances between loading places

Demand volume of oils

Limits of loading volume and unloading volume at each place

Capacity of tankers

Decision variable

$x_{i,j,k} \in \{0,1\}$ visiting sequence

$a_i$ loading volume

$\delta_{k,i} \in \{0,1\}$ assignment

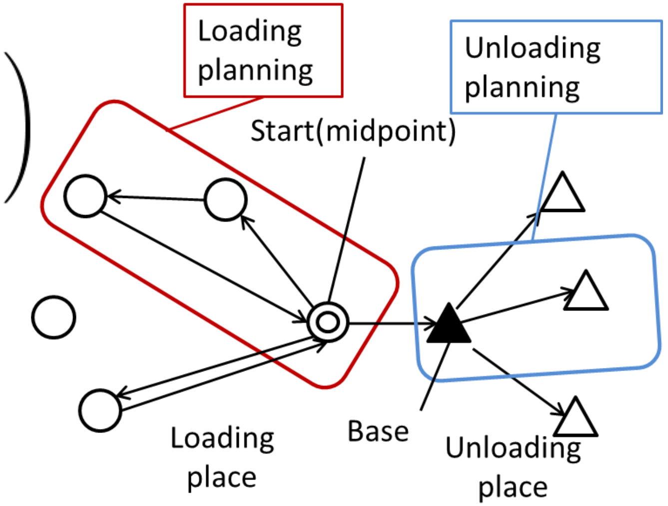
$b_i$ unloading volume

Objective function: to minimize

the total distances and port charge

# Problem formulation

$$\min \ w_1 \left( \sum_{k\in T} \sum_{i\in L} \sum_{j\in L} d_{i,j} v_{k,i,j} \right) + w_2 \left( \sum_{k\in T} \sum_{i\in L} c_i \delta_{k,i} \right)$$




$$\sum_{k\in T} \sum_{i\in L} a_{k,i,o} = D_o$$ Demand constraint

$$\sum_{i\in L\cup\{s\}} x_{k,i,j} = \delta_{k,j}$$

$$\sum_{j\in L\cup\{s\}} x_{k,i,j} = \delta_{k,i}$$ Assignment constraint

$$\sum_{i\in L} \delta_{k,i} \le \Gamma_m$$ Limitation of visiting time

$$t_{k,i} + T_{k,i,j} - t_{k,j} - M(1 - x_{k,i,j}) \le 0$$ Subtour elimination

$$\delta_{k,i} a_i^{\min} \le \sum_{o\in O} \eta_{i,o} a_{k,i,o} \le \delta_{k,i} a_i^{\max}$$ Minimum and maximum Loading volume

$$\sum_{k\in T} \delta_{k,i} = H_i$$ Required number of tanker from demands

$$\sum_{i'\in U} w_{k,i',j'} = \varepsilon_{k,j'}$$

$$\sum_{j'\in U} w_{k,i',j'} = \varepsilon_{k,i'}$$

$$\sum_{i\in L} q_{k,i,o} = \sum_{i'\in U} b_{k,i',o}$$

$$\sum_{k\in T} \sum_{o\in O} b_{k,i',o} \ge D_{i'}^{ul}$$

The problem is known to be NP-complete

# Dantzig-Wolfe reformulation

$\alpha_k^p$ takes a value of 1 if plan p is adopted for tanker k

$$\min\left(w_1 \sum_{k\in T}\sum_{p}\sum_{i}\sum_{j} d_{i,j} x_{i,j}^p \alpha_k^p + w_2 \sum_{k\in T}\sum_{p}\sum_{i} c_i \delta_{k,i}^p \alpha_k^p\right)$$

$$\sum_{k\in T}\sum_{p}\sum_{i} a_{k,i,o}^p \alpha_k^p = D_o$$ → Set partitioning constraints

$$\sum_{p} \alpha_k^p \le 1$$

$$\alpha_k^p \in \{0,1\}$$ → $$0 \le \alpha_k^p \le 1$$

# Column generation and Lagrangian relaxation

Continuous Relaxation of DW reformulation

$$\min \left(w_1 \sum_{k\in T}\sum_p\sum_i\sum_j d_{i,j} x_{i,j}^p \alpha_k^p + w_2 \sum_{k\in T}\sum_p\sum_i c_i \delta_{k,i}^p \alpha_k^p\right)$$

$$\sum_{k\in T}\sum_p\sum_i q_{k,i,o}^p \alpha_k^p = D_o$$

$$\sum_p \alpha_k^p \le 1 \qquad 0 \le \alpha_k^p \le 1$$

Lagrangian Dual of Original Problem

$$\max \sum_{k\in T}\eta_k - \sum_o \lambda_o D_o$$

$$\eta_k \le w_1 \sum_{k\in T}\sum_p\sum_i\sum_j d_{i,j} x_{i,j}^p \alpha_k^p + w_2 \sum_{k\in T}\sum_p\sum_i c_i \delta_{k,i}^p \alpha_k^p + \sum_i\sum_o \lambda_o q_{k,i,o}$$

$$\forall k \in K$$

Dual

**Simplex algorithm**

Lower bound

Column generation heuristics

Upper bound

**Subgradient algorithm**

Lower bound

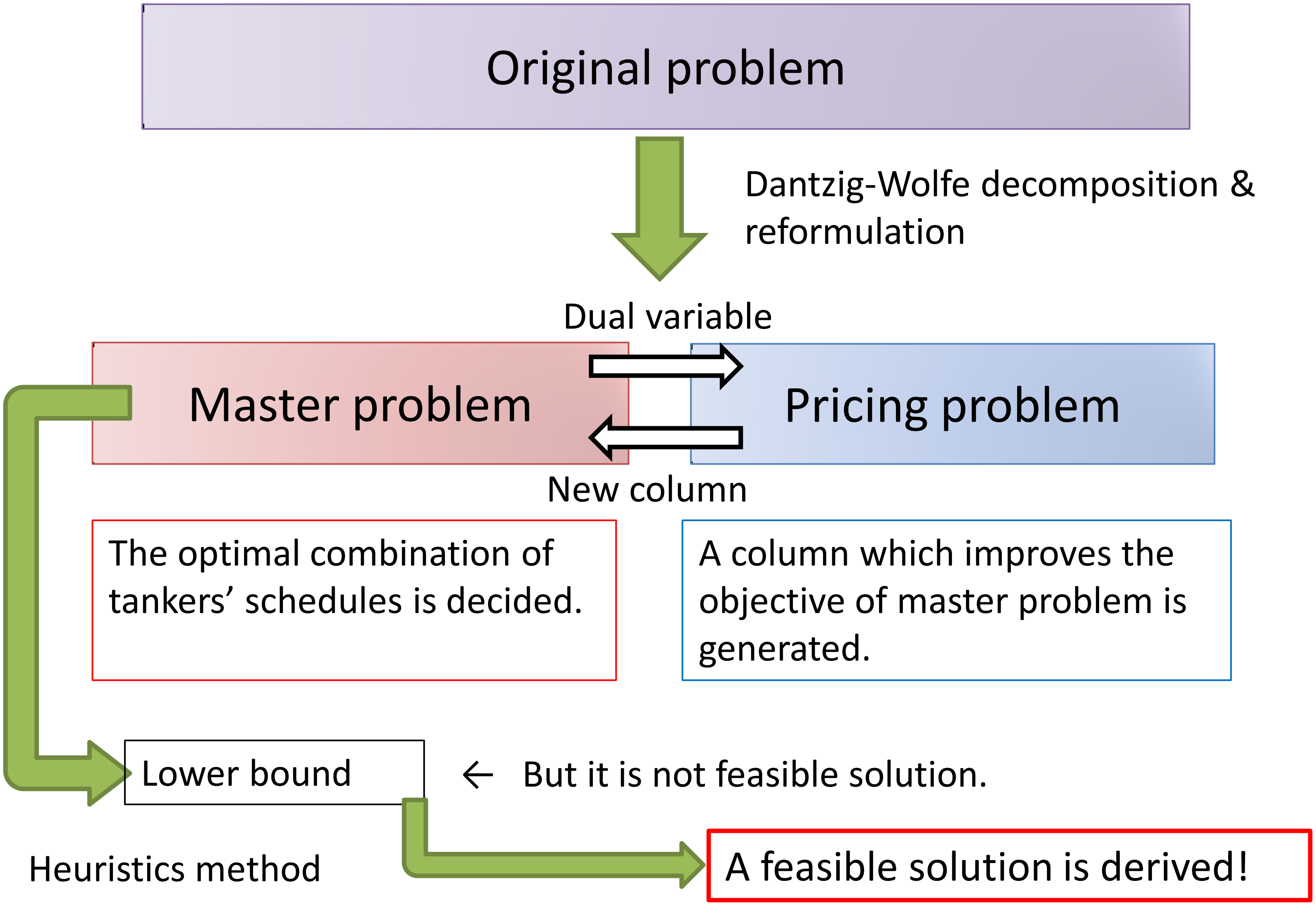
Which is better bound ?

Lagrangian relaxation heuristics

Upper bound

Lower bounds are theoretically equal.
but simplex algorithm Is better than subgradient method

# Column generation approach



# Restricted master problem

Total distance

Total port charge

$$(LRSP)\quad \min\ w_1 \sum_{k\in T}\sum_{p\in \bar{P}_k}\sum_{i\in L}\sum_{j\in L} d_{i,j} v^p_{k,i,j} \alpha^p_k + w_2 \sum_{k\in T}\sum_{p\in \bar{P}_k}\sum_{i\in L} c_i \delta^p_{k,i} \alpha^p_k$$

Weighted coefficient

Constraints

s.t.

- Total demand constraints
- One routing plan can be adopted for each tanker
- Relaxation of binary constraints on $\alpha^p_k$

⇓

Linear Problem(LP)

**Decision variable**

$\alpha^p_k$ Whether plan p for k is adopted or not.

**Coefficients dependent routing plans**

$v^p_{k,i,j}$ Whether tanker k of plan p visits from i to j or not.

$\delta^p_{k,i}$ Whether tanker k of plan p visits i or not

$a^p_{k,t,o}$ Loading volume

# Pricing problem

The objective is to minimize the reduced cost.
Plan p of tanker k is considered.

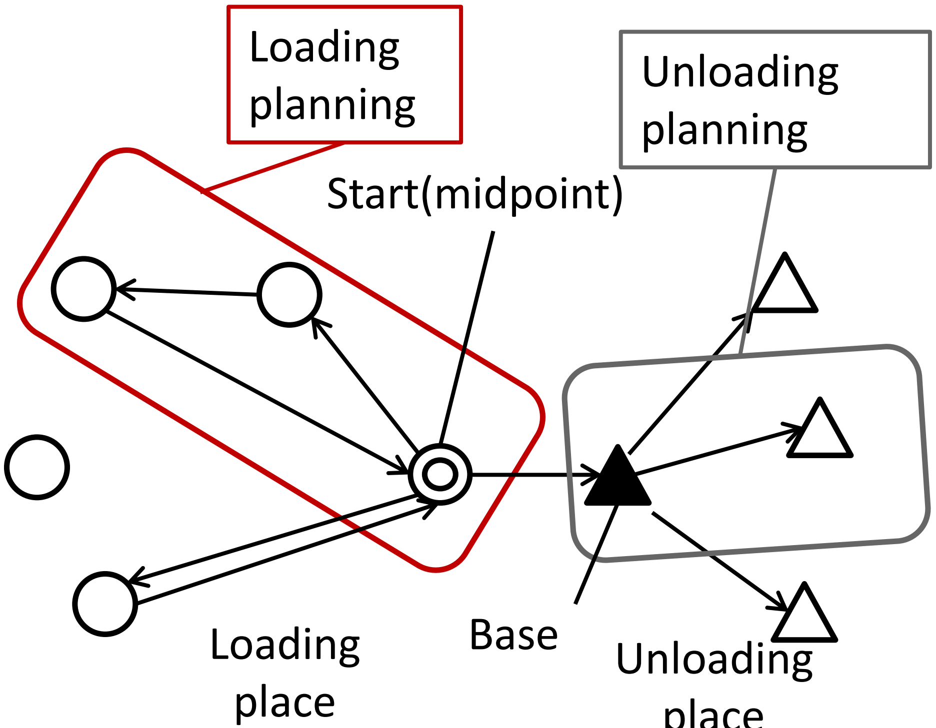
Dual variable of *RMP*

$$(SP_k) \ \min w_1 \sum_{i \in L} \sum_{j \in L} d_{i,j} v^p_{k,i,j} + w_2 \sum_{i \in L} c_i \delta^p_{k,i} - \sum_{t=1}^{2} \sum_{o \in O} \pi^*_o a^p_{k,t,o} - \lambda^*_k$$

There are constraints all for each single tanker.

$$v^p_{k,i,j} \ \delta^p_{k,i} \quad a^p_{k,t,o} \quad w^p_{k,i',j'} \ \epsilon^p_{k,i'} \quad b^p_{k,i',o}$$

To determine a sequence, loading volume, and unloading volume.



# Construction of an initial solution

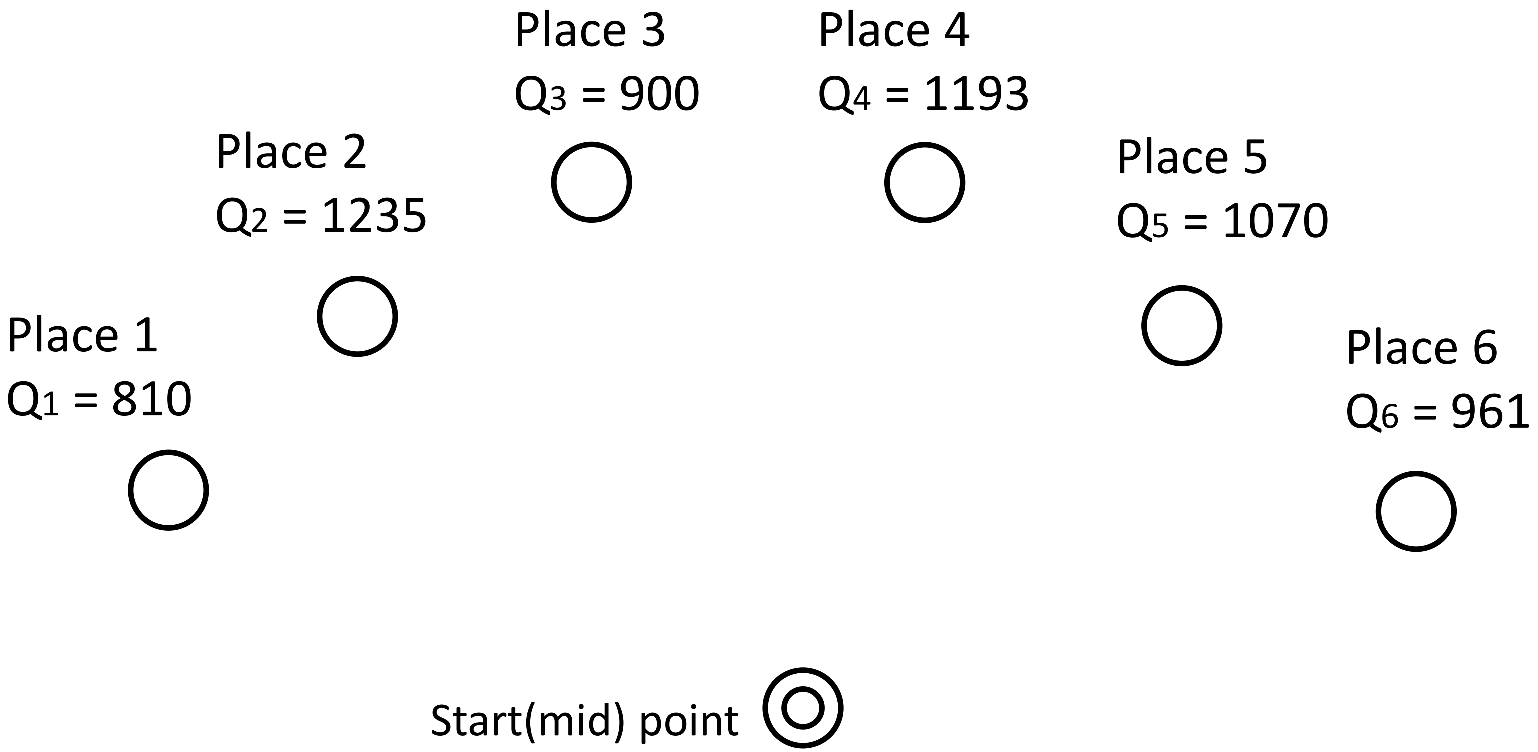
- When using column generation, an initial feasible solution is needed.
- In this problem, real cases are supposed. So we cannot increase the number of available tankers.

Challenge

- To derive an initial solution in fixed number of tankers is difficult because of set partitioning constraints (demand constraints) and a tanker cannot visit more than two loading places.

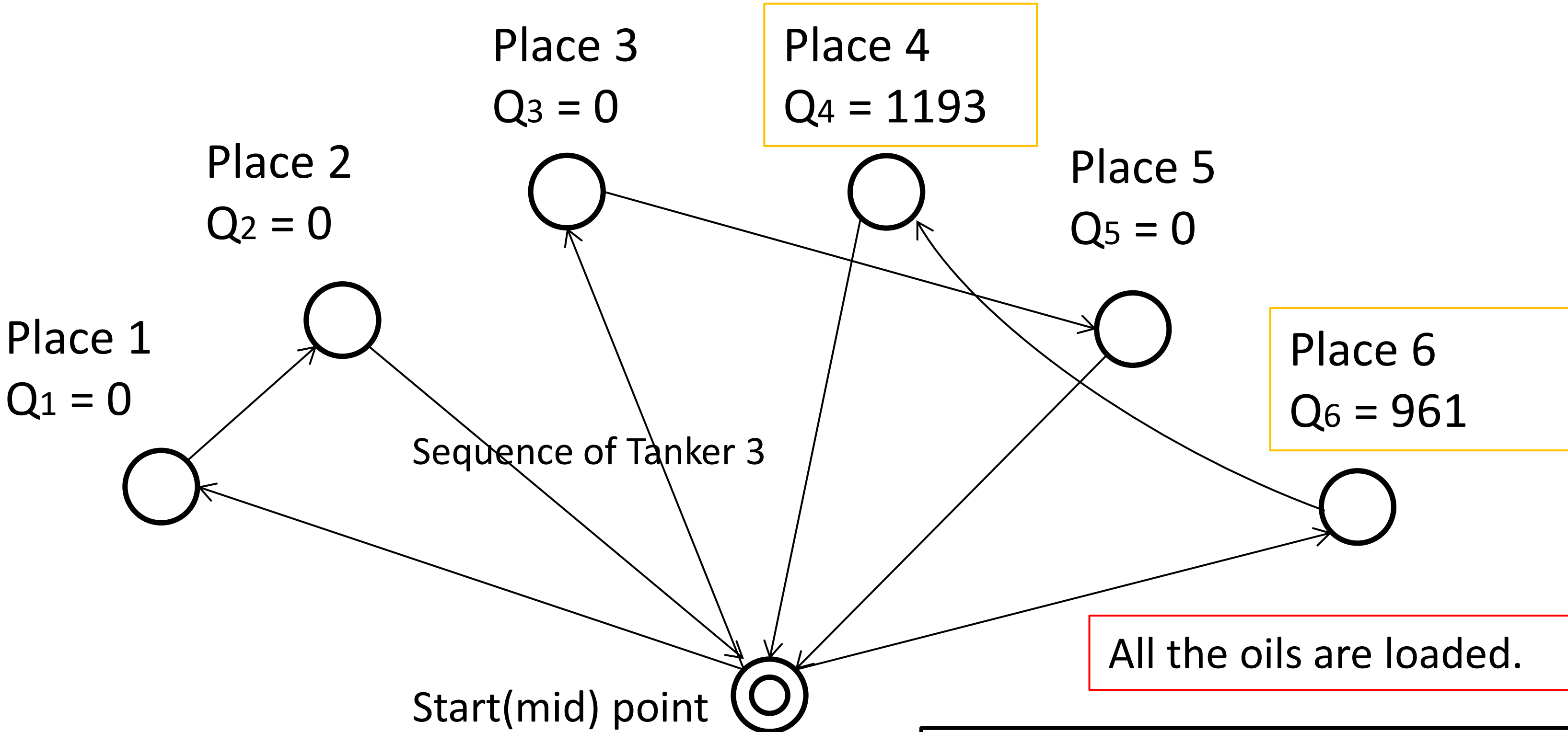
We propose a method constructing an initial solution by using only existing tankers.

# Construction of an initial solution




| Tanker | Capacity | Total loading volume |
|---|---|---|
| 1 | 2160 | |
| 2 | 2017 | |
| 3 | 2171 | |

# Construction of an initial solution




All the oils are loaded.

| Tanker | Capacity | Total loading volume |
|---|---|---|
| 1 | 2160 | 2045 |
| 2 | 2017 | 1970 |
| 3 | 2171 | 2154 |

An added constraint:
Each tanker has to leave at least a threshold units of oil or no oil in each place.
This threshold is determined by gradually increasing the value.

# Construction of a feasible solution

- In the master problem, the optimal combination of plans is decided.

The decision variable $\alpha_k^p$ takes

$$\begin{cases} 1 & \text{if plan p is adopted.} \\ 0 & \text{if plan p is not adopted.} \end{cases}$$

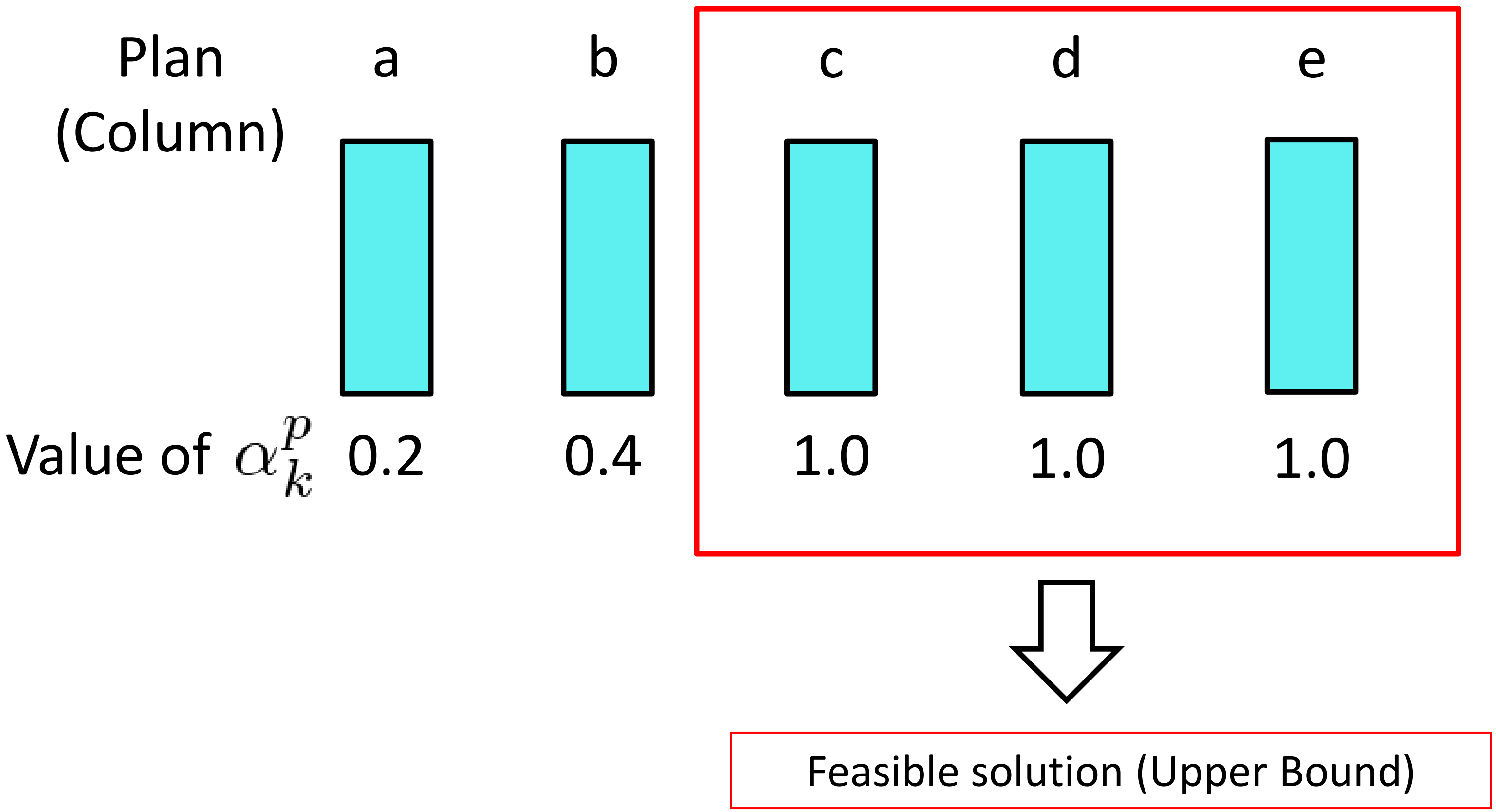
- However, the master problem has LP relaxation, so $\alpha_k^p$ takes fractional value.

⇨ We cannot decide whether the plan is adopted or not.

We propose an algorithm to derive a feasible solution from fractional solution in the master problem.

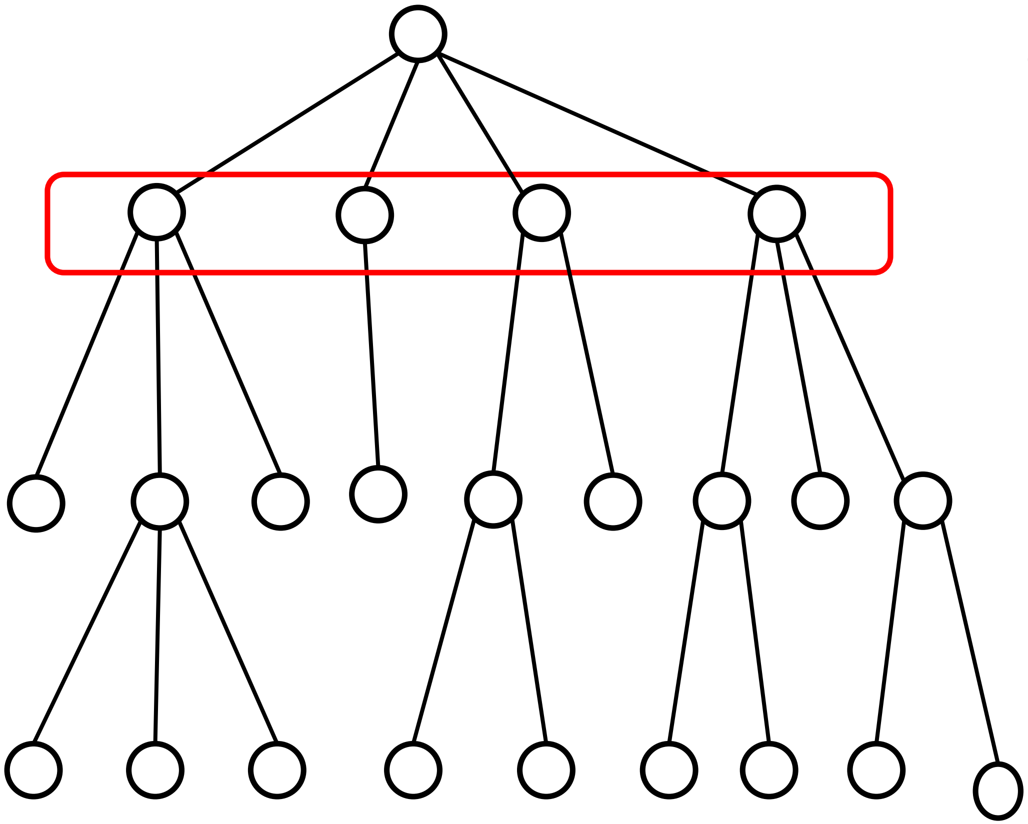
# Construction of a feasible solution

The linear relaxed master problem



# Column Generation Heuristic

Basic Idea: specify threshold value to eliminate non-promising nodes



(i) Solve an initial problem

(ii) Delete all the columns not fixed
And execute column generation again

(iii) If the solution is infeasible and the lower bound is larger than upper bound
Backtrack and solve another problem

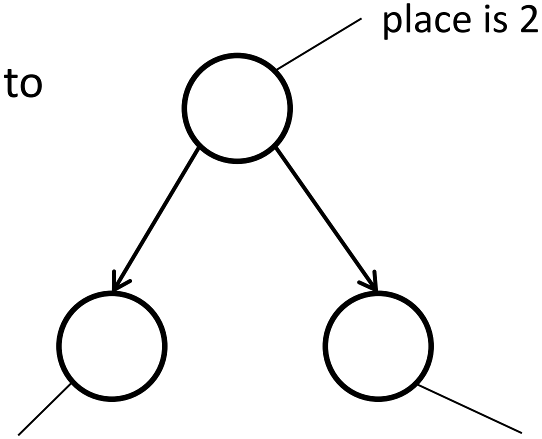
(iv) Output the solution which has the best objective in all of combinations

# Branch and price

- Branching operation
    1. Number of tankers visiting at a loading place
    2. Number of tankers visiting two loading places sequentially
    3. A tanker visits a loading place or not
    4. A tanker visits two loading places sequentially or not

number of visiting time for tanker k to loading place i is calculated

$$\sum_{k \in T} \sum_{p \in \bar{P}_k} \delta_{k,i}^{p} \alpha_k^p$$



If the number of visiting to a loading place is 2.5

Constraint is added such that the number of visiting is less than 2

Constraint is added such that the number of visiting is greater than 3

# Reduction of computational effort

- Lower bound of column generation takes a lot of computing time. We utilize continuous relaxation of the original problem for bounding procedure before column generation. It can reduce computing time.

- If the solution of restricted master problem is 1.0, the solution after the fixing of the variable is the same. It can eliminate column generation procedure.

# Computational experiments

- Case study with practical data

  Loading planning (SDVRP)

  - Small scale: 4 tankers, 22 loading places, 26oils, 8 unloading places
  - Medium scale: 13 tankers, 22 loading places, 26 oils, 8 unloading places

  Loading and unloading planning (SPDVRP)

  - Large scale: 18 tankers, 22 loading places, 26 oils, 8 unloading places

- Computational environment
  - Intel Core(TM)2 Duo 3.06GHz with 2GB memory is used for the computations.
  - *RMP* and $SP_k$ are solved by IBM ILOG CPLEX12.1.

# Computational results (small scale instance)

## 4 Tankers instances

| | Branch and bound | Column generation ＋heuristics | Branch and price |
|---|---|---|---|
| Number of tankers | 4 | 4 | 4 |
| Upper bound | 557045 | 557045 | 557045 |
| Lower bound | 557045 | 556866.539 | 557045 |
| DGap(%)※1 | 0 | 0.032 | 0 |
| Computation time[s] | 1.55 | 40.92 | 528.66 |

- ※1 : DGap = 100 × (UB - LB) ／ LB

# Computational results(medium scale instance)

## 13 Tankers instances

| Method | UB | LB | DGAP | columns | time[s] |
|---|---|---|---|---|---|
| Initial | 3784780 | - | - | - | 0.06 |
| CG-BB | 3784780 | 3144164 | 20.37 | 130 | 23.38 |
| PM(0.5) | 3168860 | 3144164 | 0.79 | 27526 | 3600* |
| PM(0.6) | 3169900 | 3144164 | 0.82 | 27142 | 3600* |
| PM(0.7) | **3164940** | 3144164 | 0.66 | 20209 | 2666.42 |
| PM(0.8) | 3230620 | 3144164 | 2.75 | 3624 | 555.34 |
| PM(0.9) | 3367260 | 3144164 | 7.10 | 635 | 120.86 |
| BB | 3165260 | 2907393 | 8.87 | - | 3600* |
| Operator | 3773060 | - | - | | |

( ): Threshold value parameter for selecting nodes

20% cost reduction by the proposed method

Table 2.4: Result of ship routing and schedule generated by PM(0.7)

| | start | 1st loading place | 2nd loading place | capacity | loading volume | volume rate (%) | loading oils |
|---|---|---|---|---|---|---|---|
| tanker 1 | 0 | 2 | 16 | 2201 | 2201 | 100 | OIL 12: 1940.0<br>OIL 1: 261.0 |
| tanker 2 | 0 | 4 | 16 | 2175 | 2175 | 100 | OIL 18: 500.0<br>OIL 1: 1675.0 |
| tanker 3 | 0 | 19 | 20 | 2173 | 2173 | 100 | OIL 17: 810.0<br>OIL 10: 1363.0 |
| tanker 4 | 0 | 3 | 16 | 2172 | 2172 | 100 | OIL 22: 600.0<br>OIL 1: 1019.0 OIL 2: 553.0 |
| tanker 5 | 0 | 6 | 15 | 2172 | 2172 | 100 | OIL 8: 1408.0 OIL 9: 264.0<br>OIL 21: 500.0 |
| tanker 6 | 0 | 16 | - | 2171 | 2171 | 100 | OIL 2: 2171.0 |
| tanker 7 | 0 | 13 | - | 2171 | 2171 | 100 | OIL 14: 621.0 OIL 16: 1550.0 |
| tanker 8 | 0 | 16 | - | 2160 | 2160 | 100 | OIL 1: 765.0 OIL 3: 930.0 OIL 4: 465.0 |
| tanker 9 | 0 | 2 | - | 2160 | 2160 | 100 | OIL 12: 2160.0 |
| tanker 10 | 0 | 13 | 16 | 2113 | 2049 | 97.0 | OIL 14: 154.0 OIL 15: 310.0<br>OIL 2: 1585.0 |
| tanker 11 | 0 | 10 | 9 | 2111 | 2095 | 99.2 | OIL 6: 1095.0<br>OIL 19: 1000.0 |
| tanker 12 | 0 | 7 | - | 2031 | 1922 | 94.6 | OIL 13: 1922.0 |
| tanker 13 | 0 | 6 | 20 | 2017 | 1926 | 95.5 | OIL 8: 797.0<br>OIL 10: 1129.0 |

# Practical constraints

- In order to create a plan with full capacity, the priority of column generation heuristic is set to $\alpha_k^p \times$ (Loading volume) in the plan.
- Some specific oils should be delivered in a specified ratio. We included the constraints in the pricing problem.
- The demanded items of oils are given as priority from the database given from expert operator.

# Computational results (large scale instance)

18 Tankers instance

| Method | UB | LB | DGAP | Number of columns | time[s] |
|---|---|---|---|---|---|
| Initial | 57109284 | - | - | - | 0.11 |
| CG-BB | 57109284 | 26682138 | 114.04 | 108 | 28.11 |
| PM(0.5) | 32819319 | 26682138 | 23.00 | 19761 | 3600* |
| PM(0.6) | 31739540 | 26682138 | 18.95 | 19738 | 3600* |
| PM(0.7) | 32528065 | 26682138 | 21.91 | 19473 | 3600* |
| PM(0.8) | 32764574 | 26682138 | 22.80 | 19726 | 3600* |
| PM(0.9) | 55007664 | 26682138 | 106.15 | 108 | 28.61 |
| BB | - | 15966800 | - | - | 3600* |

B&B method cannot derive a feasible solution

(   ): Threshold value parameter for selecting nodes

# Conclusion and future works

- Conclusion
  - A column generation approach has been proposed to solve the split pickup and delivery vehicle routing problem for crude oil transportation.
  - We proposed a practical algorithm to generate a feasible solution with column generation.
  - The case study has demonstrated that the effectiveness of the proposed method compared with human operator's result.

- Future works
  - We should consider more detailed constraints in unloading planning.
  - We will try to apply branch-and-price algorithm for this problem.
  - Integration of production planning and ship scheduling will be required.