Planning and scheduling of PPG glass production, model and implementation.

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Project description

Objective:

- Development of a Mixed Integer Linear Programming (MILP) model for the **planning and scheduling of the glass production**
    - **Capture the essence** of the process that **is not considered** in the Master Production Schedule
        - Management of waste glass (cullet)

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- The model was extended to consider cullet management with 2 continuous production lines
- Implementation of the model into a user-friendly application based on Excel and GAMS.
- Collaboration with the supply chain group to help on strategic business decisions.
- Efficient solution of the model by improving the linear relaxation.

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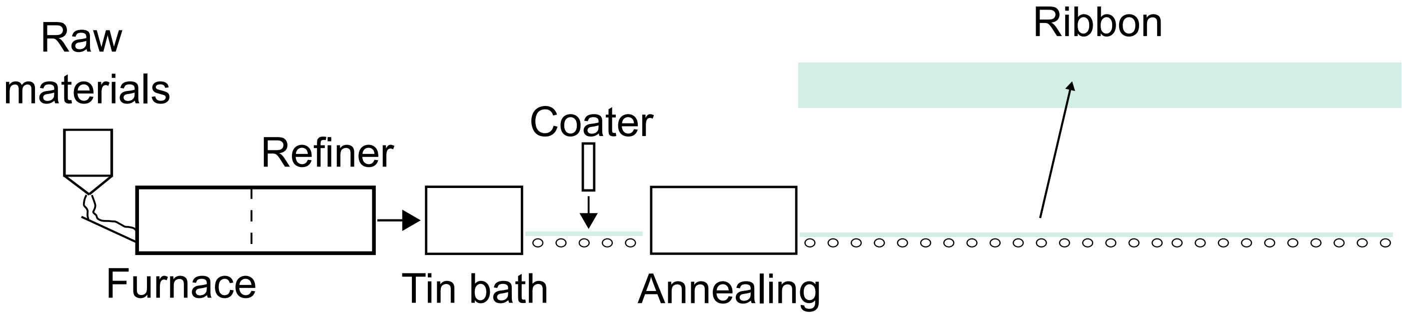
Research challenge

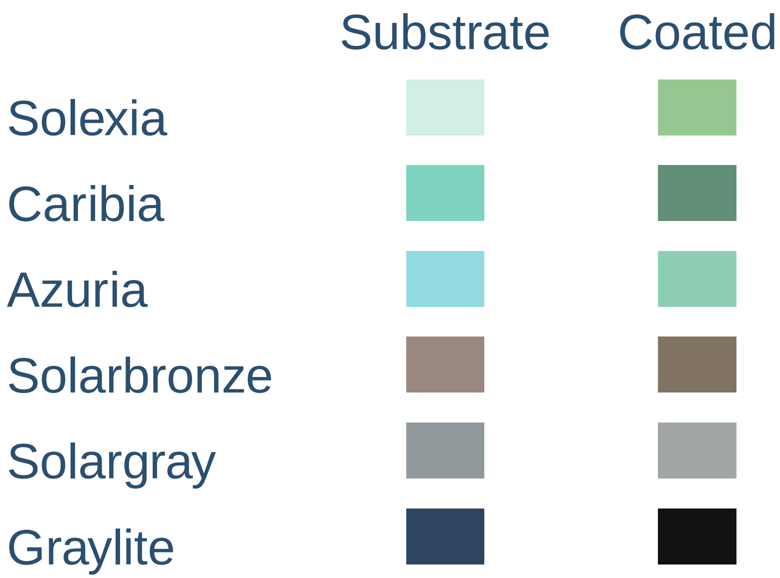
- Develop an efficient solution strategy for a planning and scheduling model considering waste glass management
  - Synchronization of the production and consumption of waste glass with 2 production lines using a slot based model.

Process and products

Continuous process:



Products defined by color



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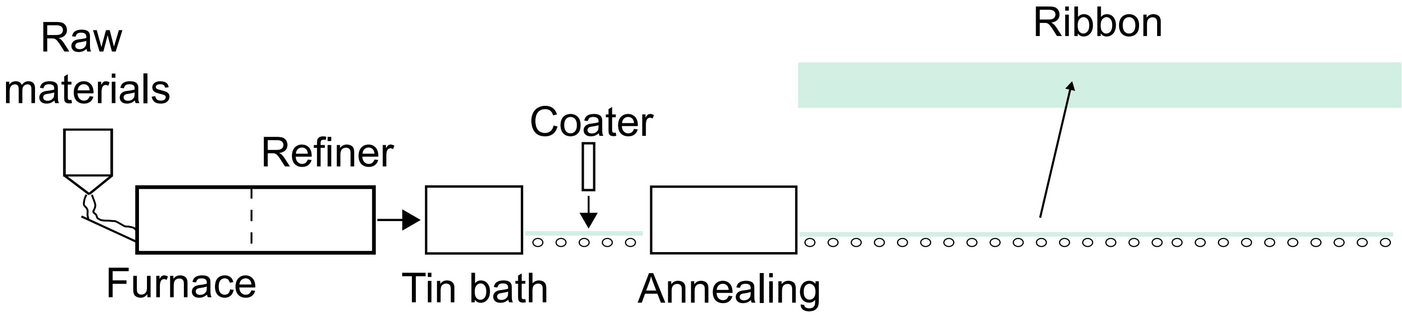

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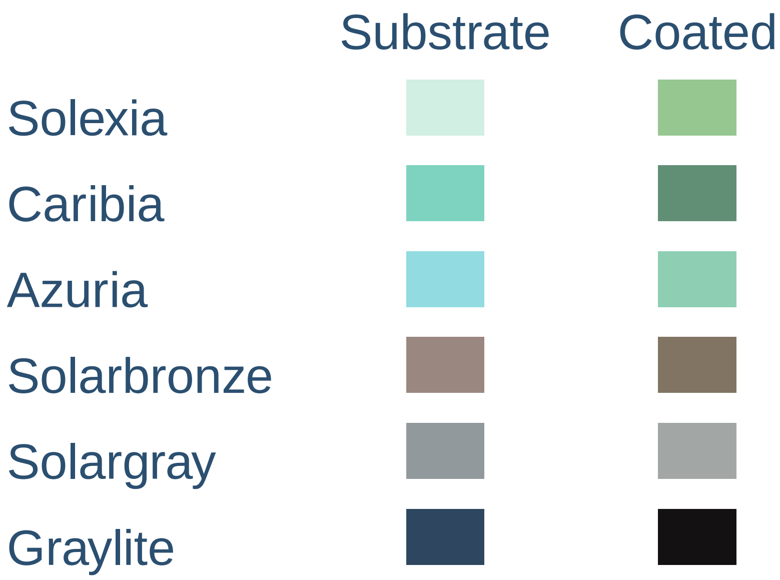

Features

- Raw materials define color of the substrate
- Sequence dependent changeovers between substrates
- No transition times between substrate and coated products

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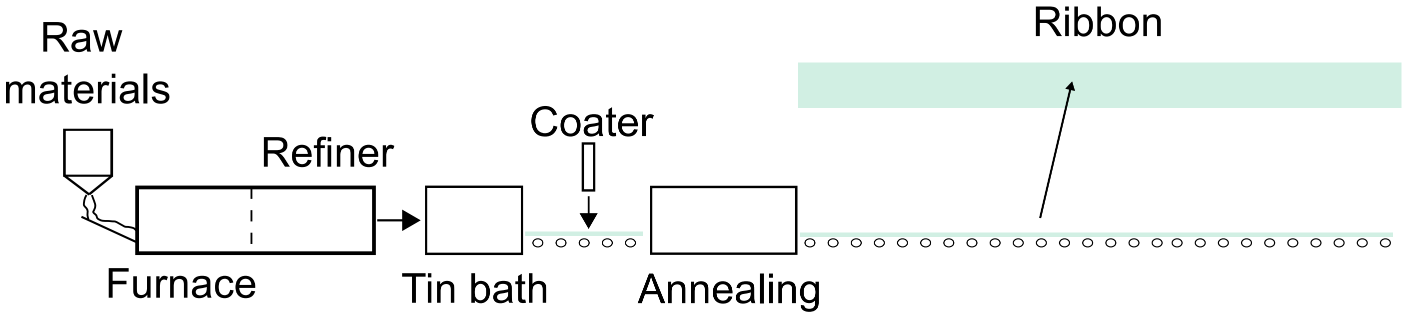

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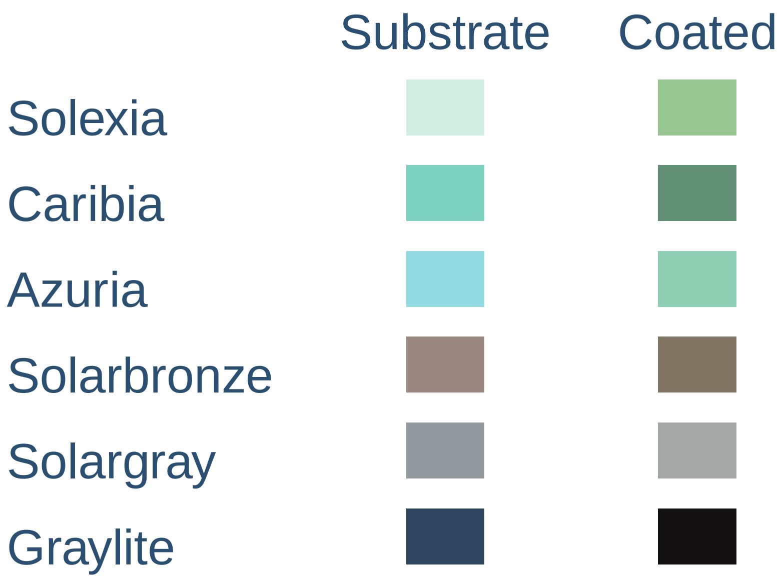

Features

- Raw materials define color of the substrate
- Sequence dependent changeovers between substrates
- No transition times between substrate and coated products
- Long transition times (order of days)
- High transition costs
- Continuous operation during changeover
- Minimum run length (days)

Cullet management

Cullet: waste glass

Good cullet: generated during the production run

Dilution cullet: produced during changeover from one substrate to other substrate

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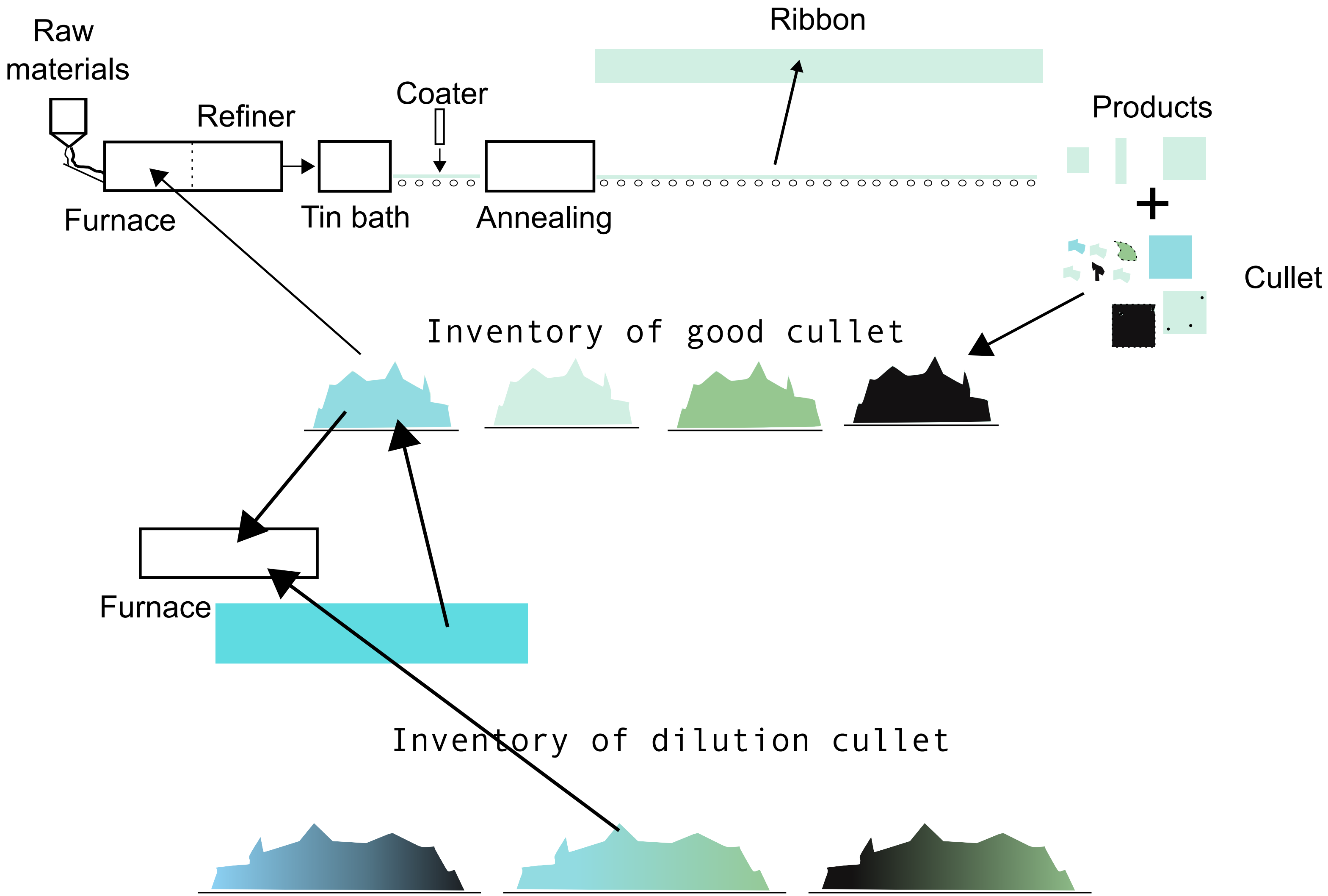

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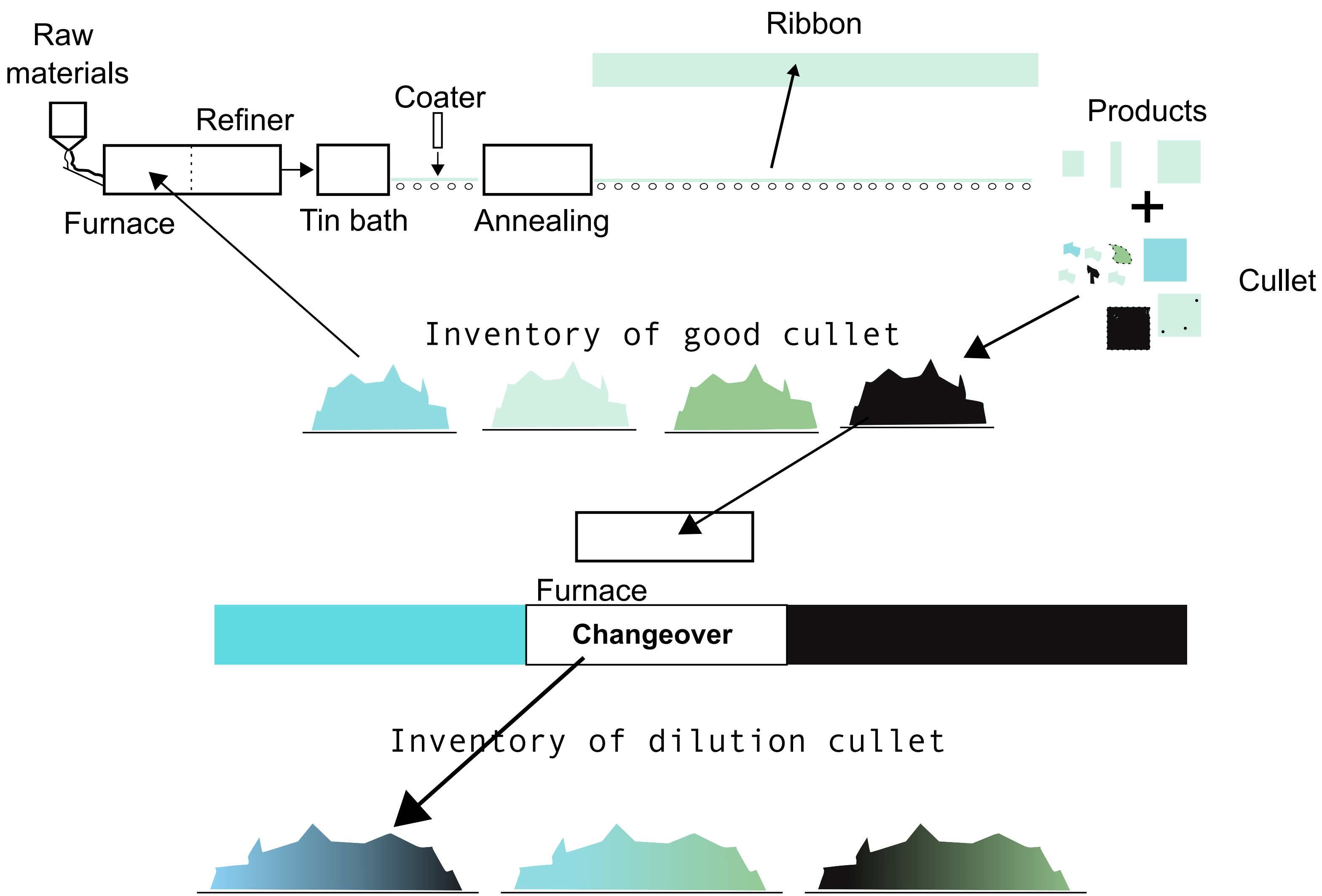

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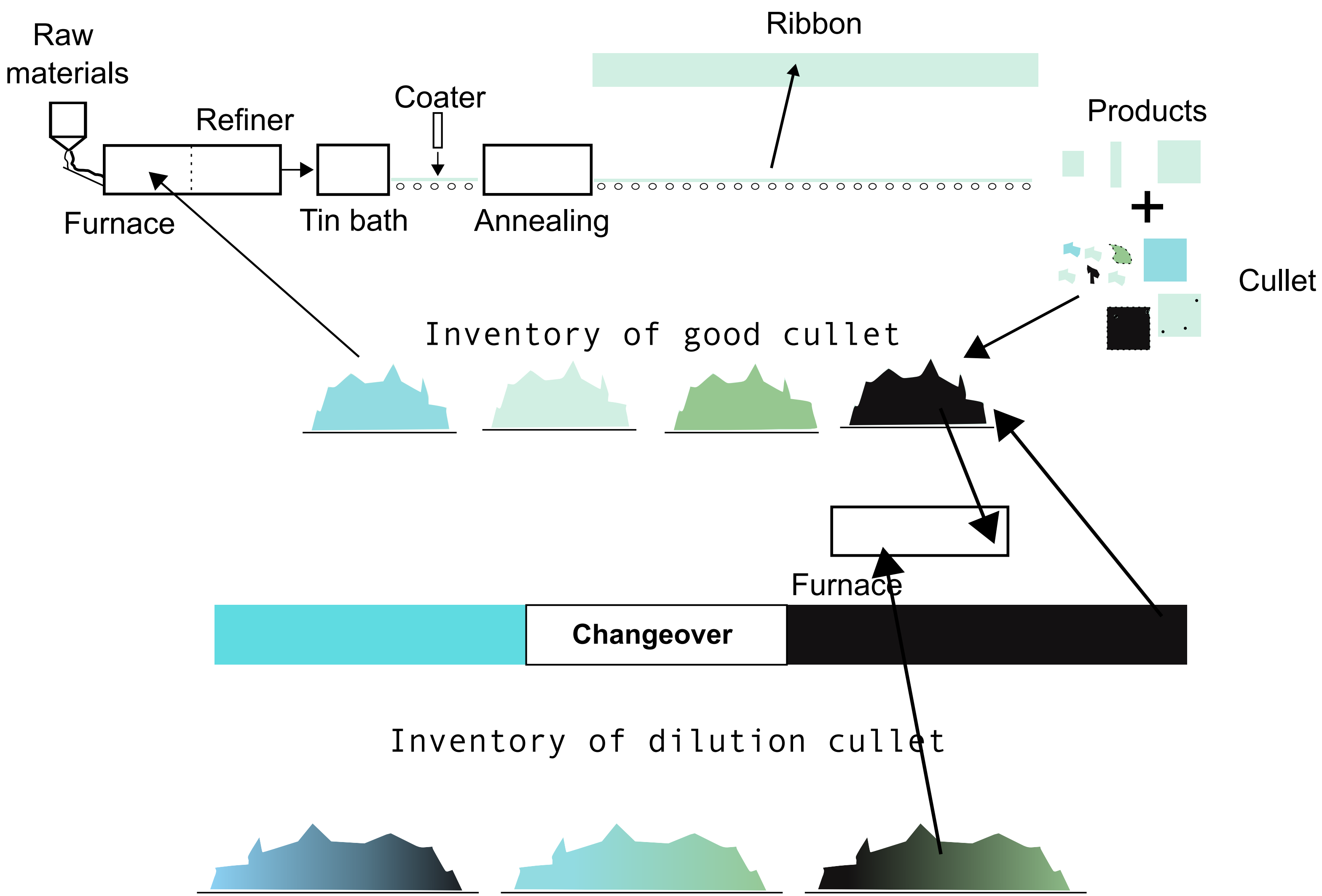

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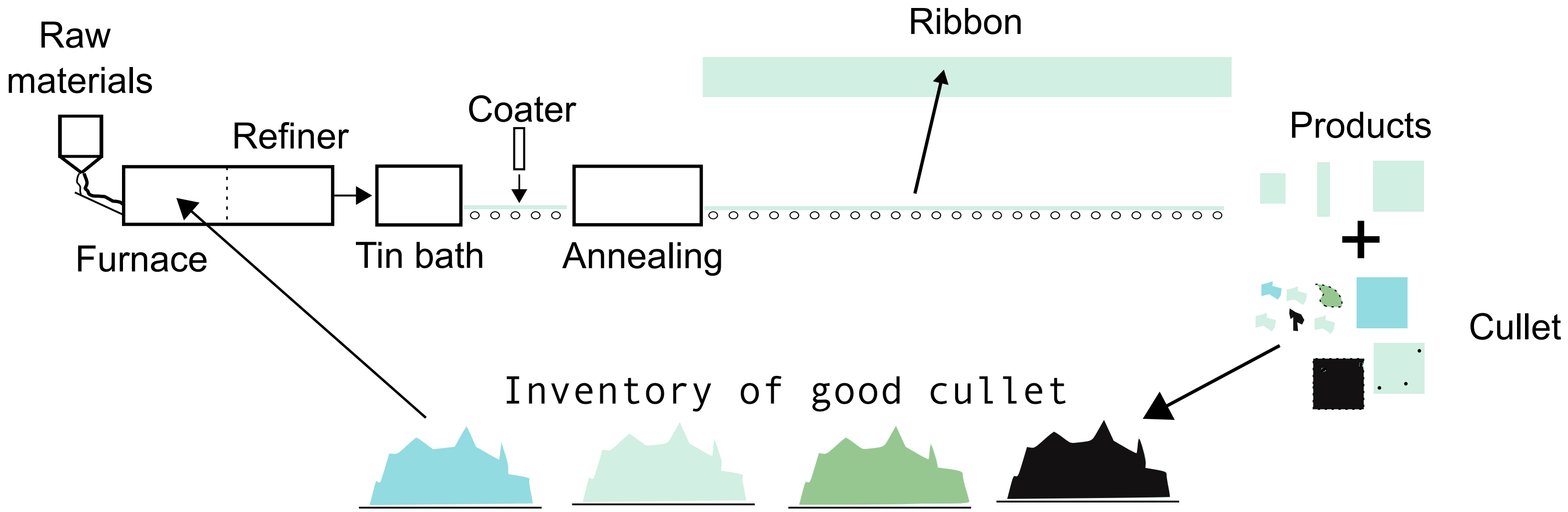


- The schedule determines the amount of cullet generated.
- The cullet in stock must meet the schedule requirements.

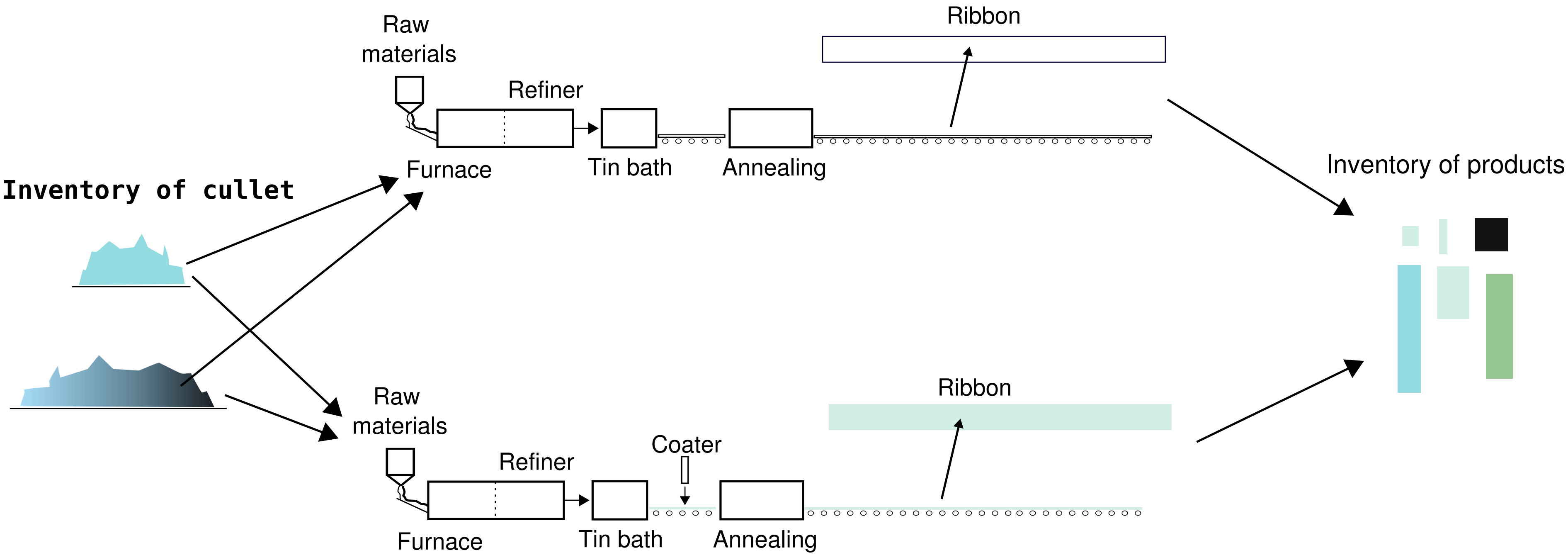
Cullet management

Cullet: waste glass

Good cullet: generated during the production run

Dilution cullet: produced during changeover from one substrate to other substrate

The tinted glass production can be distributed by two lines, which are integrated by a common set of products, cullet production, consumption and storage, and by the glass products storage.



Cullet model

- Mass balance to the process (good and dilution cullet)
  - The consumption of cullet is a nonlinear function of the production length
  - δ-form **piecewise linear formulation (PLF)** to approximate the cullet consumption

$$tc1_{i,l,m,t} = b_{i,0,t} ZNS_{i,l,m,t} + \left(\frac{b_{i,1,t} - b_{i,0,t}}{a_{i,1,t} - a_{i,0,t}} \right) \delta^1_{i,l,m,t} + \left(\frac{b_{i,2,t} - b_{i,1,t}}{a_{i,2,t} - a_{i,1,t}} \right) \delta^2_{i,l,m,t}$$

$$\delta^1_{i,l,m,t} \leq (a_{i,1,t} - a_{i,0,t}) ZNS_{i,l,m,t}$$

$$\delta^1_{i,l,m,t} \geq (a_{i,1,t} - a_{i,0,t}) ZCS_{i,l,m,t}$$

$$\delta^2_{i,l,m,t} \leq (a_{i,2,t} - a_{i,1,t}) ZCS_{i,l,m,t}$$

$$ZNS_{i,l,m,t} \geq ZCS_{i,l,m,t}$$

Cullet model

- Mass balance to the process (good and dilution cullet)
  - The consumption of cullet is a nonlinear function of the production length
  - δ-form **piecewise linear formulation (PLF)** to approximate the cullet consumption
  - The cullet consumption profile has **2 operating modes** for the total, good, and dilution cullet, ($tc1$,$dc1$,$gc1$) or ($tc2$,$dc2$,$gc2$), represented with a disjunction between piecewise linear formulations:

$$\left[\begin{array}{c} Z1_{i,l,t} \\ tc1_{i,l,m,t} = b_{i,0,t} ZNS_{i,l,m,t} + \left(\dfrac{b_{i,1,t} - b_{i,0,t}}{a_{i,1,t} - a_{i,0,t}}\right) \delta^1_{i,l,m,t} + \left(\dfrac{b_{i,2,t} - b_{i,1,t}}{a_{i,2,t} - a_{i,1,t}}\right) \delta^2_{i,l,m,t} \\ dc1_{i,l,m,t} \leq db_{i,0,t} ZNS_{i,l,m,t} + \left(\dfrac{db_{i,1,t} - db_{i,0,t}}{a_{i,1,t} - a_{i,0,t}}\right) \delta^1_{i,l,m,t} + \left(\dfrac{db_{i,2,t} - db_{i,1,t}}{a_{i,2,t} - a_{i,1,t}}\right) \delta^2_{i,l,m,t} \end{array}\right] \vee$$

$$\left[\begin{array}{c} \neg Z1_{i,l,t} \\ tc2_{i,l,m,t} = b'_{i,0,t} ZNS_{i,l,m,t} + \left(\dfrac{b'_{i,1,t} - b'_{i,0,t}}{a'_{i,1,t} - a'_{i,0,t}}\right) \delta^1_{i,l,m,t} + \left(\dfrac{b'_{i,2,t} - b'_{i,1,t}}{a'_{i,2,t} - a'_{i,1,t}}\right) \delta^2_{i,l,m,t} \\ dc2_{i,l,m,t} \leq db'_{i,0,t} ZNS_{i,l,m,t} + \left(\dfrac{db'_{i,1,t} - db'_{i,0,t}}{a'_{i,1,t} - a'_{i,0,t}}\right) \delta^1_{i,l,m,t} + \left(\dfrac{db'_{i,2,t} - db'_{i,1,t}}{a'_{i,2,t} - a'_{i,1,t}}\right) \delta^2_{i,l,m,t} \end{array}\right]$$

The above disjunction was tested and modeled using the convex-hull and Big-M formulations.

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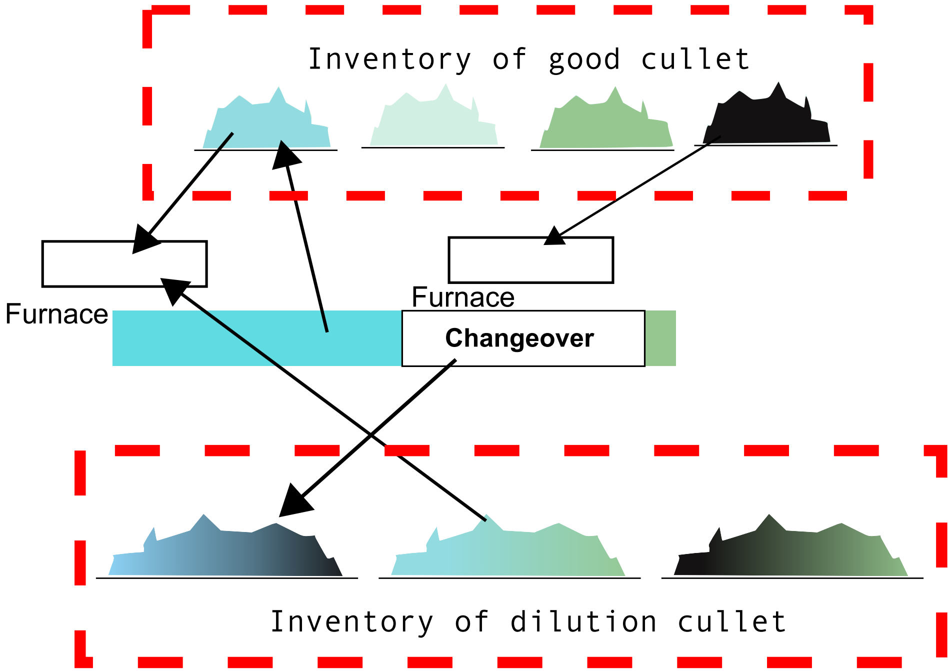
  - Mass balance to the inventory (good, and dilution cullet) on a slot basis in the scheduling model and time period basis in the planning model
  - Cullet inventory constraints
  - Option to sell cullet with a penalty



Cullet synchronization

Assumptions:

1. **Good cullet** is consumed by **both production lines**.
2. **Dilution cullet** is only consumed by **one production line**.
3. If product i is produced in line m, it is not produced in line m'.
4. Dilution cullet produced in time period t in line m cannot be used in the line m' in the time period t, however at the end of the time period t the inventory levels are synchronized.
5. Good cullet of color i is only used in the production of the product i.
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Cullet resources cannot be simultaneously used by both production lines, and therefore, synchronization of the cullet consumption and production does not require new binary variables.

Problem statement

Given:

- Time horizon of **18 months**
- Transition, and inventory costs
- Set of products
  - deterministic **demand**
  - initial, minimum, and maximum **inventory** levels
  - **production rates**
  - sequence dependent **transitions**
  - operating **costs**
  - **selling prices**
- **Cullet**
  - initial, minimum, and maximum **inventory** levels
  - **production and consumption** rates
  - compatibility matrix between colors
  - **selling price**

Determine:

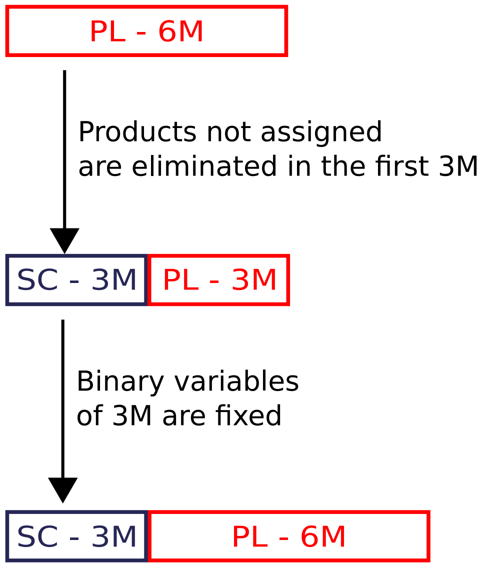
- **sequence of production** (production times and amounts)
- **inventory levels** of products during and at the end of the time horizon
- **inventory levels** of **cullet** during and at the end of the time horizon
- **economic** terms: total operating, transition, inventory costs

That **maximize the profit**

Solution approach

Rolling horizon algorithm

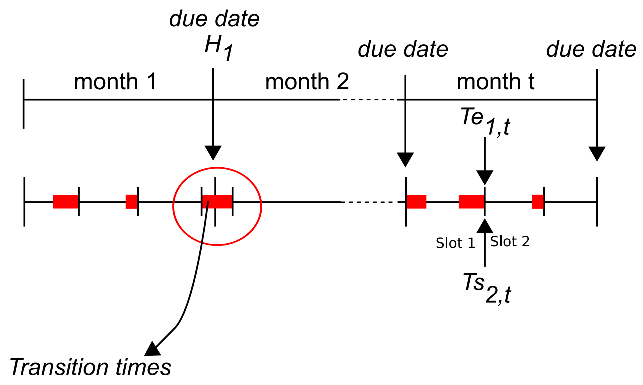
- 2 models, scheduling and planning
- Elimination of the variables/equations related with cullet management (big-M/convex-hull constraints for products not assigned are eliminated)
- Parameters that may influence the quality of the final solution:
  - length of the planning and scheduling horizons.
  - number of sub-problems to solve.
  - strategy used to fix the binary variables from the planning to the scheduling model for the same time periods.
  - strategy utilized to fix the binary variables after the solution of the scheduling model.

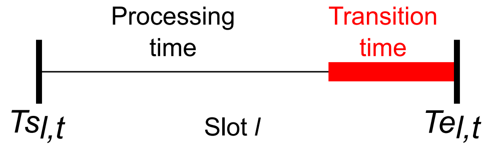


MILP planning and scheduling models

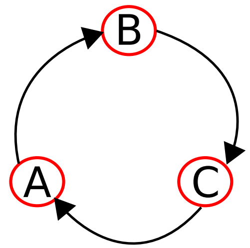
Scheduling model: slot based continuous time model

- detailed timing of the schedule





Planning model: traveling salesman sequence based model



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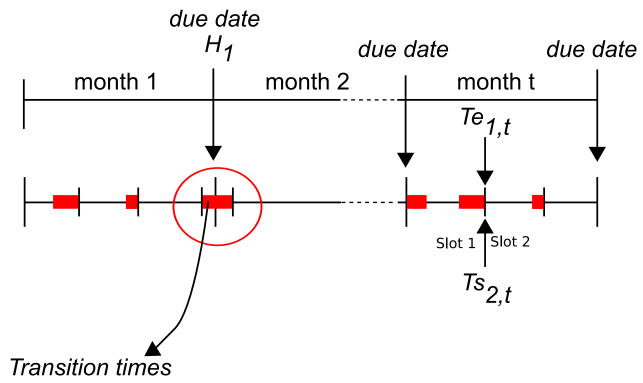

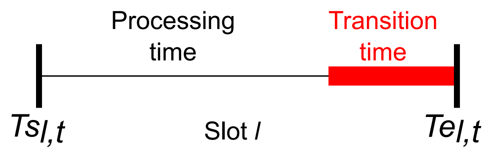

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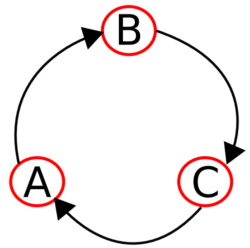

Generic new features integrated in **both** models:

- aggregation of products for changeovers with no transition time
- transitions across due dates
- length of the production run and minimum run lengths across due dates

Specific features:

- cullet management

Case studies

Current work has been focused on helping the PPG Performance Glazings Supply Chain Group in order to study **different scenarios**:

- **Different mix of products**, change the portfolio of products offered, and its impact on the profit.
- How to maximize the profit of the production lines under different demand conditions.

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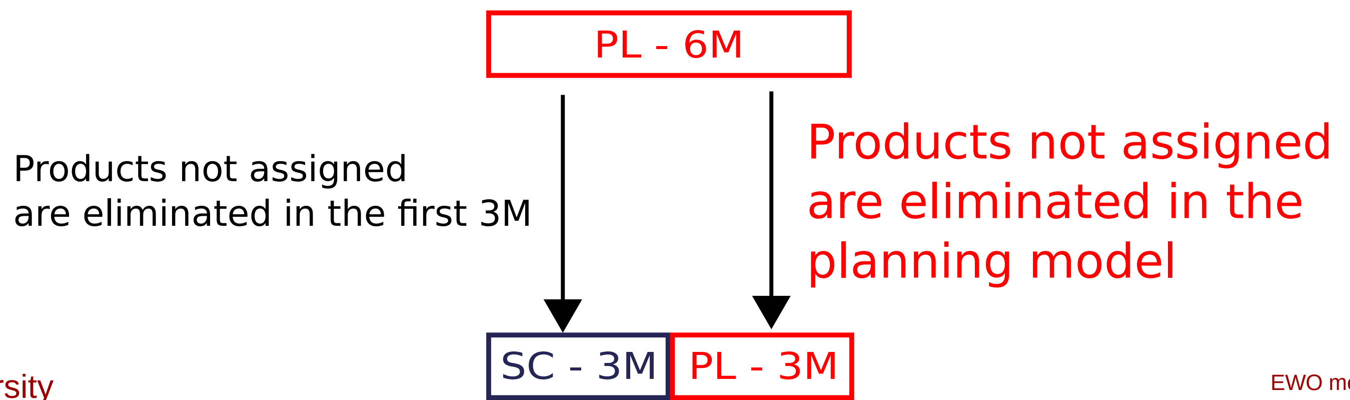
- How to maximize the profit of the production lines under different demand conditions.

The need to solve several scenarios has urged the need to improve the efficiency of the solution approach.

Case studies

Goal: show the improvement on the computational performance

- Case 1 - Base model.
- Case 2 - Case 1 plus additional cuts based on the cullet model.
- Case 3 - Case 2, but additional binary variables fixed in the rolling horizon.



Results

Profit for the three cases and operating results.

	Case 1	Case 2 4.8%	Case 3 3.4%
Profit ($)	0.707	0.743	0.732
# Transitions in line 1:	23	22	20
# Transitions in line 2:	9	16	16
# Transition days in line 1:	94	86	79
# Transition days in line 2:	0	0	0
Tons produced in the first year in line 1	138,429	142526	149,197
Tons produced in the first year in line 2	188,707	188707	188,707
Idle time (days):	97	101	101
Amount below min. (ton):	0.01	0.03	0.03
Total backlog (ton):	0	0	0
Amount above max capacity (ton):	0	0	0

- Comparing with Case 1, Case 2 improves the solution in 3.4%, and Case 3 in 4.8%.
- Non-production days 191/180/187

Statistics and results for each subproblem of the RH

Case 1

Iter	Equations	Variables	0-1 Variables	Slots	CPU (s)	RGap (%)	Best Obj	Obj
iter1	11,496	13,440	3,350	-	7,200	19.2	8,343	6,997
iter2	15,189	21,026	2,445	9	7,200	**1,305.3**	**4,400**	**-365**
iter3	21,097	27,441	4,616	8	7,200	29.4	4,263	3,295
iter4	22,279	31,425	2,653	16	7,200	43.6	-2,256	-3,996
iter5	28,270	37,855	4,849	15	7,200	2,324.4	1,695	-76
iter6	28,659	40,822	2,786	22	2,200	0.0	-6,066	-6,066
iter7	35,424	48,274	5,047	22	7,200	99.1	-22	-2,404
iter8	37,066	53,015	3,068	30	7,200	40.7	-3,128	-5,270
iter9	42,518	58,493	5,254	28	7,200	19.3	-1,354	-1,678
iter10	44,337	63,462	3,310	37	7,200	14.7	-4,151	-4,867

- Large relative gaps due to a weak linear relaxation
- The integer solution of iter2 has a small integrality gap, however in this case the relaxation is weak.

Statistics and results for each subproblem of the RH

Case 2

Iter	Equations	Variables	0-1 Variables	Slots	CPU (s)	RGap (%)	Best Obj	Obj
iter1	10,072	12,516	3,350	-	3,600	14.5	7,943	6,936
iter2	13,977	19,500	2,397	9	3,600	15.8	**-2,504**	**-2,973**
iter3	20,155	26,402	4,462	9	3,600	21.2	627	517
iter4	21,733	31,172	2,525	17	3,600	2.3	-4,022	-4,115
iter5	27,996	38,106	4,634	17	3,600	30.5	-212	-305
iter6	28,928	41,926	2,655	24	3,600	1.4	-3,105	-3,150
iter7	35,381	48,916	4,813	24	3,600	14.6	807	704
iter8	36,991	53,715	2,889	32	3,600	2.2	-2,312	-2,365
iter9	43,382	60,677	5,005	32	3,600	7.8	1,389	1,289
iter10	44,284	64,486	3,040	39	3,600	2.4	-1,796	-1,841

- The relative gaps decreased comparing with Case 1.

Statistics and results for each subproblem of the RH

Case 3

Iter	Equations	Variables	0-1 Variables	Slots	CPU (s)	RGap (%)	Best Obj	Obj
iter1	10,072	12,516	3,350	-	3,600	14.1	7,913	6,936
iter2	13,977	19,500	2,384	9	100	0.0	**-2,970**	**-2,970**
iter3	20,155	26,402	4,473	9	3,600	5.0	560	534
iter4	21,823	31,205	2,576	17	200	0.0	-6,136	-6,136
iter5	28,213	38,177	4,673	17	3,600	1.7	-2,091	-2,127
iter6	28,461	41,052	2,629	23	100	0.0	-5,290	-5,290
iter7	34,914	48,042	4,821	23	3,600	5.4	-1,347	-1,424
iter8	36,546	52,836	2,893	31	3,600	0.6	-4,765	-4,794
iter9	43,062	59,844	5,032	31	3,600	6.2	-1,090	-1,162
iter10	43,416	62,750	3,004	37	3,600	0.7	-4,259	-4,291

- Much lower relative gaps than in Case 1 and Case 2.
- Lower CPU times are needed.

Concluding remarks

- The model was **extended** to deal with the cullet consumption/production/storage for **both lines**.
- The **process characteristics allow the use of a slot based model** without adding new binary variables to handle the cullet management.
- The complexity of the model required the **modification of the rolling horizon algorithm**:
  - smaller time horizons for both models
  - fix the assignment variables for the planning model
- **The performance of the rolling horizon algorithm was improved** without considerable deterioration of the quality of the solution.
- Using the rolling horizon algorithm global optimality is not guaranteed.

δ-form

$$x = a_0 + \sum_{l=1}^{k} y_l$$

$$\hat{\phi}(x) = b_0 + \sum_{l=1}^{k} \frac{b_l - b_{l-1}}{a_l - a_{l-1}} y_l$$

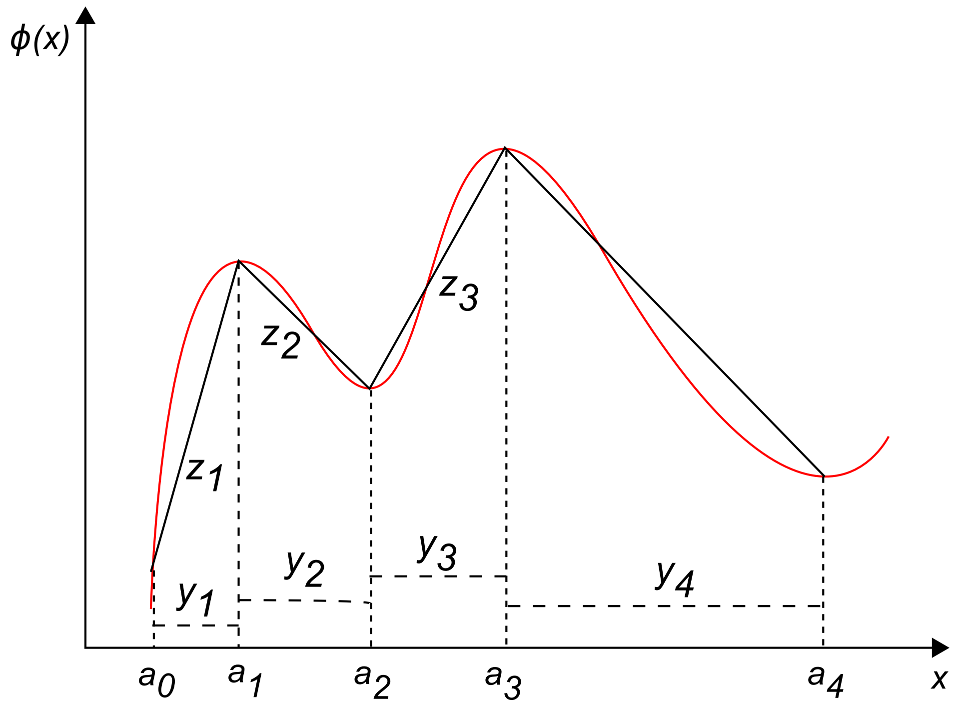
$$y_1 \leq a_1 - a_0$$

$$y_k \geq 0$$

$$y_l \geq (a_l - a_{l-1}) z_l$$

$$y_{l+1} \leq (a_{l+1} - a_l) z_l, \; 1 \leq l \leq k-1$$

$$z_l \in \{0, 1\}, \; 1 \leq l \leq k-1$$



$$1 \geq z_1 \geq z_2 \geq ... \geq z_{k-1} \geq 0$$