Planning and long-term scheduling for PPG glass production

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Introduction

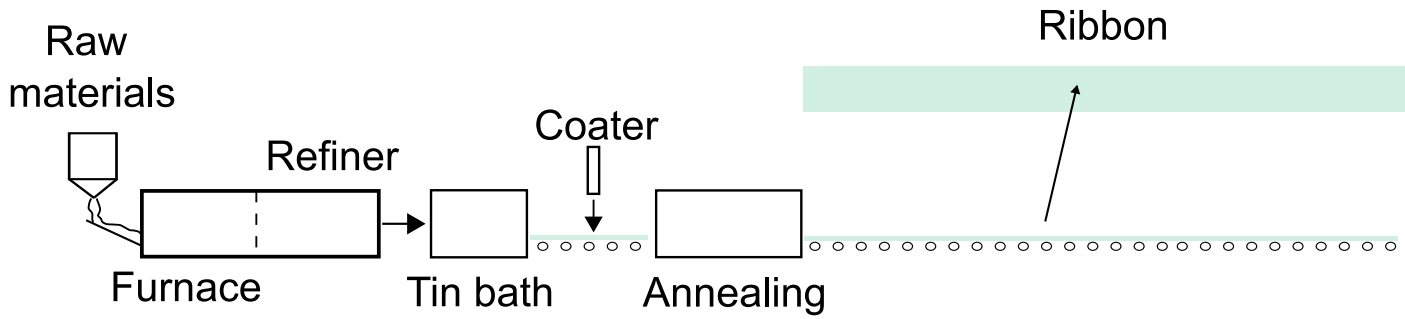
Goals of the project:

- Development of a Mixed Integer Linear Programming (MILP) model for the **planning and scheduling of PPG tinted glasses production**
  - **capture the essence** of the process that **is not considered** in the Master Production Schedule (MPS)
    - cullet management
    - changeovers driven by SKUs

      that may lead to infeasible schedules
- develop a decision making production tool that can influence real world decisions

Process description

Continuous process:



Features

- Raw materials define color of the substrate
- Sequence dependent changeovers between substrates
- Zero length transitions between substrate and coated products
- Long transition times (order of days)
- High transition costs
- Continuous operation during changeover
- Minimum run length (days)

Products defined by color

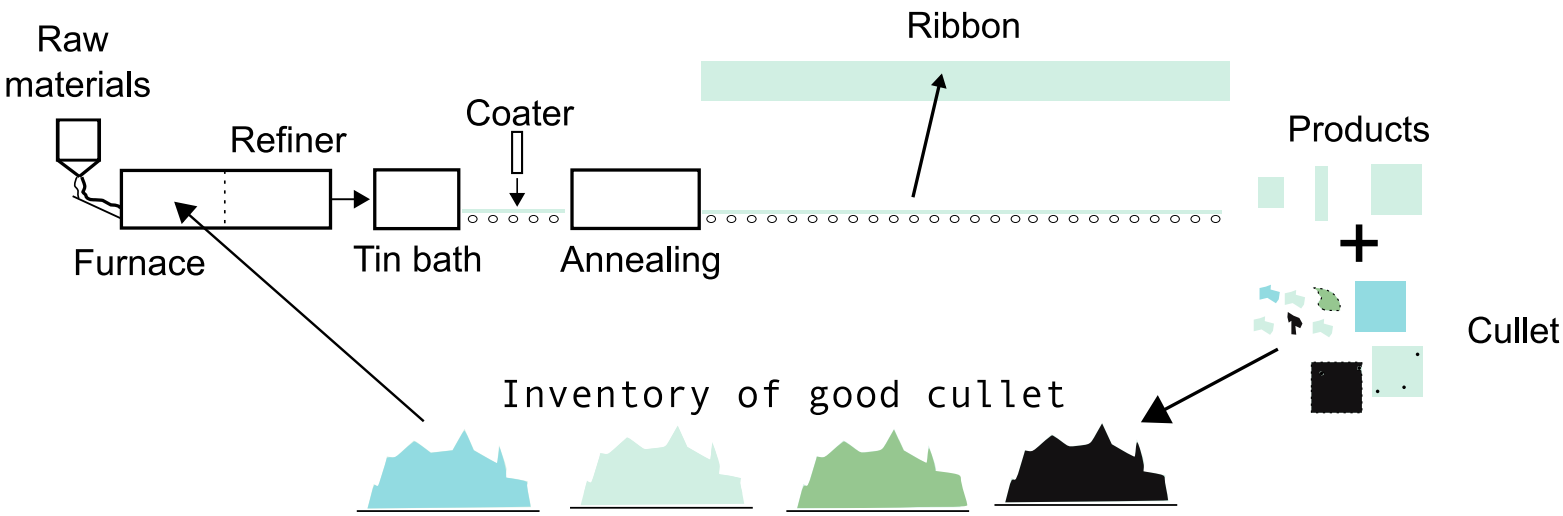
	Substrate	Coated*
Solexia		
Caribia		
Azuria		
Solarbronze		
Solargray		
Graylite		

Process description: cullet management

Cullet: waste glass

Good cullet: generated during the production run

Dilution cullet: produced during changeover from one substrate to other substrate



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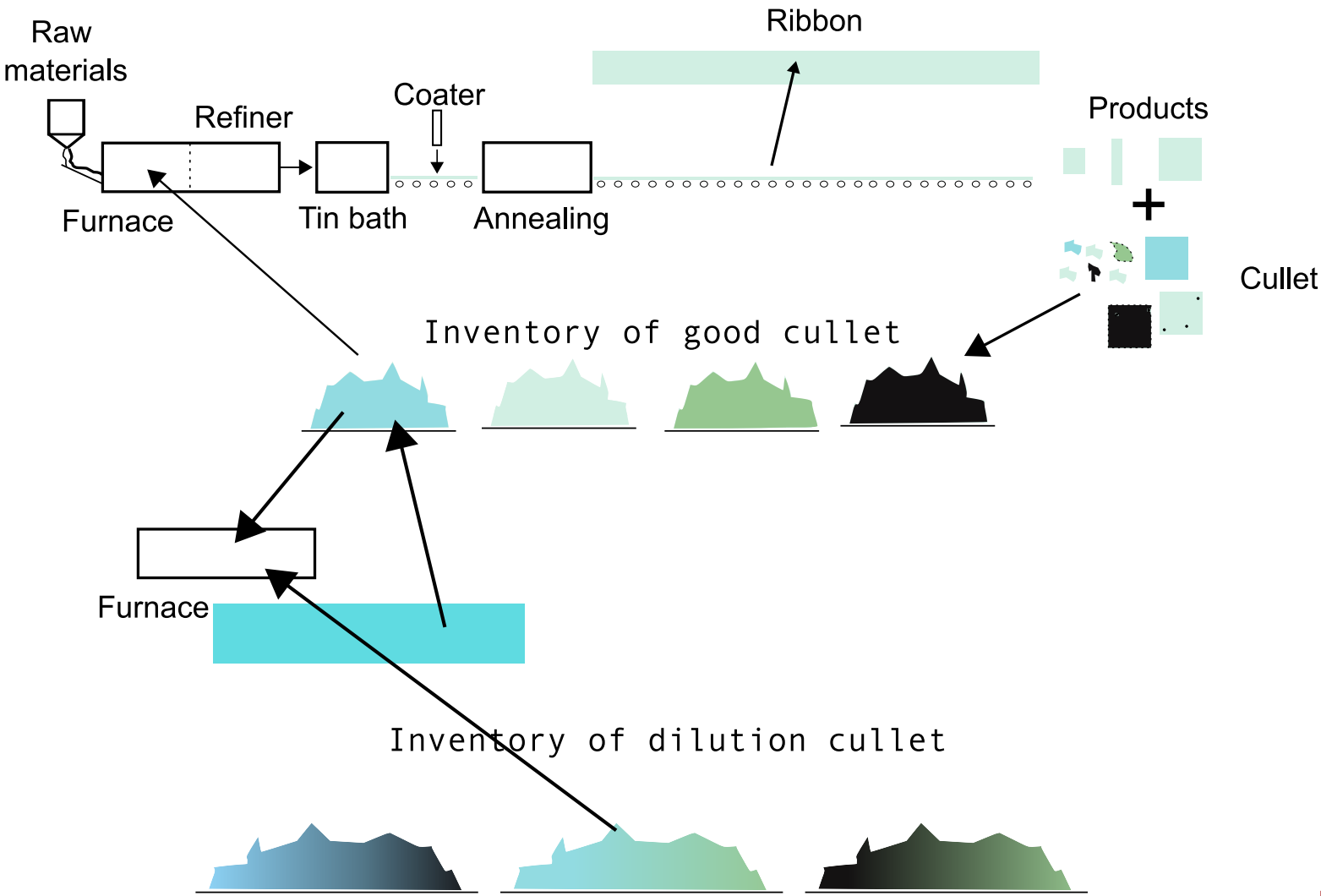

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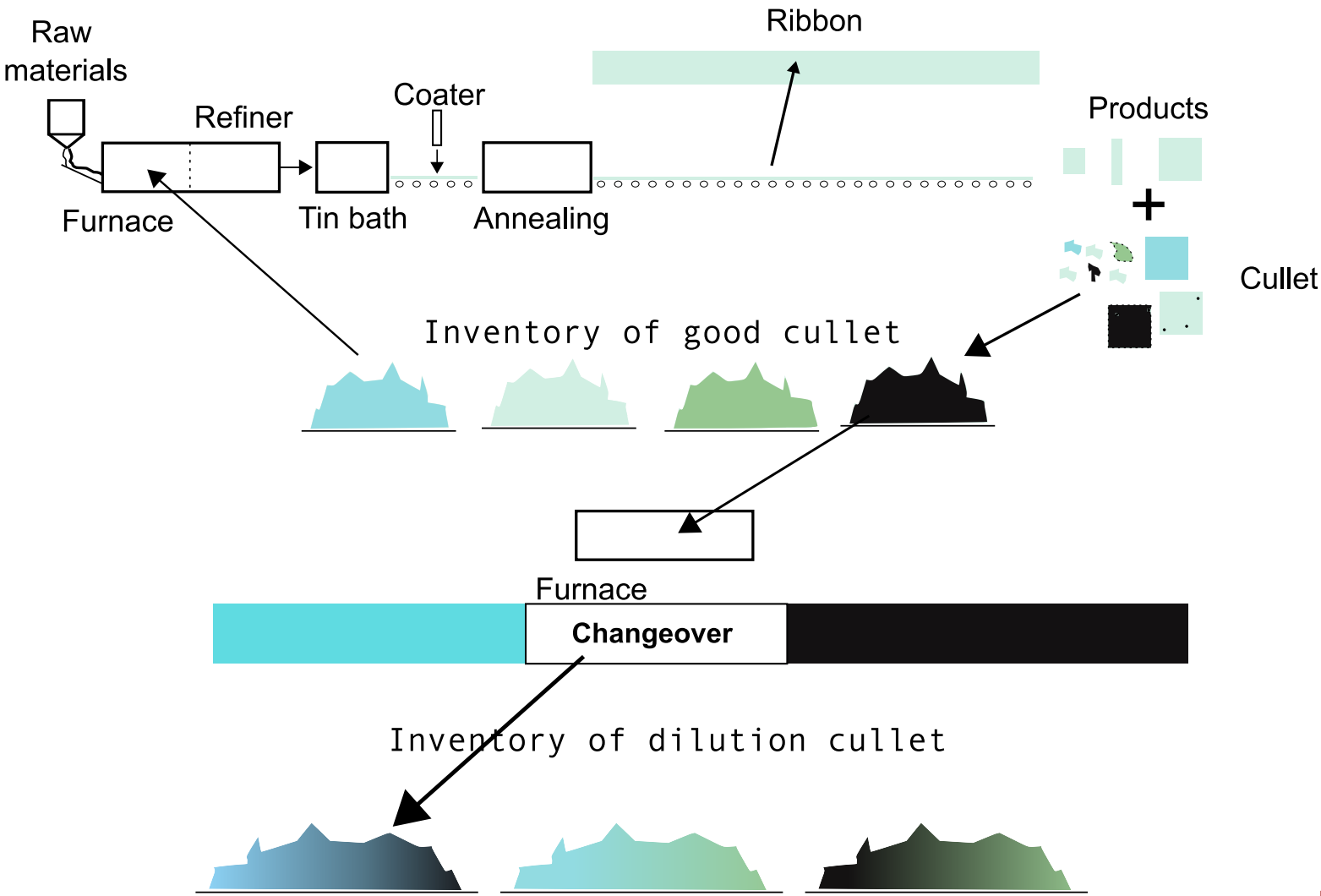

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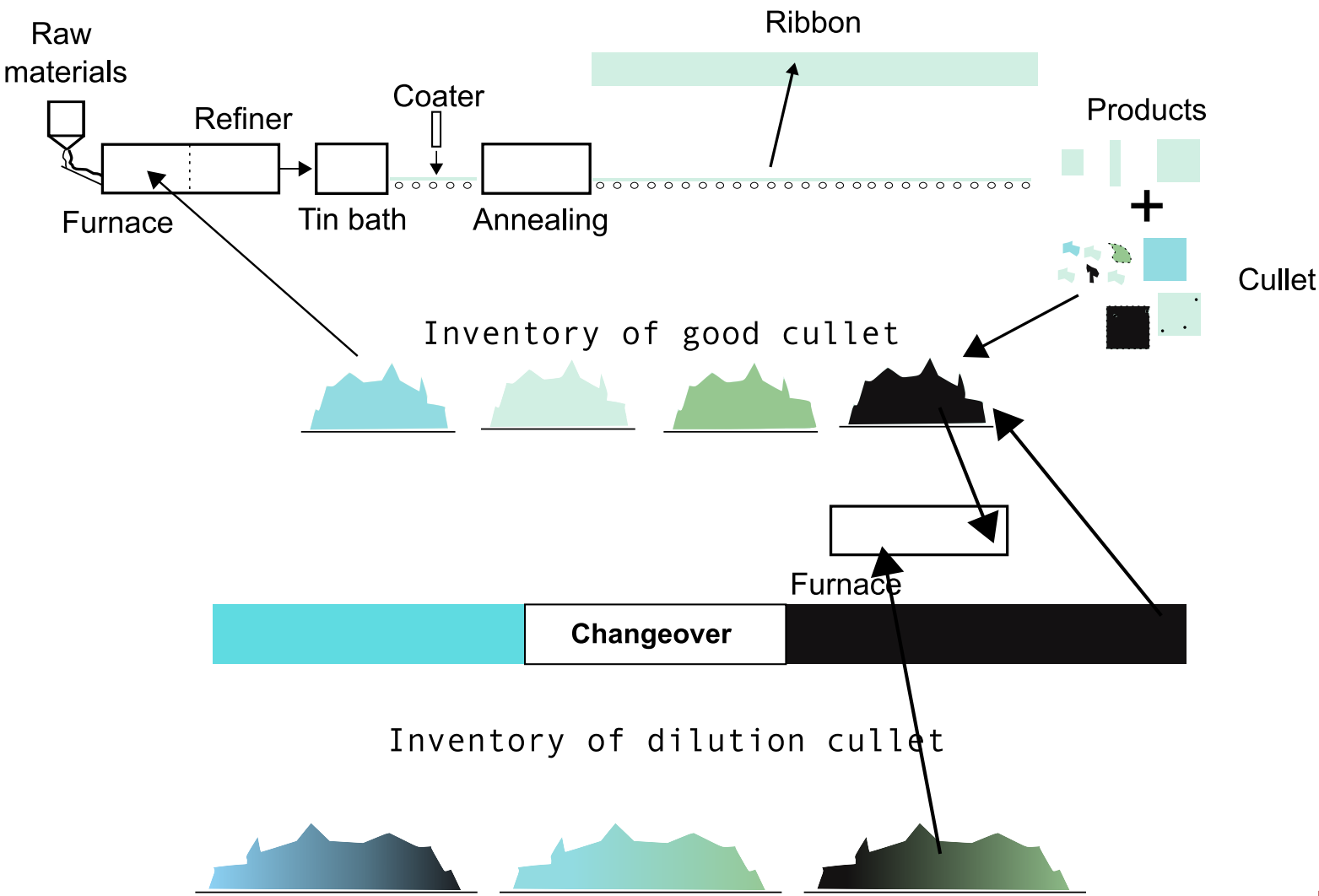

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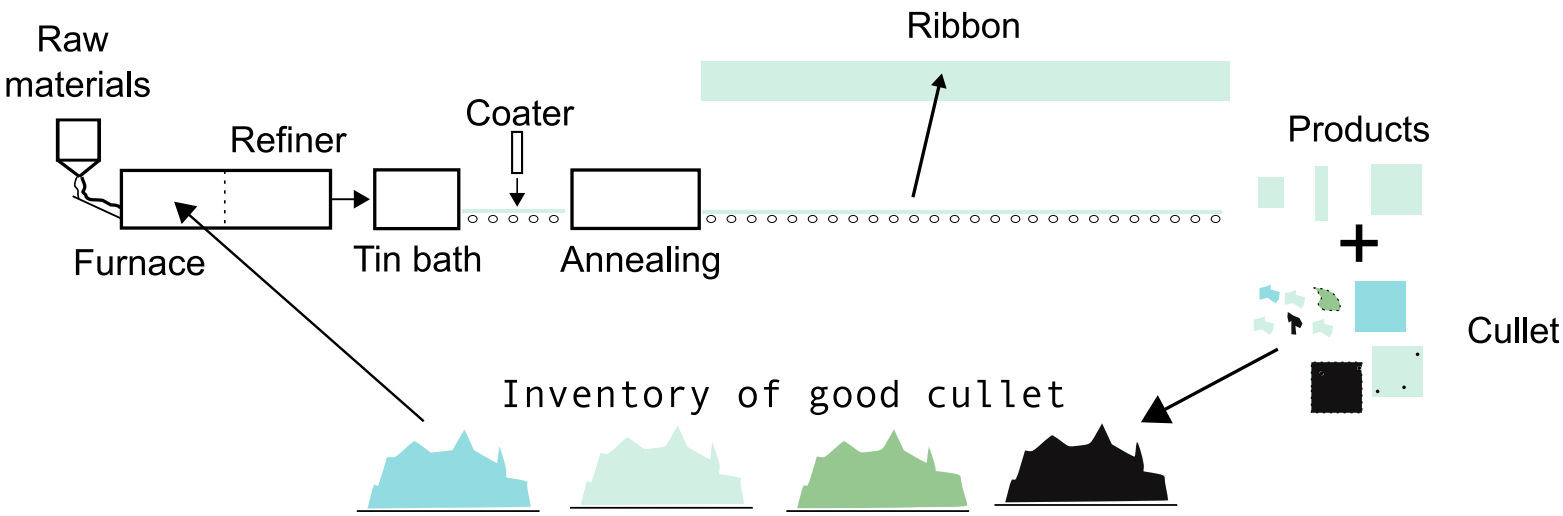


- For each color it is necessary to have in stock the same color of cullet available.
- Changeovers generate dilution cullet at the same operating conditions of production.
- There is a maximum storage capacity for cullet, extra cullet is sold below the production cost.
- The schedule determines the amount of cullet generated.

Problem statement

Given

- Time horizon of **18 months**
- Transition, and inventory costs
- Set of products
  - deterministic **demand**
  - initial, minimum, and maximum **inventory** levels
  - **production rates**
  - sequence dependent **transitions**
  - operating **costs**
  - **selling prices**
- Cullet
  - initial, minimum, and maximum **inventory** levels
  - **production rates**
  - consumption rates
  - compatibility matrix between colors
  - **selling price**

Determine

- sequence of production (production times and amounts)
- inventory levels during and at the end of the time horizon
- economic terms: total operating, transition, inventory costs

That **maximize the profit**

Mathematical programming model

Full space MILP model

- Detailed scheduling: slot based continuous time model (Erdirik-Dogan and Grossmann (2008))
- Planning model: traveling salesman sequence based model (Erdirik-Dogan and Grossmann (2008))

Proposed features

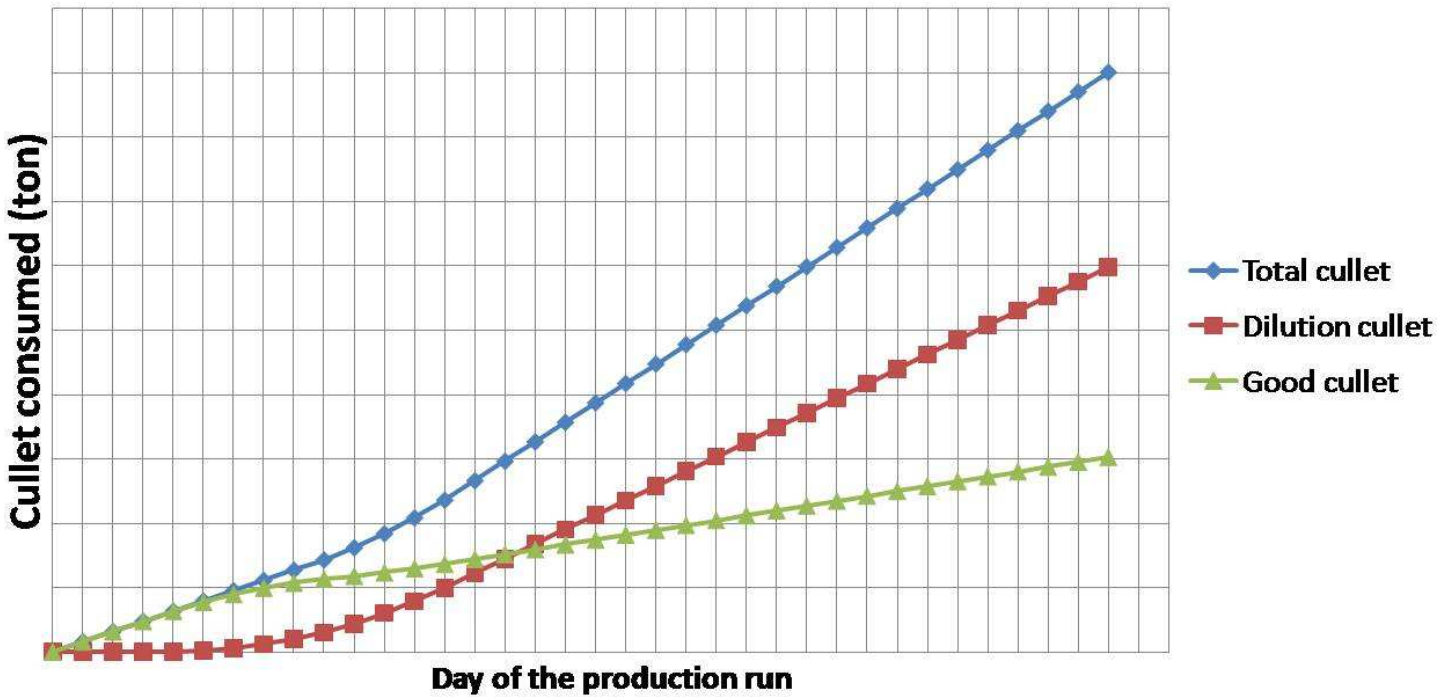
- Changeovers across due dates
- Minimum run lengths across due dates
- Aggregation of the products to cope with redundancy from zero length transition times
- Moving forward terminal constraints for minimum inventory levels at the end of the time horizon
- **Cullet management**

Cullet model

- Consumption profiles for good and dilution cullet
  - by group of colors
  - 2 operating conditions in terms of % of cullet fed to the furnace
    High % of cullet fed to the furnace, when there is enough cullet in stock
    Low % of cullet fed to the furnace, when the cullet in stock is close to the minimum

Cullet consumed in function of the length of the run

High % of cullet



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- New variables
  - Length of the production run
  - 0-1 variable to define if a new production run starts

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- New variables
  - Length of the production run
  - 0-1 variable to define if a new production run starts
- New constraints
  - Mass balance to the process for each production run, for good and dilution cullet
  - Inventory balances for good and dilution cullet
  - Minimum inventory levels for each color of cullet
  - Total maximum capacity storage
  - Penalty for selling cullet

Cullet consumption model

δ-form **piecewise linear formulation (PLF) to approximate cullet consumption**

$$\overline{\theta}_{i,l,t} = MRT_i\, ZNS_{i,l,t} + \delta 1s_{i,l,t} + \delta 2s_{i,l,t} \qquad \forall i \in I, l \in LM, t \in T$$

$$tc1_{i,l,t} = b_{i,0}\, ZNS_{i,l,t} + \frac{b_{i,1} - b_{i,0}}{a_{i,1} - a_{i,0}} \delta 1s_{i,l,t} + \frac{b_{i,2} - b_{i,1}}{a_{i,2} - a_{i,1}} \delta 2s_{i,l,t}$$

$$dc1_{i,l,t} = db_{i,0}\, ZNS_{i,l,t} + \frac{db_{i,1} - db_{i,0}}{a_{i,1} - a_{i,0}} \delta 1s_{i,l,t} + \frac{db_{i,2} - db_{i,1}}{a_{i,2} - a_{i,1}} \delta 2s_{i,l,t}$$

$$\delta 1s_{i,l,t} \leq (a_{i,1} - a_{i,0})\, ZNS_{i,l,t}$$
$$\delta 1s_{i,l,t} \geq (a_{i,1} - a_{i,0})\, ZCS_{i,l,t}$$
$$\delta 2s_{i,l,t} \leq (a_{i,2} - a_{i,1})\, ZCS_{i,l,t}$$
$$ZNS_{i,l,t} \geq ZCS_{i,l,t}$$
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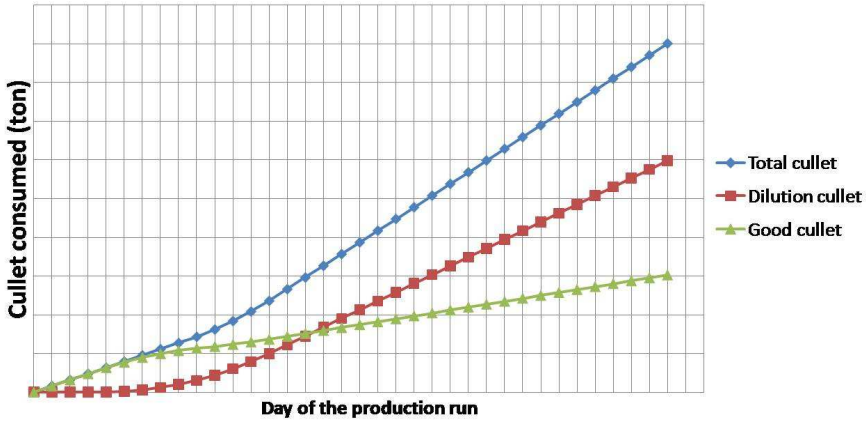
$$gcs_{i,l,t} = tcs_{i,l,t} - dcs_{i,l,t}$$



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$$gcs_{i,l,t} = tcs_{i,l,t} - dcs_{i,l,t}$$

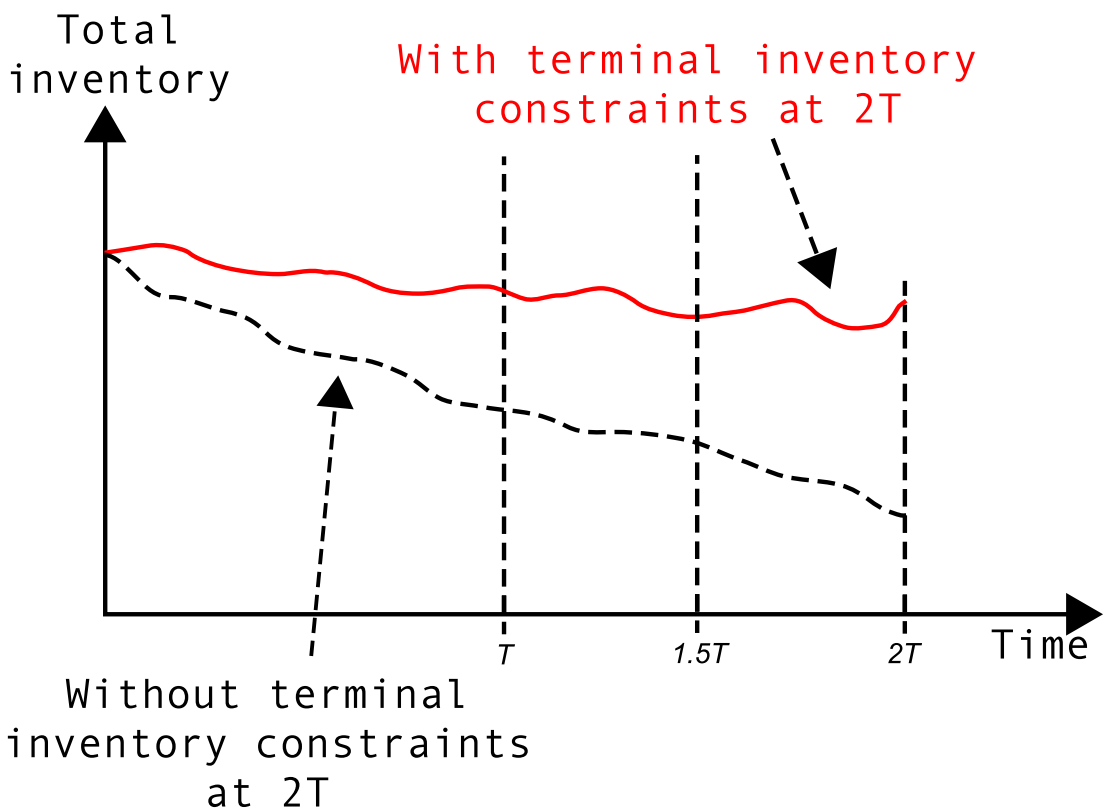
Disjunction to choose the cullet operating profile to use, $ZP25_{i,l,t}$ is a 0-1 variable to select the consumption profile

$$\begin{bmatrix} Z1_{i,l,t} \\ tc_{i,l,t} = tc1_{i,l,t} \\ dc_{i,l,t} = dc1_{i,l,t} \end{bmatrix} \vee \begin{bmatrix} \neg Z1_{i,l,t} \\ tc_{i,l,t} = tc2_{i,l,t} \\ dc_{i,l,t} = dc2_{i,l,t} \end{bmatrix} \qquad \forall i \in IM, l \in LM, t \in T$$

Solution approach

Rolling horizon algorithm (RHA)

- Time horizon = 2 years
- 2 models are used: scheduling model and a planning model
- Aggregation of time periods in the planning model
- Rolling forward terminal constraints for the minimum inventory levels at the end of the time horizon.



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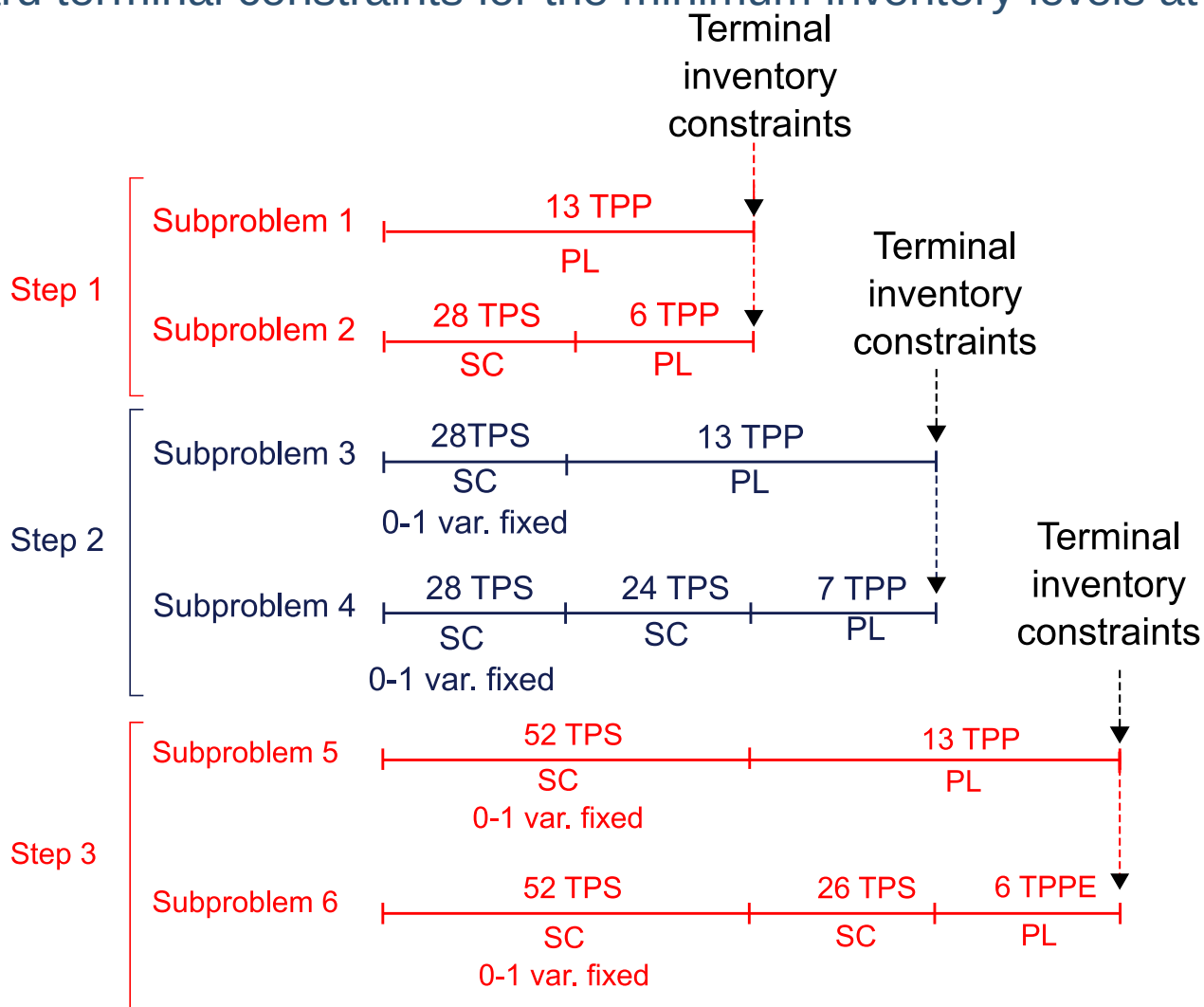

Case studies

- Case 1 - **Scheduling without terminal constraints**
  Time periods = 1 month.
  10 substrates, 9 coated products, 1 different thickness.

- Case 2 - **Scheduling with terminal constraints**
  Time periods = 28 days in the planning model, 7 days in the scheduling model.
  10 substrates, 9 coated products, 1 different thickness.

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- Case 2 - **Scheduling with terminal constraints**
  Time periods = 28 days in the planning model, 7 days in the scheduling model.
  10 substrates, 9 coated products, 1 different thickness.

- Case 3 - **Scheduling and cullet management**
  Time periods = 1 month.
  12 substrates, 12 coated products, 1 different thickness

Models implemented in GAMS and solved with Cplex in a machine running Linux with 8 CPUs Intel Xeon, 1.86GHz and 8GB of RAM

Gantt charts and performance

Case I

Profit 1st year = 4.711m.u

Objective function = -83.54m.u

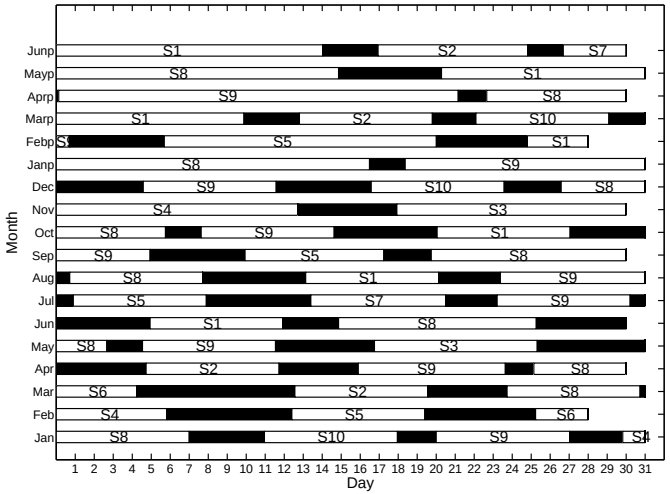
Transition days = 125.5

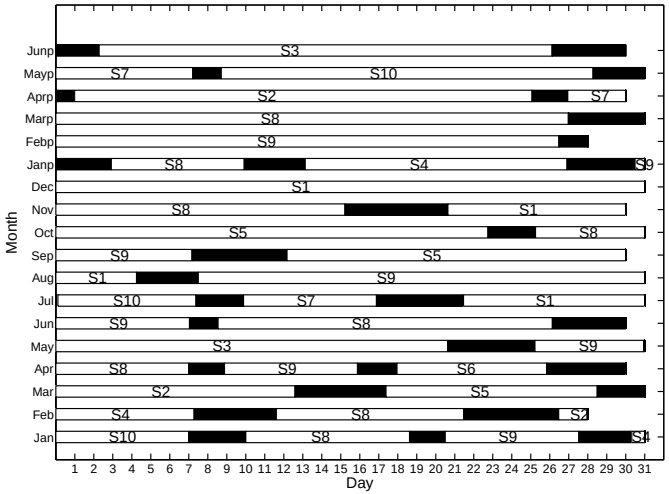
Case II

Profit 1st year = 4.131m.u.

Objective function = 7.87m.u.

Transition days = 70.5





Case II

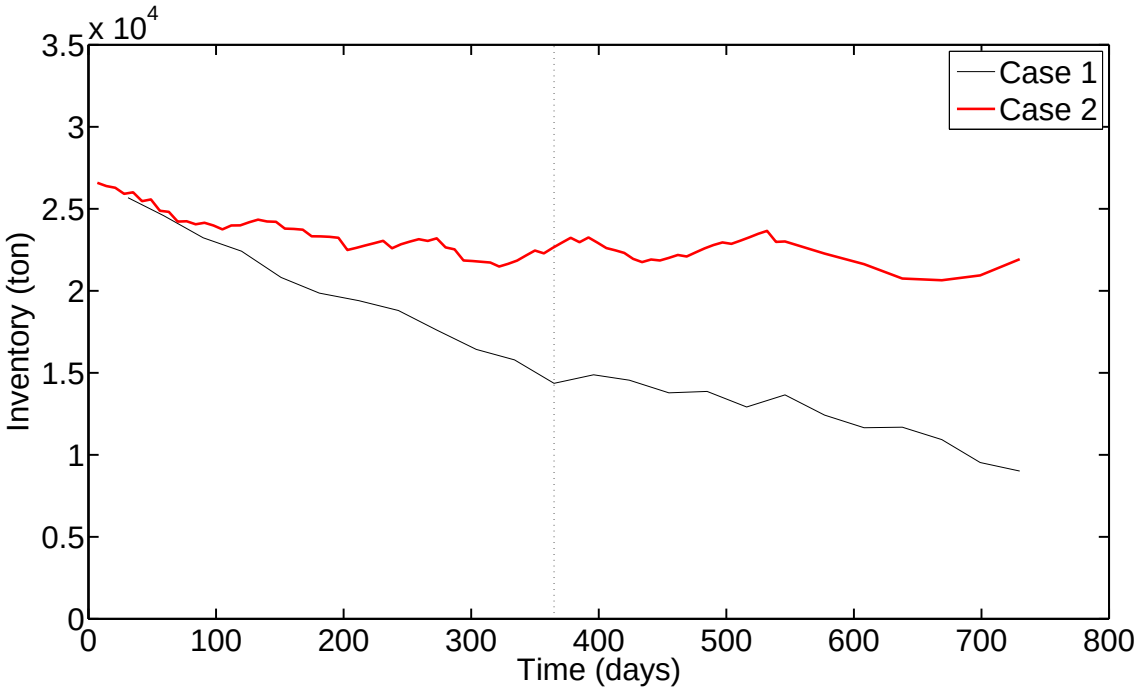
Gap*=1%	SP	# Eq.	Total	Binary	Slots	Gap (%)	RMIP†	Profit (m.u.)	CPU (s)
CPU* = 3,600s	1	9,095	8,115	3,860	-	**1.7**	4.233	4.078	3,600
CPU* = 3,600s	2	5,973	5,329	2,289	17	**0.8**	4.219	4.071	554
CPU* = 3,600s	3	10,252	9,152	3,960	17	**1.4**	6.071	5.945	3,600
CPU* = 10,800s	4	6,827	6,016	2,149	30	**1.0**	6.025	5.919	96
CPU* = 10,800s	5	11,185	9,989	3,960	30	**1.0**	8.214	8.095	3,605
CPU* = 10,800s	6	7,755	6,837	2,140	43	**0.8**	8.177	8.075	384

* - Minimum integrality gap and maximum CPU time set. SP - Subproblem. † - Linear relaxation in the root node.

Total inventory levels

Total inventory levels

Case 1 and Case 2

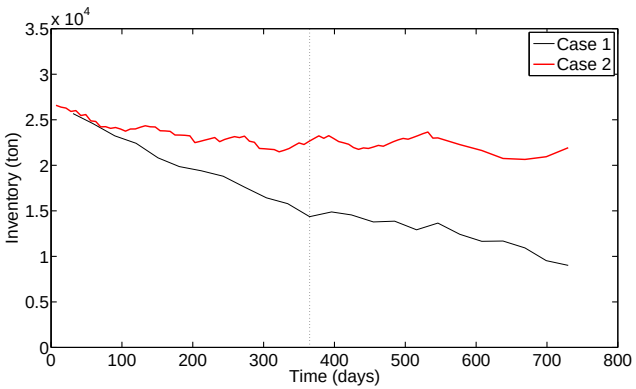


The transition cost is smaller than the production cost. If the demand can be met the model increases the number of changeovers. This leads to the depletion of inventory.

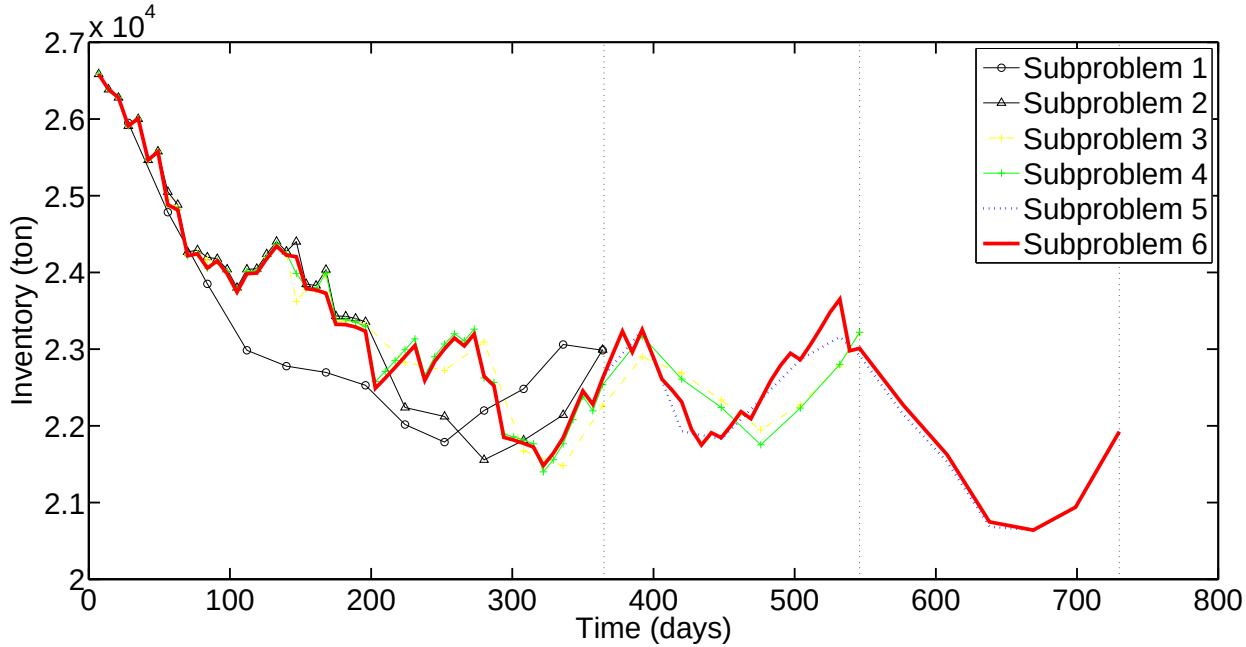
Terminal constraints for the minimum inventory levels at the end of the 2nd year prevent the depletion of inventory.

Total inventory levels

Total inventory levels
Case 1 and Case 2



Total inventory level for each subproblem
of the RHA for Case 2

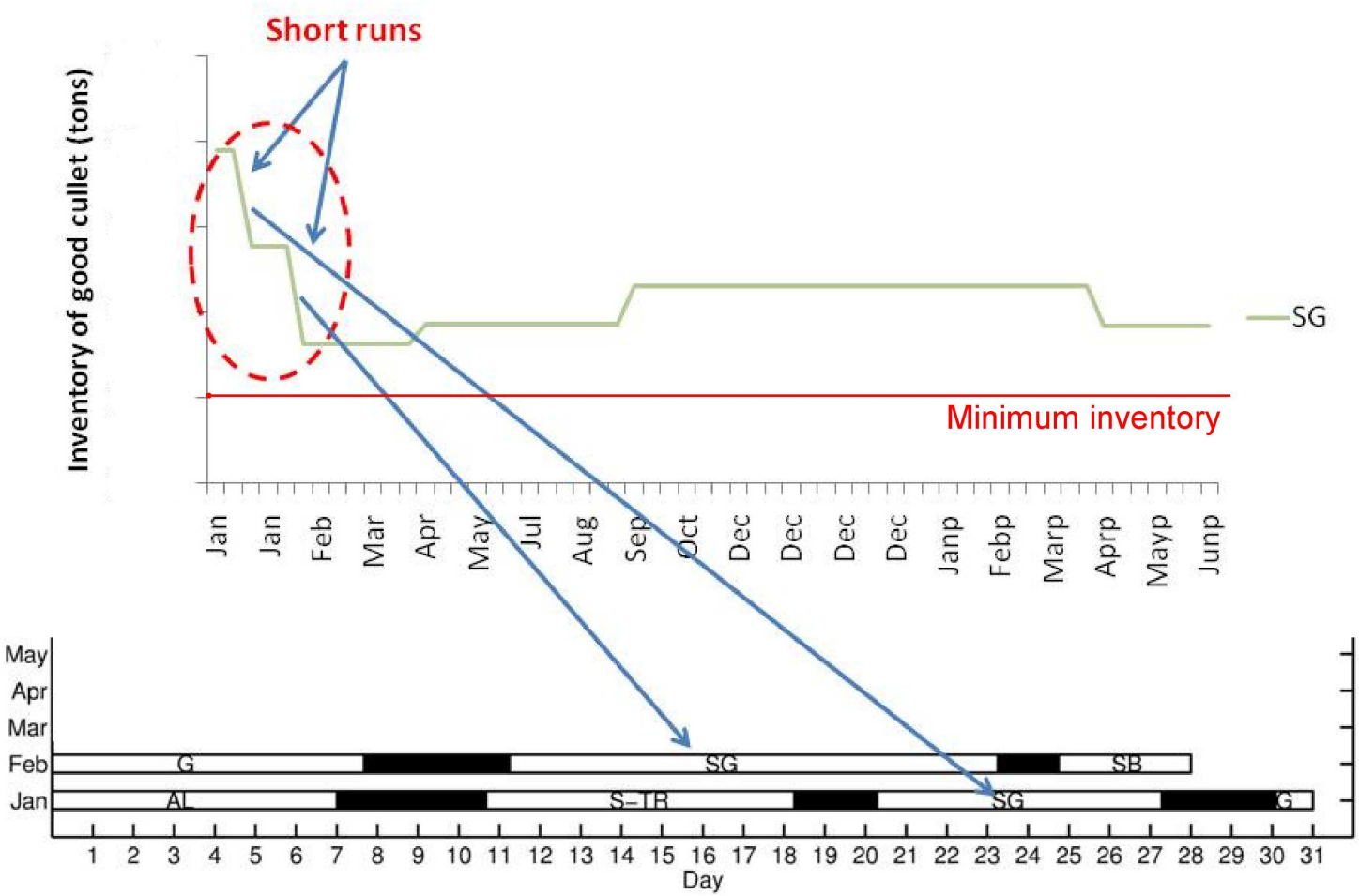


Fixing the binary variables still allows some flexibility to change the amounts produced between subproblems. With time periods of one week, the binary variables for the transitions across due dates restrict changes in the length of the production runs.

Cullet inventory and Gantt chart

Case 3 - Scheduling and culet management

Relation between the **inventory cullet profile** of one product and the Gantt chart



The trend between the length of the production run and the accumulation of cullet is captured by the model.

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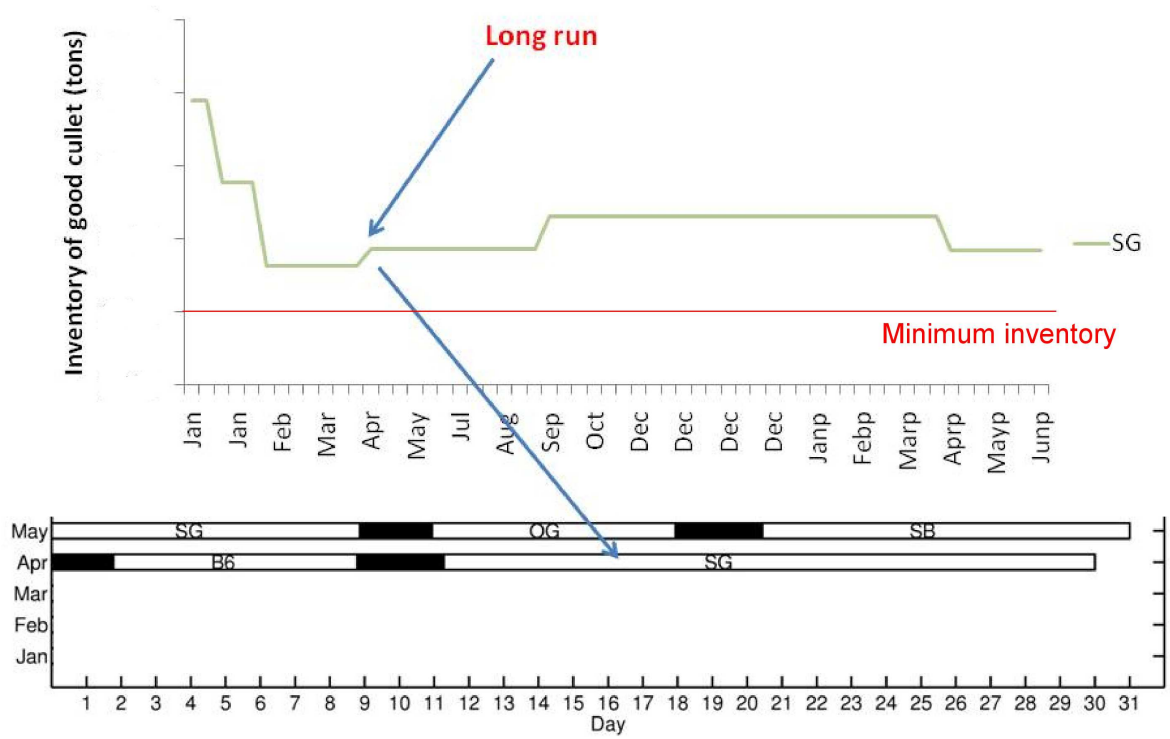

Results for subproblem 6 of the proposed RHA.

SP	# Eq.	Total	Binary	Slots	Gap (%)	RMIP†	Profit (m.u.)	CPU (s)
6	56,986	41,308	4,633	47	14.8	-3,316,534	-4,461,342	3,600

Minimum integrality gap = 5%, max CPU = 10,800/3600s for the first and remaining subproblems, respectively. SP - Subproblem. † - Linear relaxation in the root node.

Concluding remarks

- The scheduling model without cullet management can be tackled for time horizons of 18 months.
- The cullet model is in agreement with the scheduling model.
- Adding the cullet management to the scheduling model poses a challenge in terms of obtaining solutions with small integrality gaps.

Current Work

- Development of an improved solution strategy to handle the scheduling and cullet management.
- Include another production line into the model, which is able to produce some of the current products, and it is integrated through the cullet management and inventory space.