Piecewise Linear Approximation and Branch-and-Refine Algorithm for PX Tank Sizing under Uncertainty Problem

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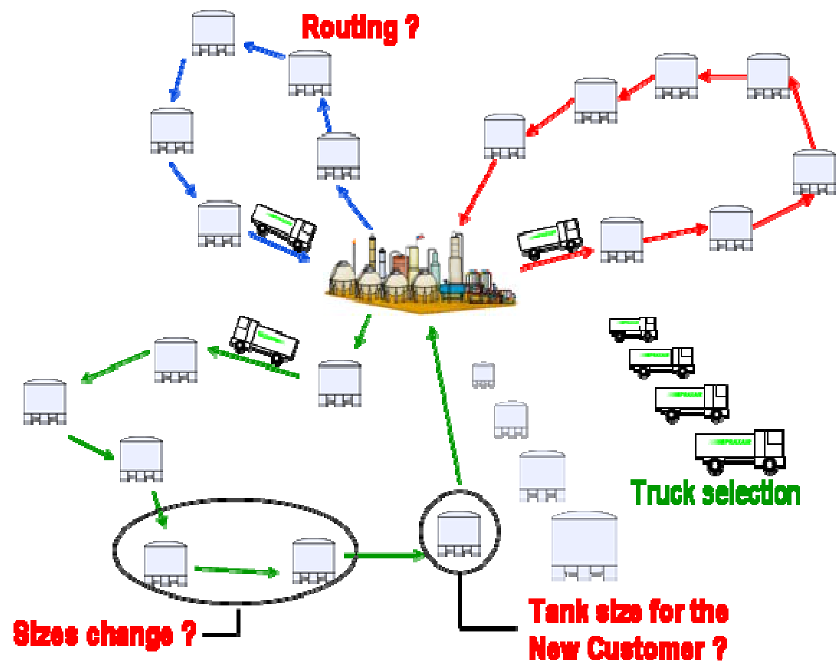
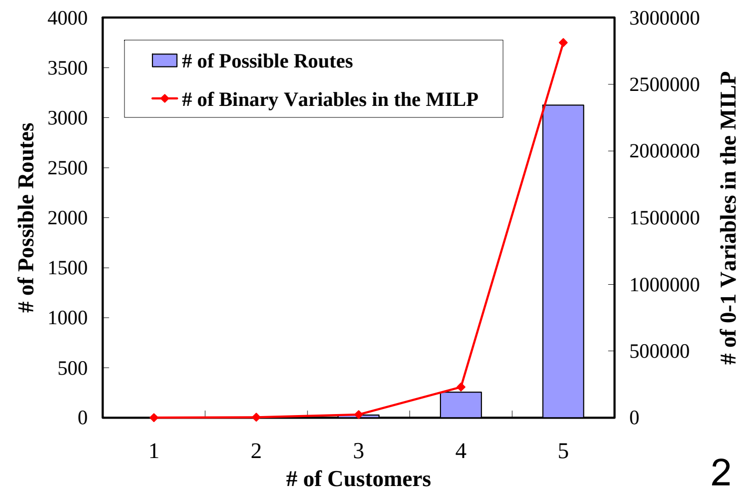
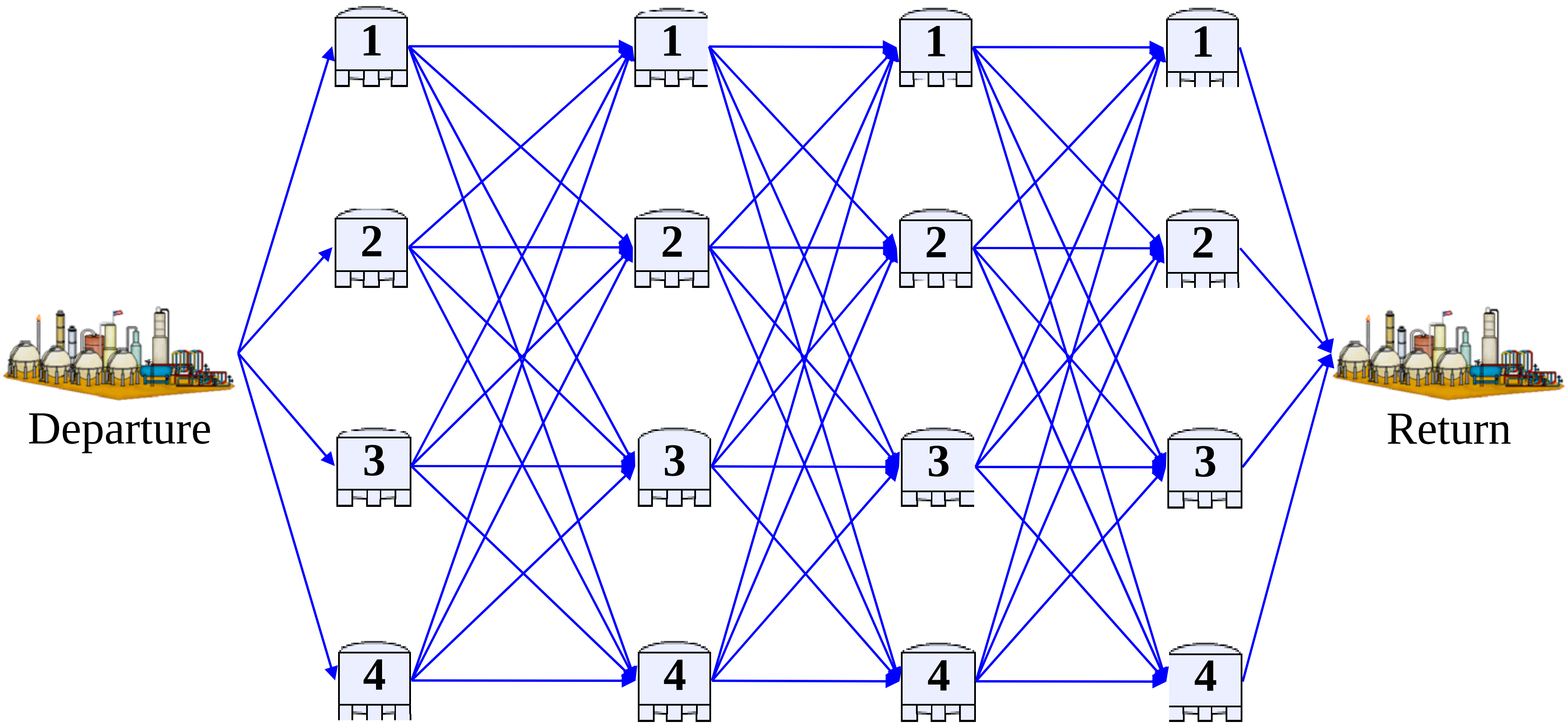
Vehicle Routing – Tank Sizing Problem

- **Tank Sizing**
  - Tank installation, upgrade & downgrade
  - Several available discrete tank sizes
  - Safety stock optimization for uncertainty
- **Vehicle Routing**
  - Several available truck sizes
  - Routing and timing decisions
- **Integration**
  - Tradeoff: operating cost vs. capital cost
  - Capture the effects of customer synergies and truck availability
  - Integration requires to solve "extended" routing problem for long term (e.g. years)
  - Integrated MILP model is **very large**





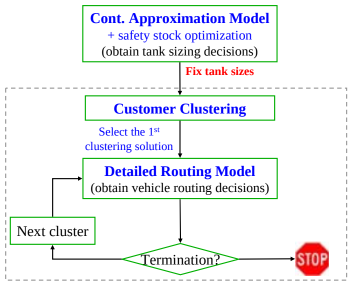
Complexity – 4 customer case



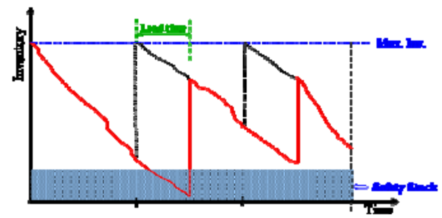
- **256 possible route**, **230,448 binary variables** in the MILP model for integrating tank sizing and vehicle routing

Modeling Challenges

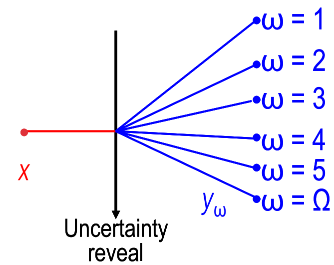
- How to effectively integrate tank sizing with vehicle routing?
  - Continuous approximation (CA) approach
    - tradeoff capital and operating cost in the strategic level
    - reduce most integer variables with some nonlinearities



- How to optimize the safety stocks for **demand uncertainty**?
  - Employ stochastic inventory model
  - Integrate safety stock optimization with tank sizing



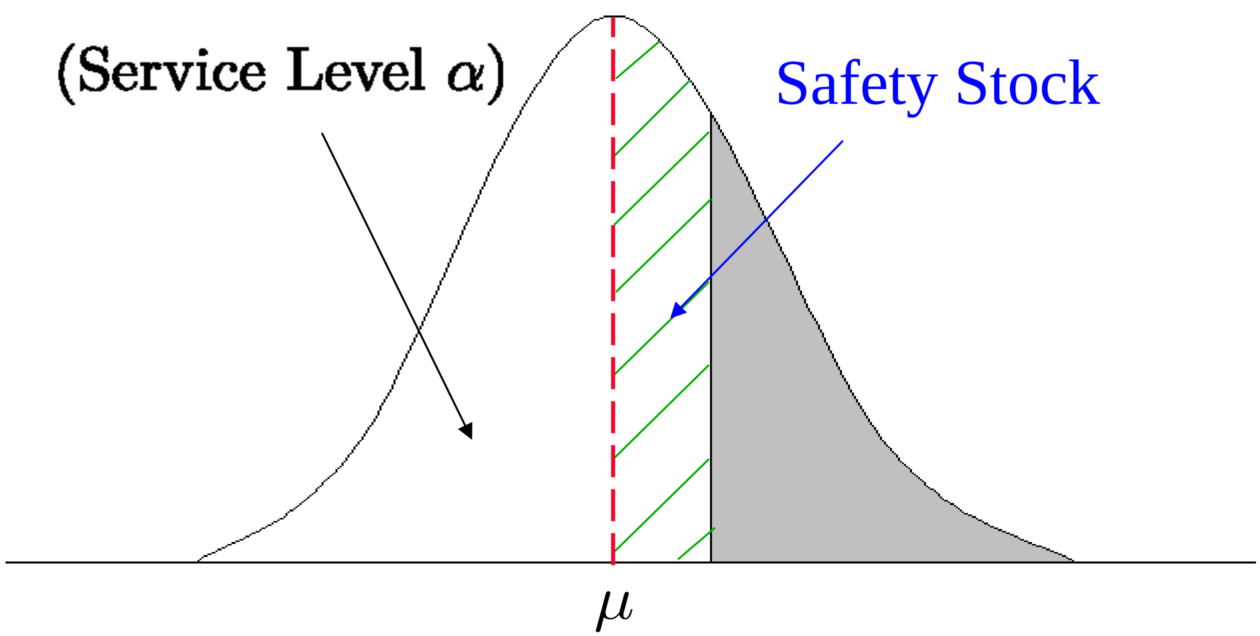
- How to model the **uncertainty of adding/losing customers**?
  - Two-stage stochastic programming
  - A network structure for each scenario in each year



Optimal Safety Stock Level

$D \sim N(\mu, \sigma^2) \Longrightarrow \text{Safety Stock} = \boxed{z_\alpha \sigma}$, $P(z \leq z_\alpha) = \alpha$ **(Service Level)**

Lead time = *T* $\Rightarrow D \sim N(T \cdot \mu, T \cdot \sigma^2) \Rightarrow \boxed{\text{Safety Stock} = z_\alpha \sigma \sqrt{T}}$

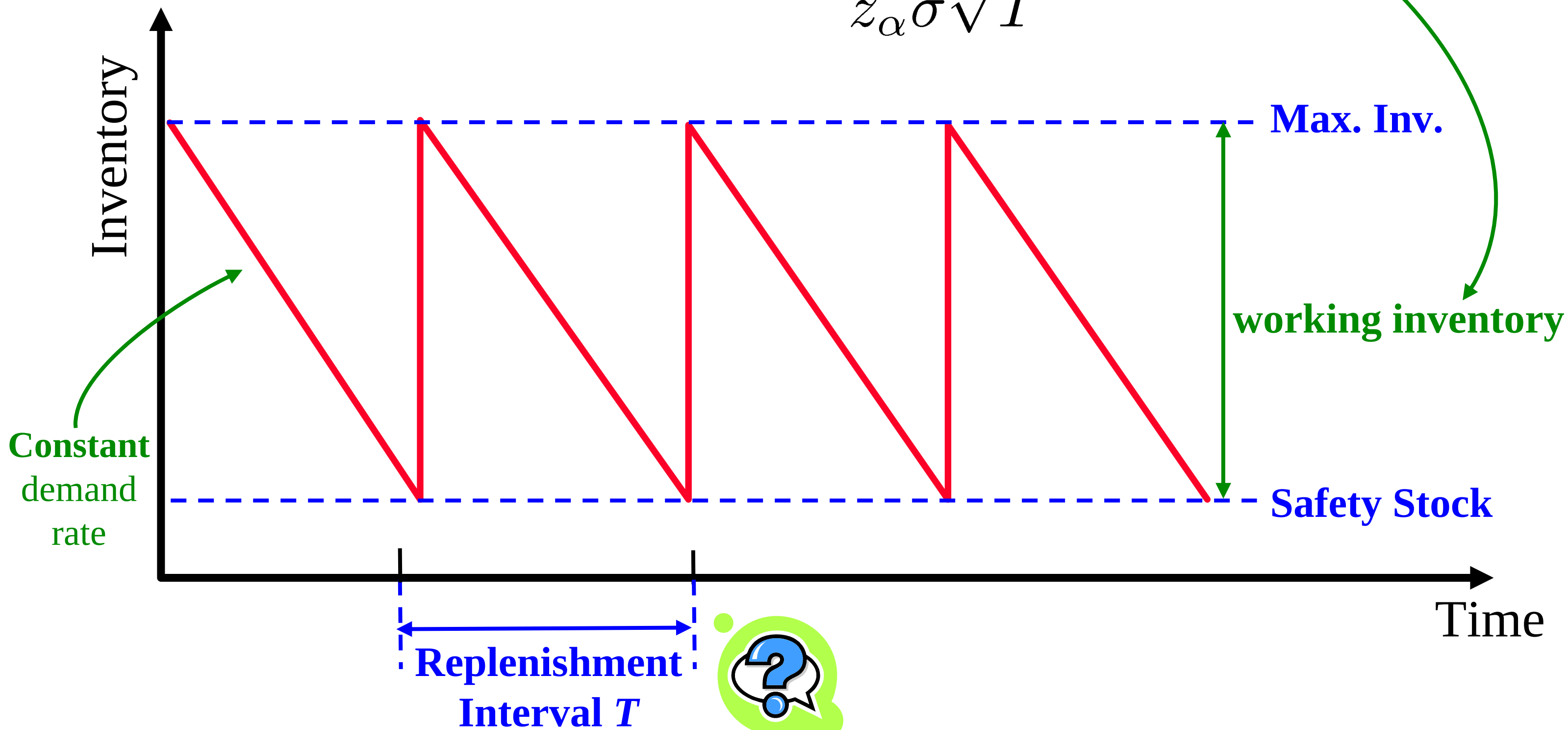


"Cyclic" Inventory-Routing

- Key Assumption: each customer is replenished in a "cyclic" way with interval T

- Required tank size ≥ max. inv. = Safety Stock + demand rate× T

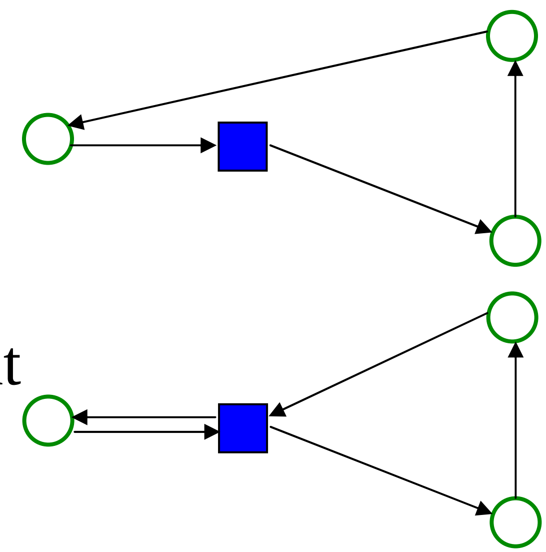
$$z_\alpha \sigma \sqrt{T}$$



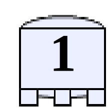
Routing & Replenishment in CAM

- $T = R$ / (ave. speed)
  - T - replenishment interval
  - R - minimum distance to visit all the customers in a cluster once
  - Average travelling speed is known

■ plant
○ customer



- If only one trip for each replenishment
  - R = **TSP** distance of the cluster & plant
- If allowing multiple trips for replenishment
  - R = ?

CAM for Capacitated Routing Problems*

- Bounds for minimum routing distance ***R***

$$R \approx 2\lceil \tfrac{n}{q} \rceil r + (1 - \tfrac{1}{q}) \cdot \mathrm{TSP}$$

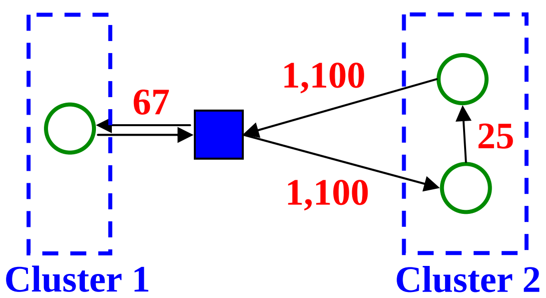
  - *n* – number of customers in the cluster
  - *q* – capacity, max. number of customers that can be visited in one trip
  - *r* – average distance from customers to the plant
  - TSP – traveling salesman distance to visit all customers once

- Examples
  - Cluster 1: *q*=1, TSP=0, *r* = 67

$$2\lceil \tfrac{n}{q} \rceil r + (1 - \tfrac{1}{q}) \cdot \mathrm{TSP} = 2r = 2 \times 67 = \boxed{134\mathrm{km}}$$

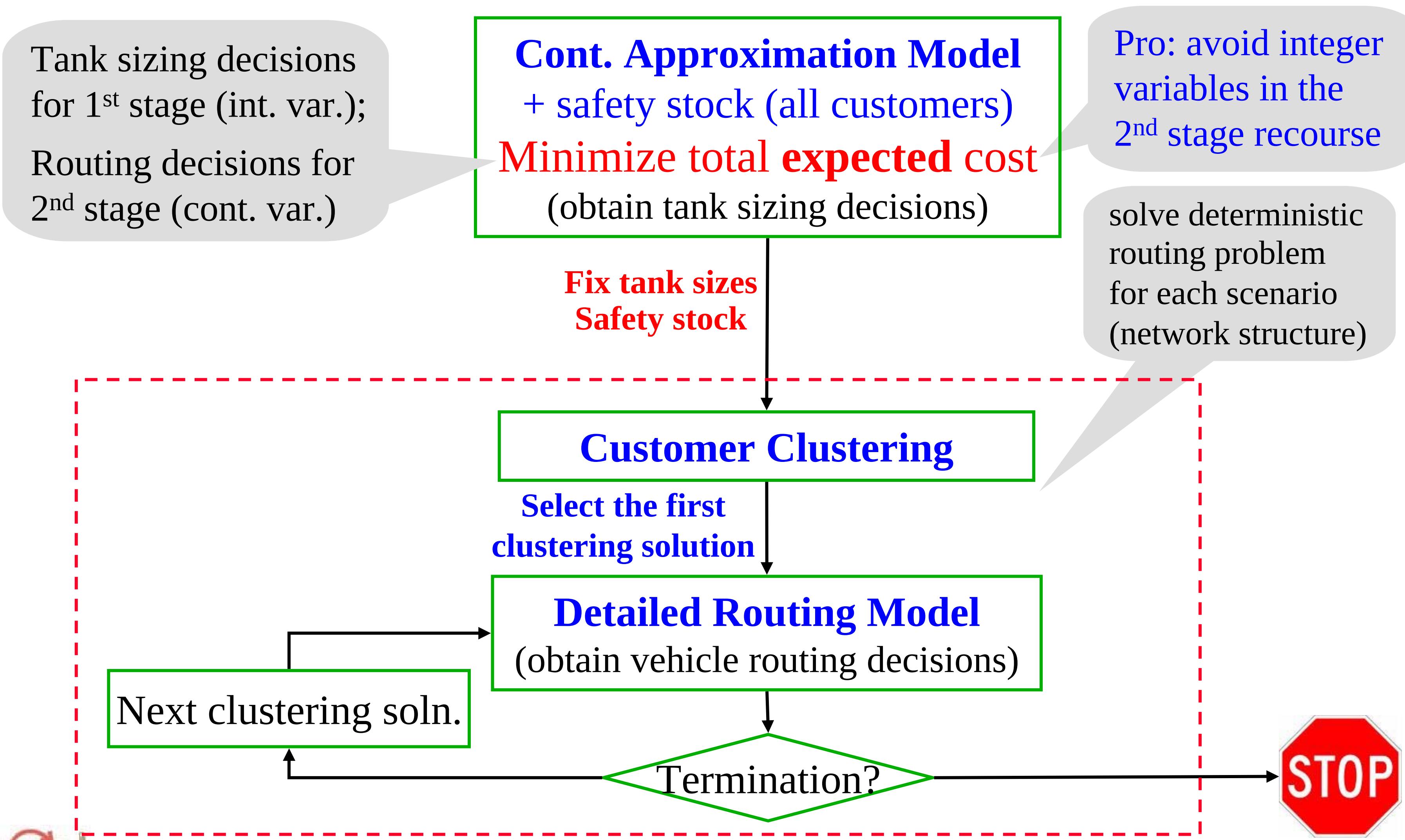
  - Cluster 2: *q*=1, same as Cluster 1, $R = \boxed{4,400\mathrm{km}}$
  - Cluster 2: *q*=2, TSP=50, *r* = 1,100

$$2\lceil \tfrac{n}{q} \rceil r + (1 - \tfrac{1}{q}) \cdot \mathrm{TSP} = 2r + \frac{\mathrm{TSP}}{2} = \boxed{2,225\mathrm{km}}$$



* *M Haimovich, AHG Rinnooy Kan, "Bounds and heuristics for CRP", Math. of Oper. Res., 1985, 10(4), 527-541*

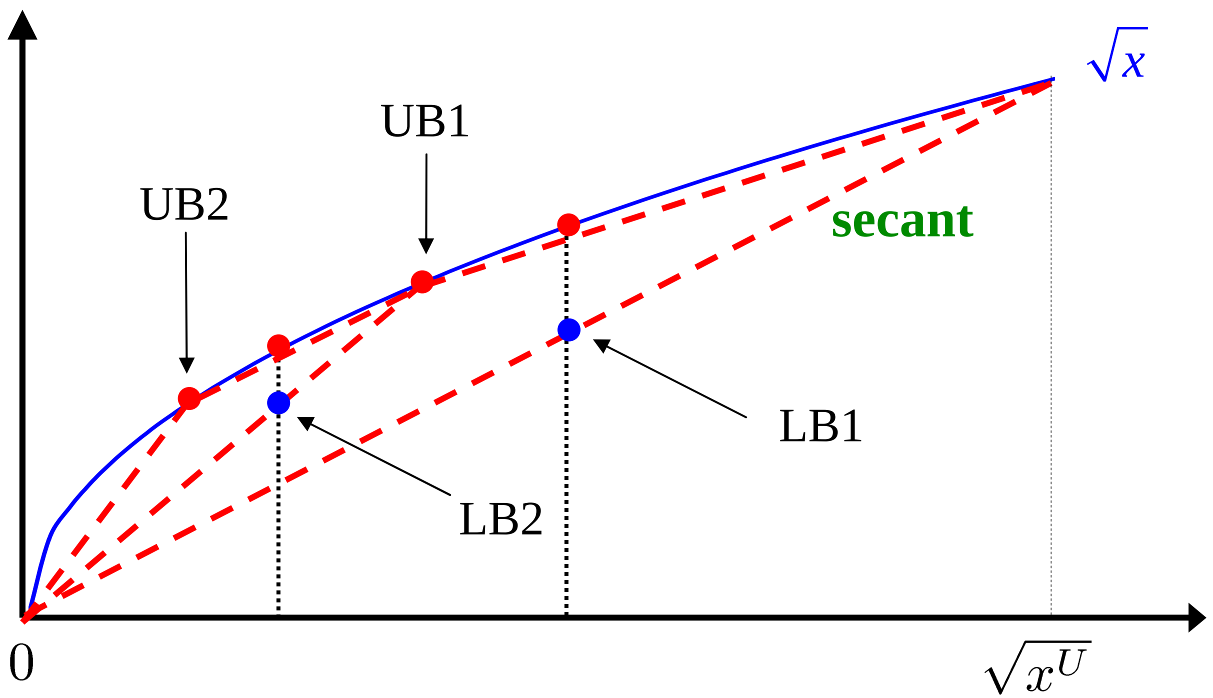
Decomposition for Scenario Planning



Each scenario has a network structure, problem size increases as scenarios increase

Branch-and-Refine Algorithm

- Global Optimization for MINLPs with *only* Univariate Concave Terms
  - Piece-wise linear approximation (MILP) provides **global** lower bounds
  - Feasible solutions provide upper bounds – solving a reduced MINLP
  - Increasing the number of pieces as iteration number increases



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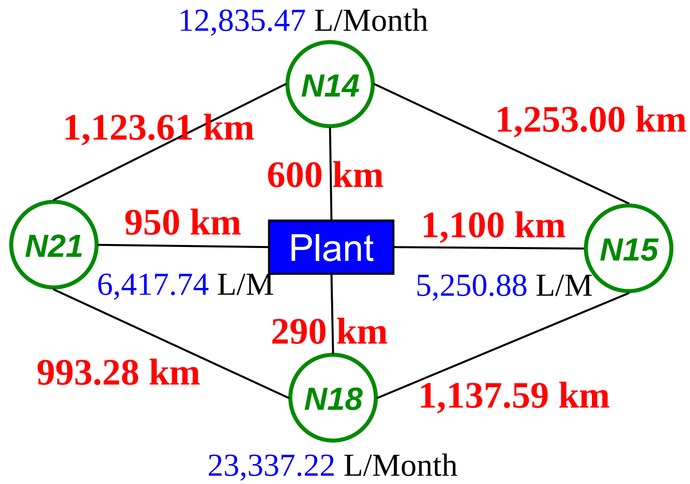
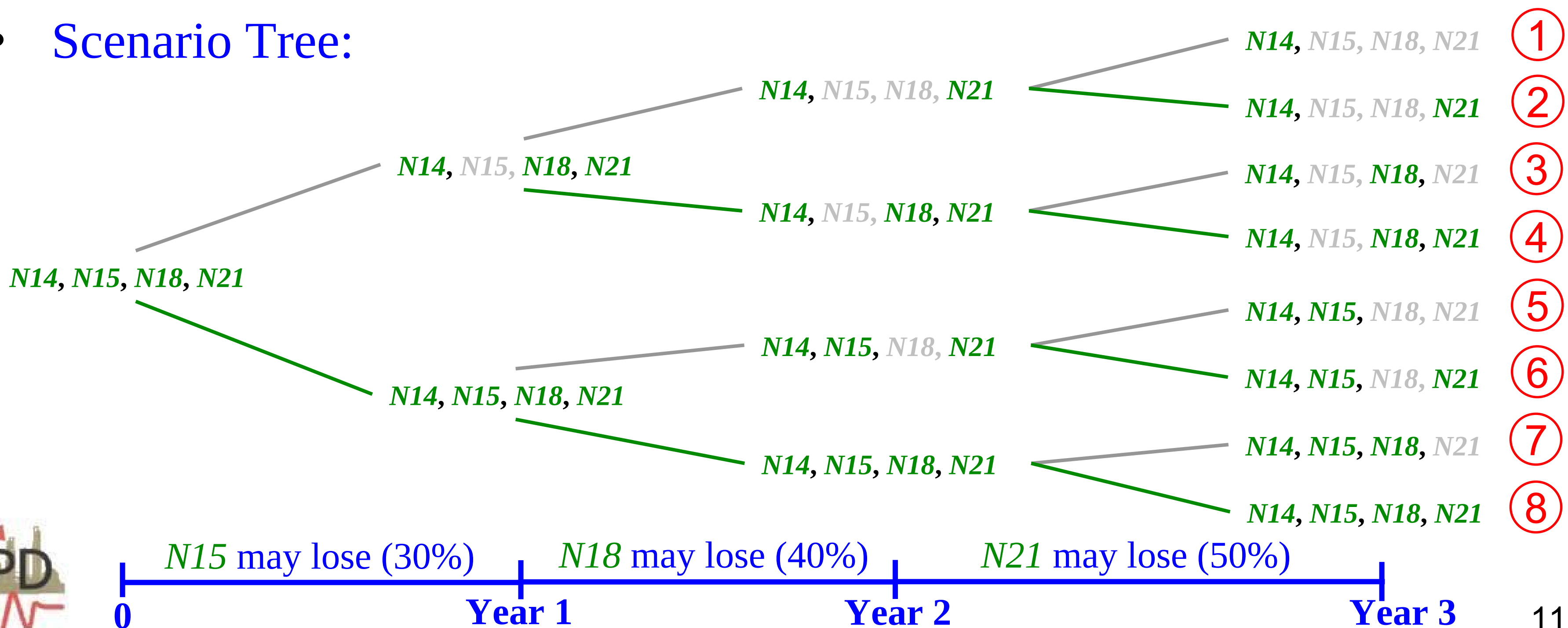
Case Study

- **Example 1:** 3-year planning horizon
  - 4 customers, 6 tank sizes, 6 types of trucks
    - *N14* will not lose by the end of Year 3
    - *N15* may lose in Year 1 with 30% chance
    - *N18* may lose in Year 2 with 40% chance
    - *N21* may lose in Year 3 with 50% chance



- Scenario Tree:



3 Existing Customer and 1 New Customer

- **Example 1:** 3-year planning horizon
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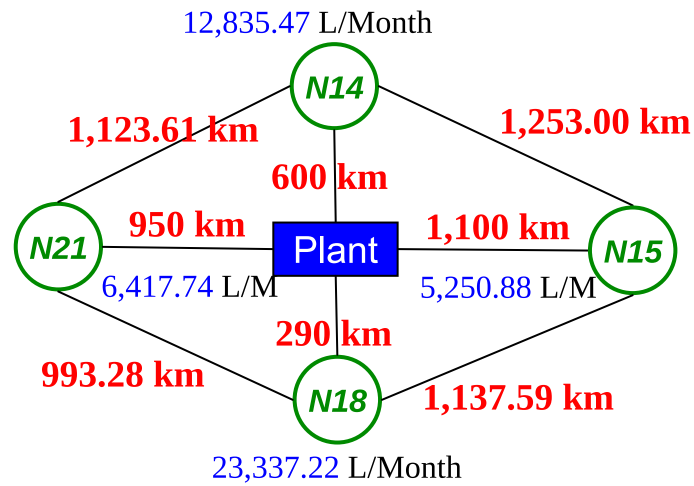
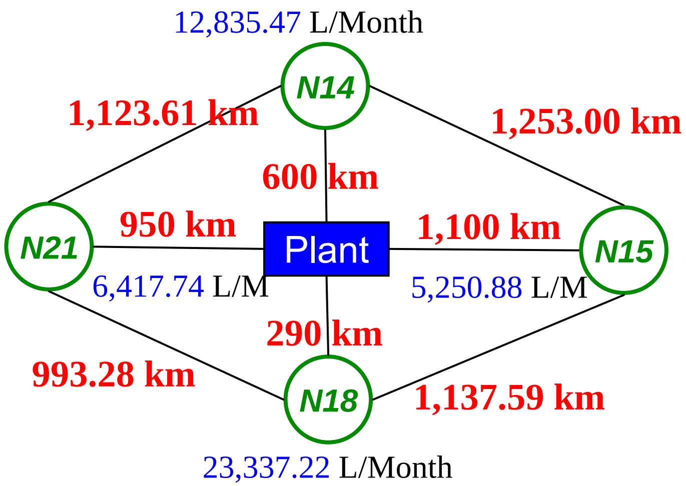
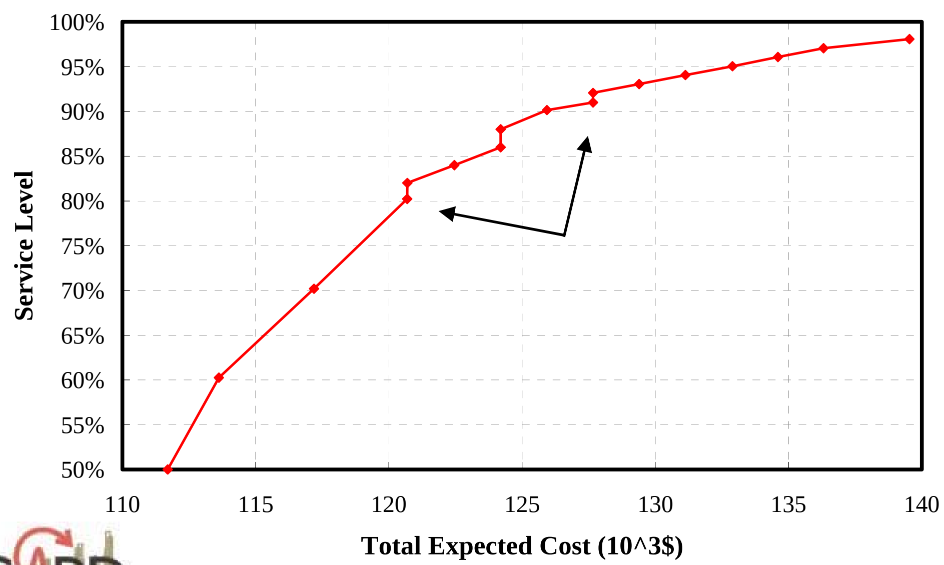


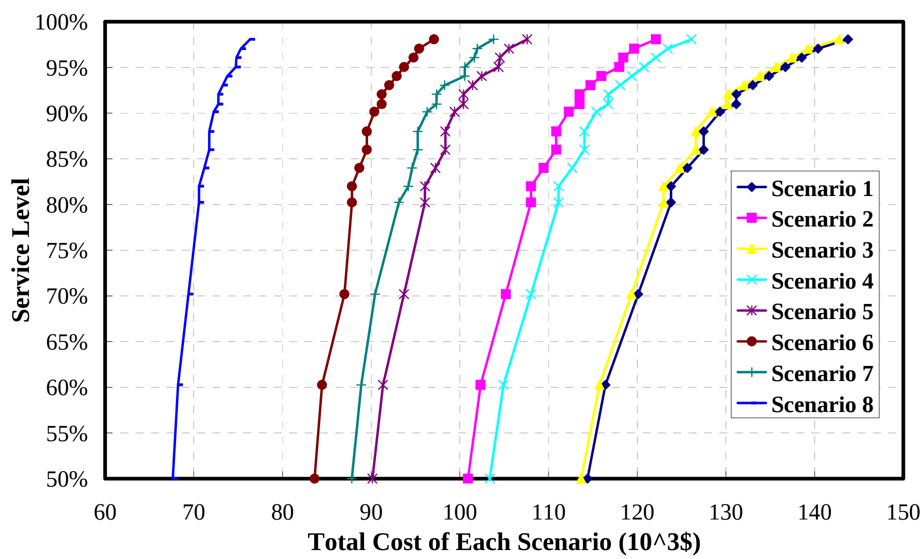
- 8 scenarios for two-stage stochastic programming
  - S1 – Y1: *N14*, *N15*, *N18*, *N21*; Y2: *N14*, *N15*, *N18*, *N21*; Y3: *N14*, *N15*, *N18*, *N21*.
  - S2 – Y1: *N14*, *N15*, *N18*, *N21*; Y2: *N14*, *N15*, *N18*, *N21*; Y3: *N14*, *N15*, *N18*, *N21*.
  - S3 – Y1: *N14*, *N15*, *N18*, *N21*; Y2: *N14*, *N15*, *N18*, *N21*; Y3: *N14*, *N15*, *N18*, *N21*.
  - S4 – Y1: *N14*, *N15*, *N18*, *N21*; Y2: *N14*, *N15*, *N18*, *N21*; Y3: *N14*, *N15*, *N18*, *N21*.
  - S5 – Y1: *N14*, *N15*, *N18*, *N21*; Y2: *N14*, *N15*, *N18*, *N21*; Y3: *N14*, *N15*, *N18*, *N21*.
  - S6 – Y1: *N14*, *N15*, *N18*, *N21*; Y2: *N14*, *N15*, *N18*, *N21*; Y3: *N14*, *N15*, *N18*, *N21*.
  - S7 – Y1: *N14*, *N15*, *N18*, *N21*; Y2: *N14*, *N15*, *N18*, *N21*; Y3: *N14*, *N15*, *N18*, *N21*.
  - S8 – Y1: *N14*, *N15*, *N18*, *N21*; Y2: *N14*, *N15*, *N18*, *N21*; Y3: *N14*, *N15*, *N18*, *N21*.

Pareto Curves and Scenario Costs

- CPU Times
  - Directly solved with DICOPT – infeasible
  - Directly solved with BARON – > 7 days
  - B&R algorithm (CPLEX 12 + DICOPT) - 285s for all the 17 instances

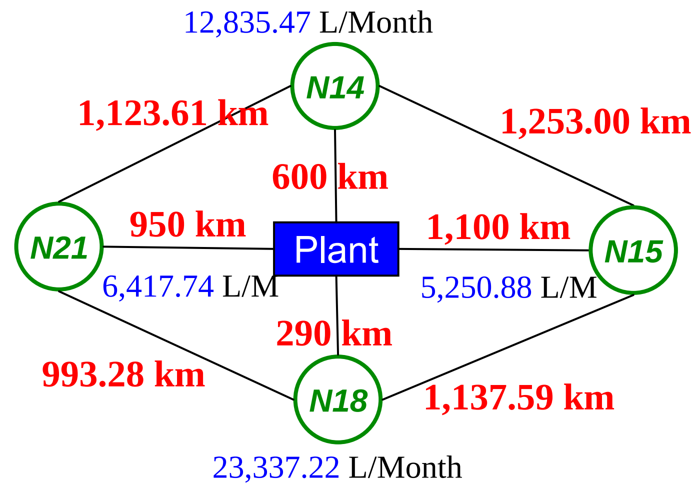


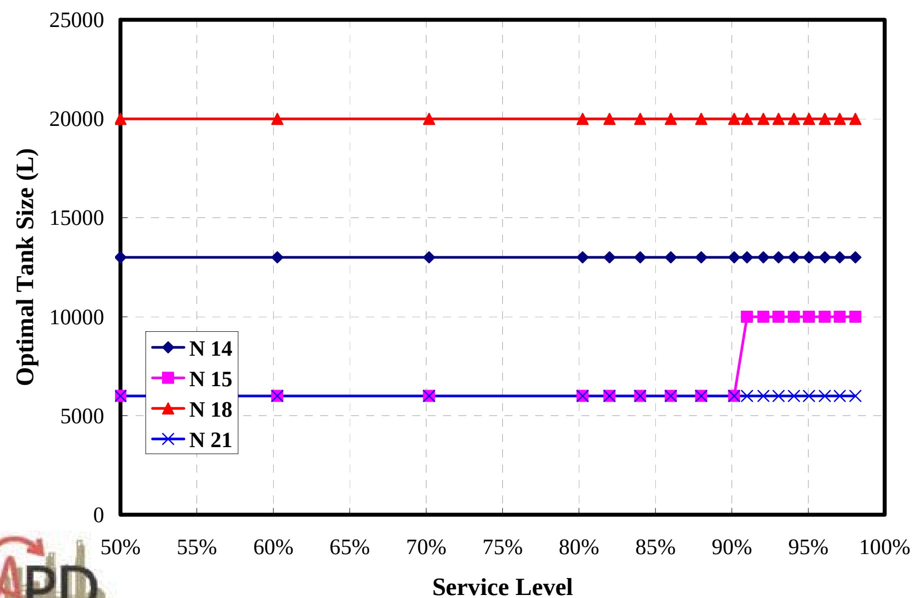


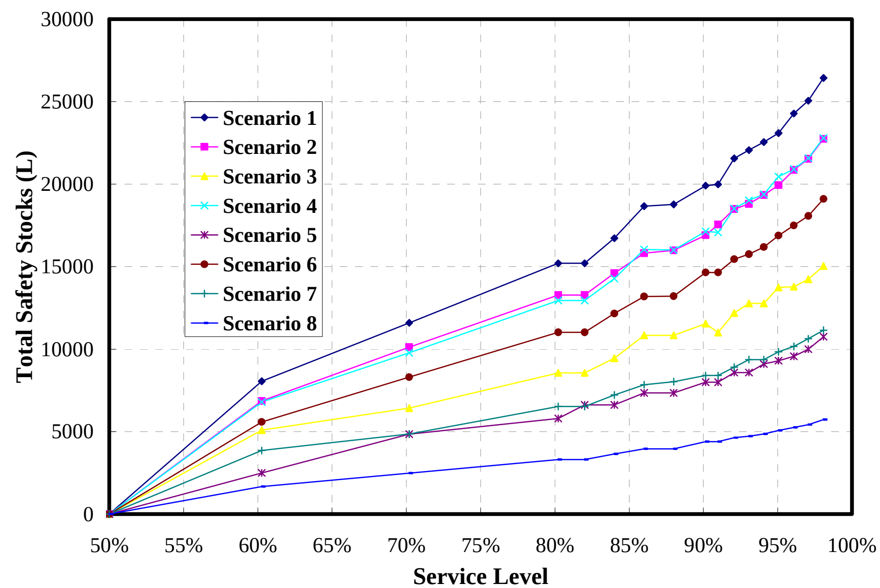


Tank Sizes and Safety Stocks

- Model Statistics
  - MINLP: 288 dis. var., 2,484 cont. var., 4,056 cons.
  - MILP model: 408 dis. var., 2,724 cont. var., 4,344 cons.; Reduced MINLP: 48 dis. var., 2,484 cont. var., 4,056 cons. (5 iter.)

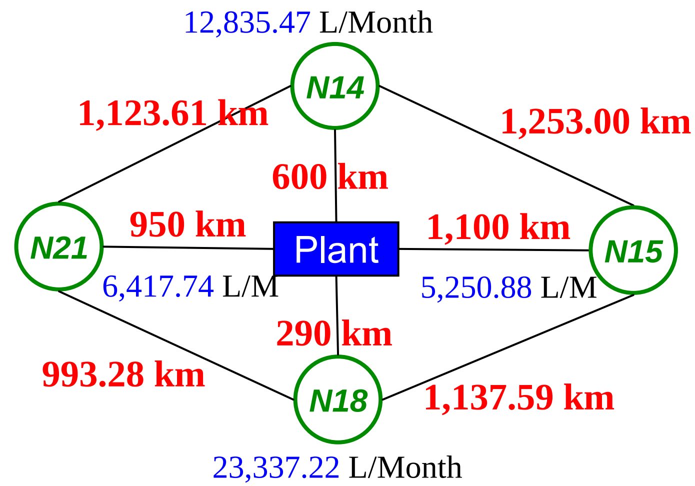


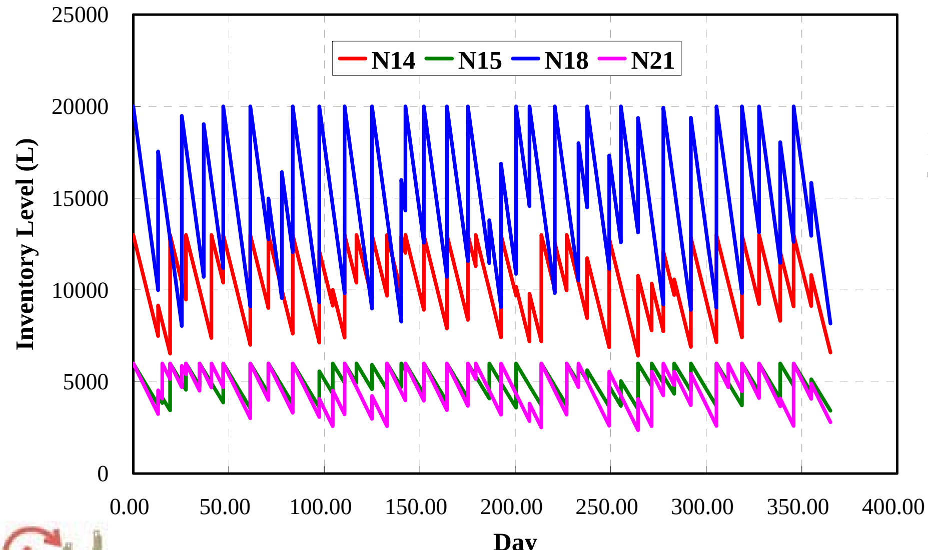




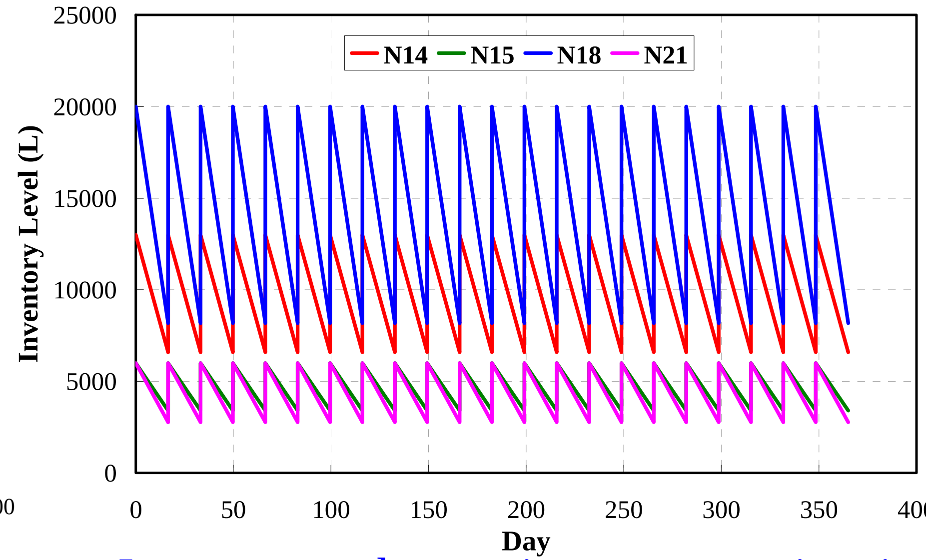
Optimal Routing Decisions

- Scenario 1 at Year 1, 90% Service Level
  - Number of visits:
    - *N14*: 17; *N15*: 17; *N18*: 37; *N21*: 18.
  - Three routes are used:
    - P → *N18* → *N15* → *N14* → *N21* → P.
    - P → *N18* → *N21* → P.
    - P → *N18* → P.





Inventory profile from detailed routing



Inventory under continuous approximation