Multi-Period Vehicle Routing with Stochastic Customers

Anirudh Subramanyam, Chrysanthos E. Gounaris

Carnegie Mellon University

Frank Mufalli, Jose M. Pinto

PRAXAIR

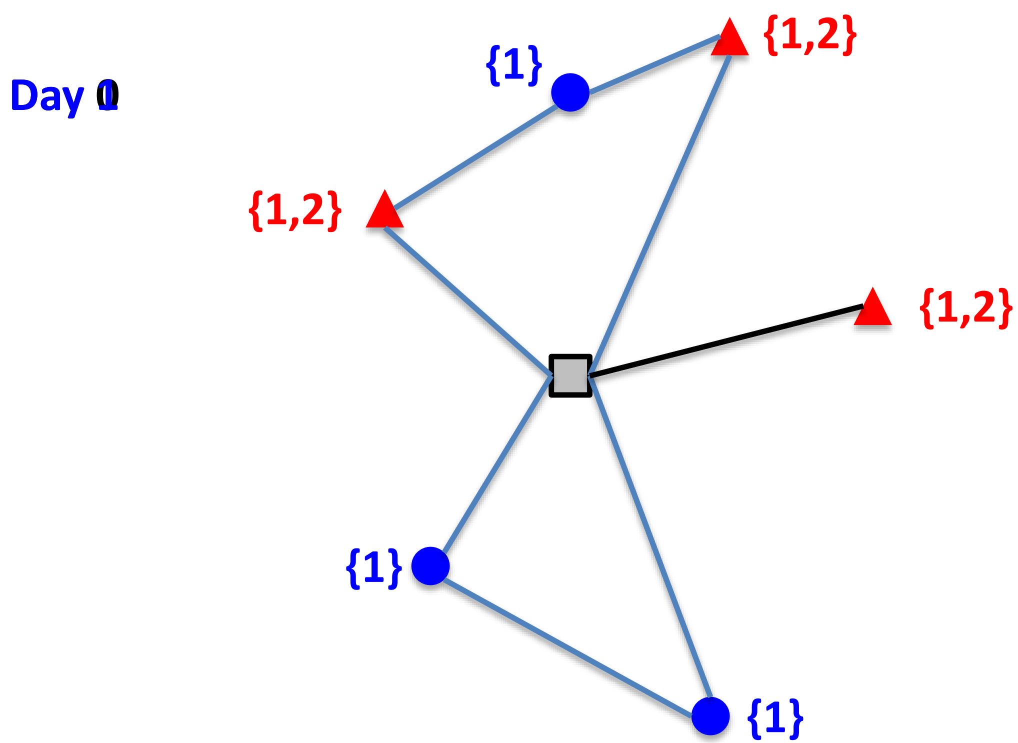
Making our planet more productive

EWO Spring Meeting

March 9, 2016

Tactical Planning in Vehicle Routing

- Given a set of customer requests
  - Assign a visit day to each customer over a short-term horizon
  - Design routes for each day in order to minimize sum of routing costs

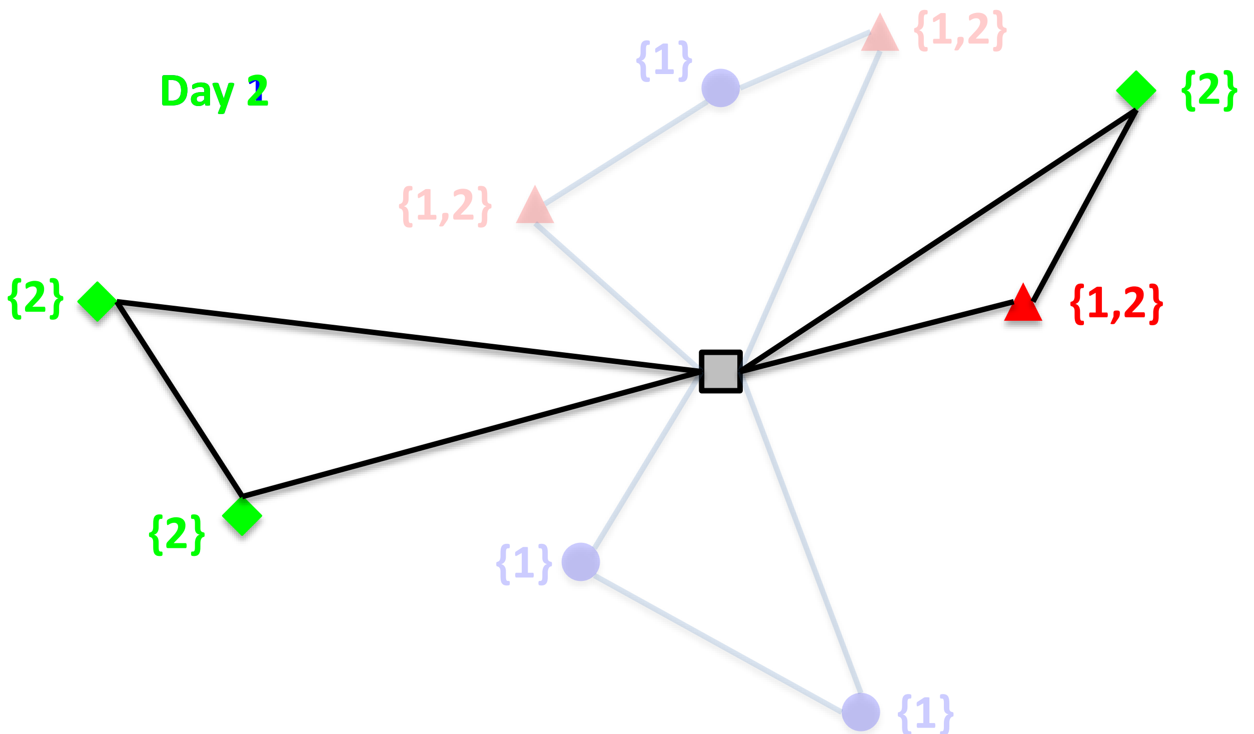


Tactical Planning in Vehicle Routing

- Given a set of customer requests
  - Assign a visit day to each customer over a short-term horizon
  - Design routes for each day in order to minimize sum of routing costs
- Plan is executed in a rolling horizon fashion
  - Routes of the first day are executed
  - New requests arrive and problem is re-solved on the updated portfolio

Tactical Planning in Vehicle Routing

- Given a set of customer requests
  - Assign a visit day to each customer over a short-term horizon
  - Design routes for each day in order to minimize sum of routing costs



Tactical Planning in Vehicle Routing

- Given a set of customer requests
  - Assign a visit day to each customer over a short-term horizon
  - Design routes for each day in order to minimize sum of routing costs

- Plan is executed in a rolling horizon fashion
  - Routes of the first day are executed
  - New requests arrive and problem is re-solved on the updated portfolio

- Single-day horizon reduces the problem to a standard VRP
  - Can be infeasible or too expensive
  - Actual setting may preclude single-day planning horizon

- Applications in chemical distribution (Wen et al, 2010), VMI systems (Coelho et al, 2012), automotive transportation (Cordeau et al, 2012)

Deterministic Tactical VRP

$$\min_{x,y,z} \sum_{p \in \mathcal{P}} \sum_{(i,j) \in E_p} c_{ij} x_{ijp}$$

s.t.

$$\sum_{j:(i,j) \in E_p} x_{ijp} = 2 \sum_{k \in \mathcal{K}} z_{ikp} = 2 y_{ip} \qquad \forall\, i \in V_p,\ \forall\, p$$

$$\sum_{j \in V_p} x_{0jp} = 2K \qquad \forall\, p \in \mathcal{P}$$

$$\sum_{p=e_i}^{l_i} y_{ip} = 1 \qquad \forall\, i \in V_C$$

$$\boxed{\sum_{(i,j) \in \delta_p(S)} x_{ijp} + \sum_{i \in S} (1 - y_{ip}) \geq 2 \left\lceil \frac{1}{Q} \sum_{i \in S} q_i \right\rceil \qquad \forall\, S \subseteq V_p,\ \forall\, p \in \mathcal{P}}$$

$$1 - x_{ijp} \geq \max\{z_{ikp} - z_{jkp}, z_{jkp} - z_{ikp}\}$$

$$z_{ikp} + \sum_{l=1}^{k} z_{jlp} \leq 3 - x_{0ip} - x_{0jp} \qquad \forall\, (i,j) \in$$

Assignment constraints

Feasible day window

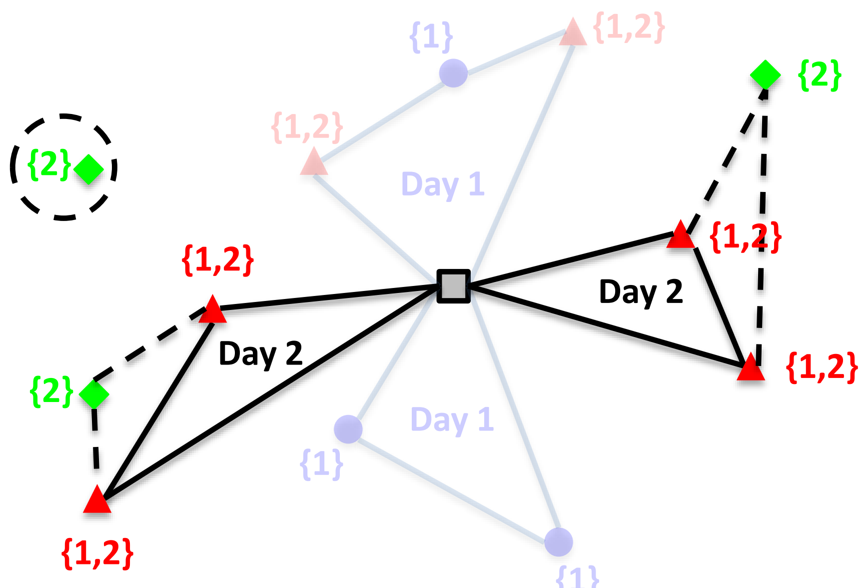
Generalized Rounded Capacity Inequalities

Vehicle-customer assignments

Call-In Customers

- Customers who are not scheduled to be visited now but might potentially “call in”, requesting service in a future time period
- The above framework is completely agnostic to future orders within the planning horizon
  - Myopically optimizes based on current information
  - Can generate routing plans that are infeasible or too expensive

Non-robust

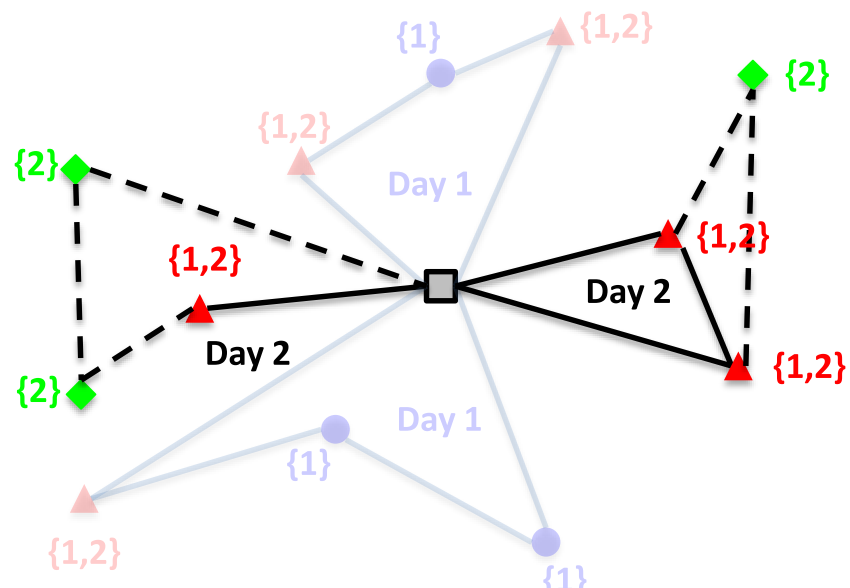


Call-In Customers

- Customers who are not scheduled to be visited now but might potentially "call in", requesting service in a future time period

- The above framework is completely agnostic to future orders within the planning horizon
  - Myopically optimizes based on current information
  - Can generate routing plans that are infeasible or too expensive

Robust



Call-In Customers

- Customers who are not scheduled to be visited now but might potentially "call in", requesting service in a future time period

- The above framework is completely agnostic to future orders within the planning horizon
  - Myopically optimizes based on current information
  - Can generate routing plans that are infeasible or too expensive

- Challenges

  … Characterize discrete (yes/no) nature of uncertainty

  … Build tractable models that insure against such uncertainty

  … Avoid being conservative as much as possible

Characterizing Discrete Uncertainty

- All call-in orders are assumed to come from a database of "potential orders" V_O

- Each call-in customer c_o places an order o, associated with
  - Call-in date d_o
  - Demand q_o
  - Service Day Window $[e_o, l_o]$

- Can account for multiple orders placed by the same customer
  - Duplicate $o_1, o_2, \ldots$ with different day windows

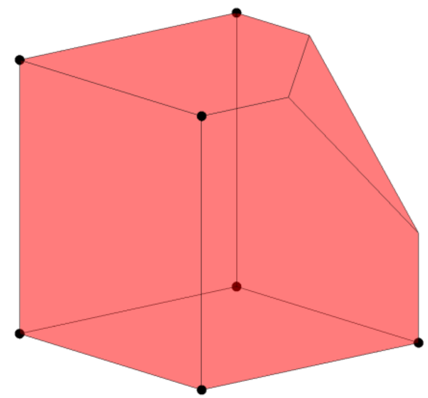
- All call-in orders satisfy $e_o \geq 2$, since all customers of day 1 ("today") are known at time of optimization

Characterizing Discrete Uncertainty

- Uncertainty set

$$\Xi = \{\boldsymbol{\xi} \in \{0,1\}^{|V_O|} : A\boldsymbol{\xi} \leq b\}$$

  - Finite collection of relevant scenarios
  - Each element is a indicates which orders can be realized together throughout the planning horizon



- The above representation can capture practically-meaningful scenarios

…budget of orders throughout the week

$$\Xi = \left\{\boldsymbol{\xi} \in \{0,1\}^{|V_O|} : \sum_{o \in V_O} \xi_o \leq \Gamma\right\}$$

…budget of calls on any day

$$\Xi = \left\{\boldsymbol{\xi} \in \{0,1\}^{|V_O|} : \sum_{o \in V_O : d_o = p} \xi_o \leq \Gamma_p \quad \forall\, p = \{1, \ldots, h\}\right\}$$

…geographical budgets

$$\Xi = \left\{\boldsymbol{\xi} \in \{0,1\}^{|V_O|} : \sum_{o \in V_O \cap B_l} \xi_o \leq b_l \quad \forall\, l = \{1, \ldots, L\}\right\}$$

…budget of orders from same customer

$$\Xi = \left\{\boldsymbol{\xi} \in \{0,1\}^{|V_O|} : \sum_{o \in V_O : c_o = c} \xi_o \leq 1 \quad \forall\, c\right\}$$

Robust Formulation

- Routing plan must remain feasible for any realization of orders

  …must have enough fleet capacity to accommodate call-in demand in future time periods

$$\forall\, \boldsymbol{\xi} \in \Xi \quad \exists\, \boldsymbol{\vartheta}(\xi) : \left[\begin{array}{l} \vartheta_{okp}(\boldsymbol{\xi}) \in \{0,1\} \quad \forall\, o \in O_p,\ \forall\, k \in K,\ \forall\, p \in P \\ \sum_{k \in K} \sum_{p=e_o}^{l_o} \vartheta_{okp}(\boldsymbol{\xi}) = \xi_o \quad \forall\, o \in V_O \\ \sum_{i \in V_p} q_i z_{ikp} + \sum_{o \in O_p} q_o \vartheta_{okp}(\boldsymbol{\xi}) \leq Q \quad \forall\, k \in K,\ \forall\, p \in P \end{array}\right]$$

- Static policy likely to be infeasible

- A first adjustable robust approach $\vartheta_{okp}(\boldsymbol{\xi}) = \theta_{okp}\xi_o,\ \forall\, o \in V_O$, where $\theta_{okp} \in \{0,1\}$
  - always assign o to the same vehicle k in the same time period p irrespective of the realization that o is a part of

Robust Formulation

- Routing plan must remain feasible for any realization of orders

  … must have enough fleet capacity to accommodate call-in demand in future time periods

- $\theta_{okp} \in \{0,1\}$ indicates which vehicle we "virtually" assign to order o

$$\sum_{p=e_o}^{l_o} \sum_{k \in \mathcal{K}} \theta_{okp} = 1 \qquad \forall\, o \in V_O$$

- Add robust capacity constraint to deterministic problem:

$$\sum_{i \in V_p} q_i z_{ikp} + \sum_{o \in O_p} q_o \theta_{okp} \xi_o \leq Q \qquad \forall\, k,\ \forall\, p,\ \forall\, \boldsymbol{\xi} \in \Xi$$

  - Resulting model is a semi-infinite MILP – cutting plane algorithm
  - *Disjoint budgets:* Can be reformulated into a finite MILP
  - Note: deterministic problem already has $\mathcal{O}(2^n)$ inequalities

Computational Results

- Database of 5 VRP benchmark instances
  - 20 – 100 fixed/deterministic customers, 2 – 4 vehicles
  - Extended into 5-period problems

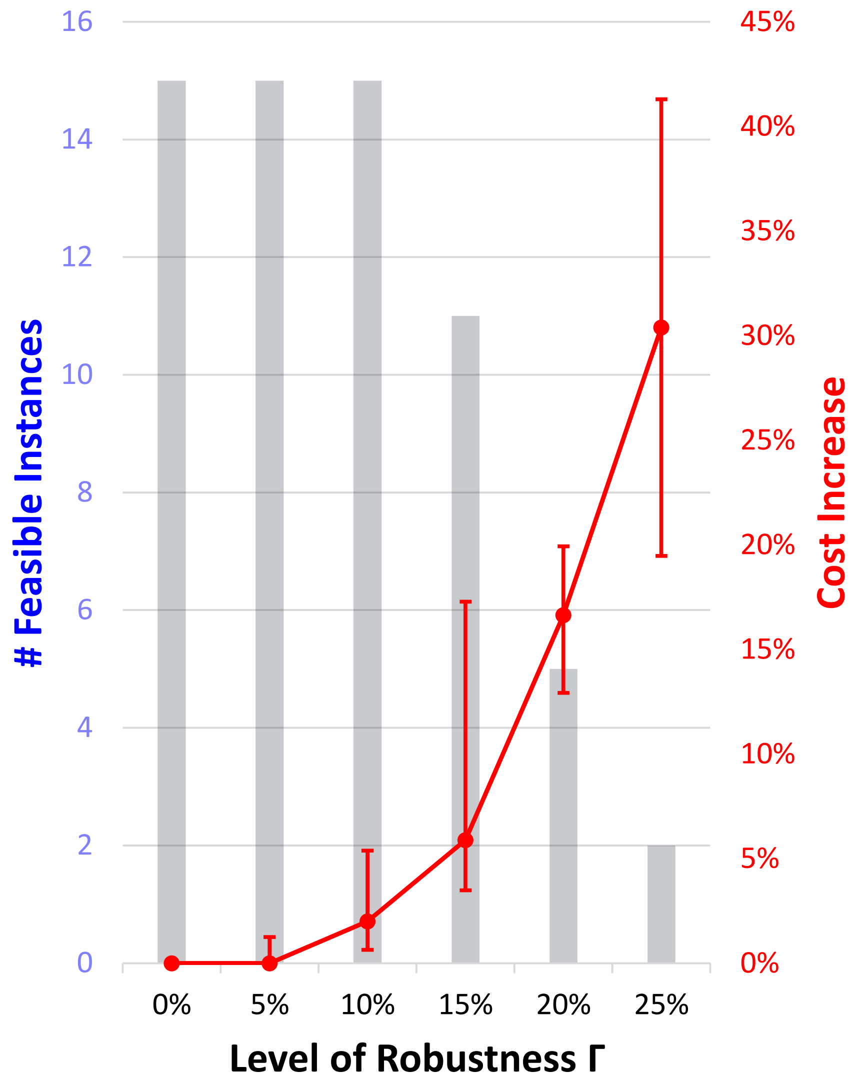
- For each instance, we constructed a large database V_O of orders
  - Day windows randomly assigned with maximum window of 3 days
  - Demands vary between $\left[\underline{q}, \overline{q}\right]$ with respect to deterministic customers

- Budget of orders throughout the week $\Xi = \left\{ \boldsymbol{\xi} \in \{0,1\}^{|V_O|} : \sum_{o \in V_O} \xi_o \leq \Gamma \right\}$
  - Γ varies between 0% and 25% of the no. of fixed customers

- 90 problems

Computational Results

C++/CPLEX 12.6 – 10 CPU minutes



Computational Results

C++/CPLEX 12.6 – 10 CPU minutes

Γ (%)	# Feasible (out of 15)	% Cost increase	# Proven optimal	t (sec)	# Reached time limit	Residual gap (%)
0	15	0.0	11	23.5	4	3.1
5	15	0.0	9	17.5	6	4.8
10	15	2.0	9	81.4	6	8.2
15	12	5.9	5	36.6	7	9.1
20	8	16.6	2	0.1	6	8.3
25	3	30.4	2	8.6	1	11.4

Table 1: Summary across 90 problems.

Conclusions

- Ignoring uncertainty in future customer requests can lead to infeasible or highly expensive routing plans in multi-period VRPs

- We developed an adjustable robust optimization framework to account for the uncertainty of future call-in customers

- A robust plan can be obtained with a small increase in routing costs

- Future work
  - *Improve conservatism* of the proposed framework
  - *Reduce the number of "relevant" realizations* Ξ' using dominance rules
  - *Better formulations/decomposition techniques* to solve larger instances
  - *Industrial case study*