Robust Multi-Period Vehicle Routing under Customer Order Uncertainty

Anirudh Subramanyam, Chrysanthos E. Gounaris
Carnegie Mellon University

Frank Mufalli, Jose L. Aguirre, Jose M. Pinto
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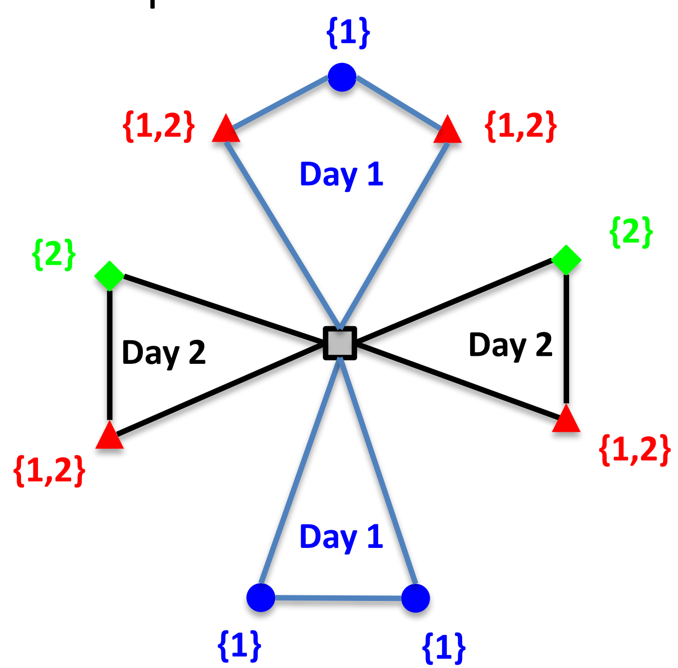
Multi-Period Vehicle Routing

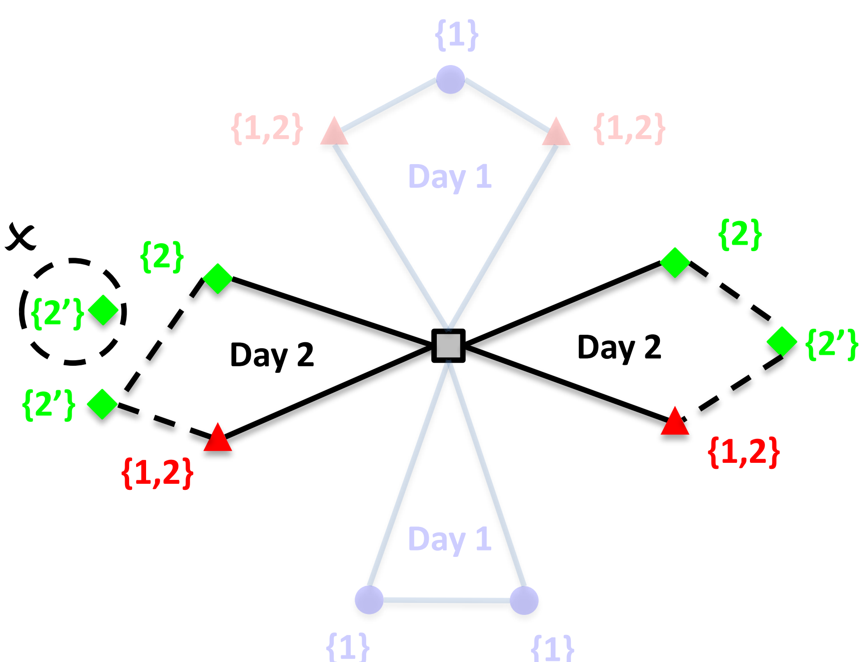
- Given a set of customer orders over multiple days
  - Assign a visit day to each customer
  - Design routes for each day in order to minimize sum of routing costs
- Plan is executed in a rolling horizon fashion
  - Routes of the first day are executed
  - New orders arrive and problem is re-solved on the updated portfolio



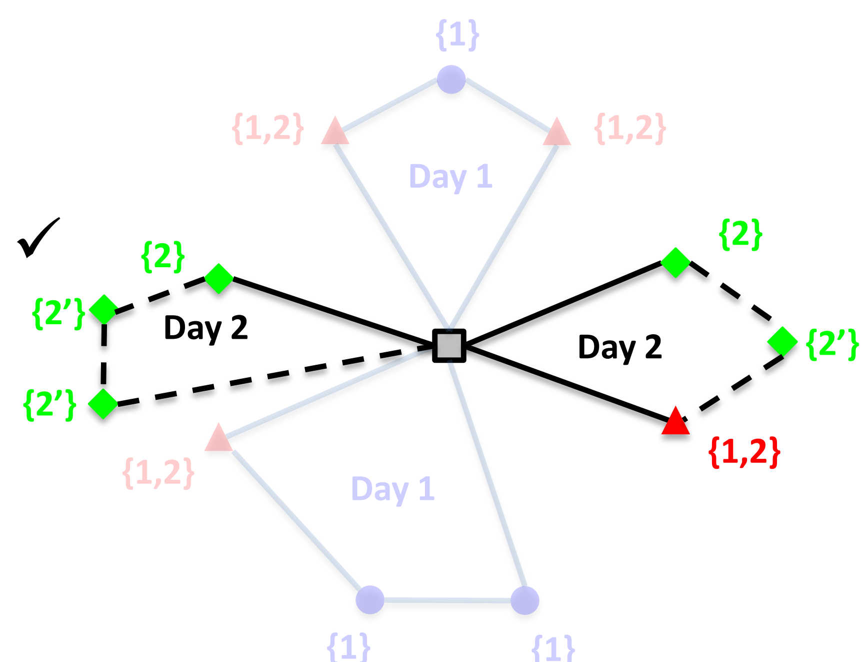
Multi-Period Vehicle Routing

- The deterministic framework myopically optimizes based on current information, and can generate infeasible or too expensive plans

Non-robust



Robust



Challenges

- Devise **suitable objective and constraints** to address problem
- Characterize **discrete nature of uncertainty**
- Build **tractable algorithms** to solve practically useful instances
- Quantify **degree of conservatism**
- Evaluate framework through **rolling-horizon simulations**

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Robust Multi-Period Vehicle Routing

- Given a set of customer orders over multiple days *AND* given potential scenarios of call-in orders over the planning horizon
  - Assign a visit day to each customer
  - Design routes for each day in order to minimize sum of routing costs
  - Ensure that each scenario can be accommodated in the designed routes

Deterministic Tactical Planning Vehicle Routing Problem

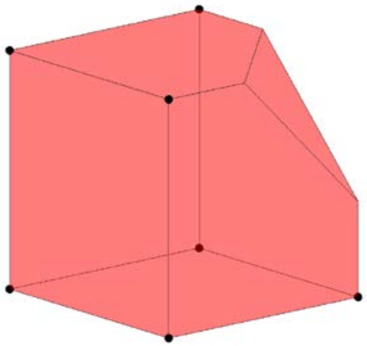
+

Robust Bin Packing Problem

- Reserve enough fleet capacity to insure against any call-in scenario
- Do not account for excess routing costs to serve call-in customers

Characterizing Discrete Uncertainty

- All orders are assumed to come from a database of potential orders V_C
  - $V_C = V_0 \cup V_1 \cup \cdots \cup V_h$ where V_0 = customers who have already called
  - V_p = customers who can potentially call on day $p \in \mathcal{P}$



- Uncertainty set $\Xi = \left\{ \boldsymbol{\xi} \in \{0,1\}^{|V_C|} : A\boldsymbol{\xi} \leq b \right\}$
  - Each element indicates which orders can be realized together throughout the planning horizon
  - This representation can capture practically-meaningful scenarios

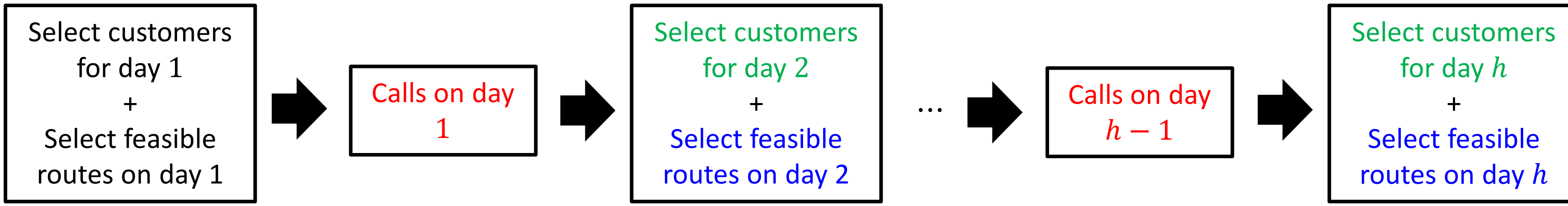
    …budget of orders throughout the week $\sum_{i \in V_C} \xi_i \leq \Gamma$

    …budget of calls on any day $\sum_{i \in V_p} \xi_i \leq \Gamma_p \quad \forall\, p \in \mathcal{P}$

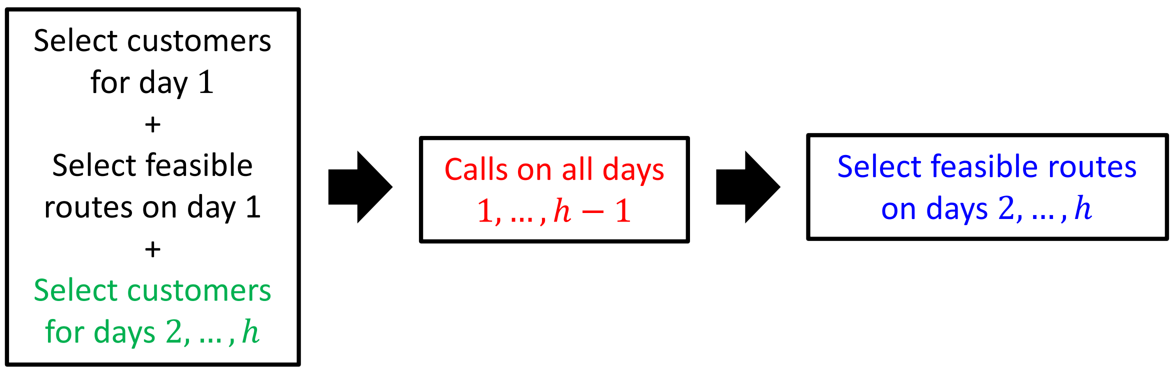
    …geographical budgets $\sum_{i \in B_g} \xi_i \leq b_g \quad \forall\, g \in \{1, \ldots, G\}$

Adaptive Robust Optimization Models

- ***Multi-stage fully adaptive model* (MSRO) – *intractable!***



- ***Two-stage non-anticipative (partially) adaptive model* ($\overline{\text{TSRO}}$) – *tractable!***



- Add the following **robust cover inequalities** to the deterministic model

$$m + \sum_{i \in S} (1 - y_{ip}) \geq \text{BPP}(S, \xi) \qquad \forall\, S \subseteq V_C,\ \forall\, p \in \mathcal{P},\ \forall\, \xi \in \Xi$$

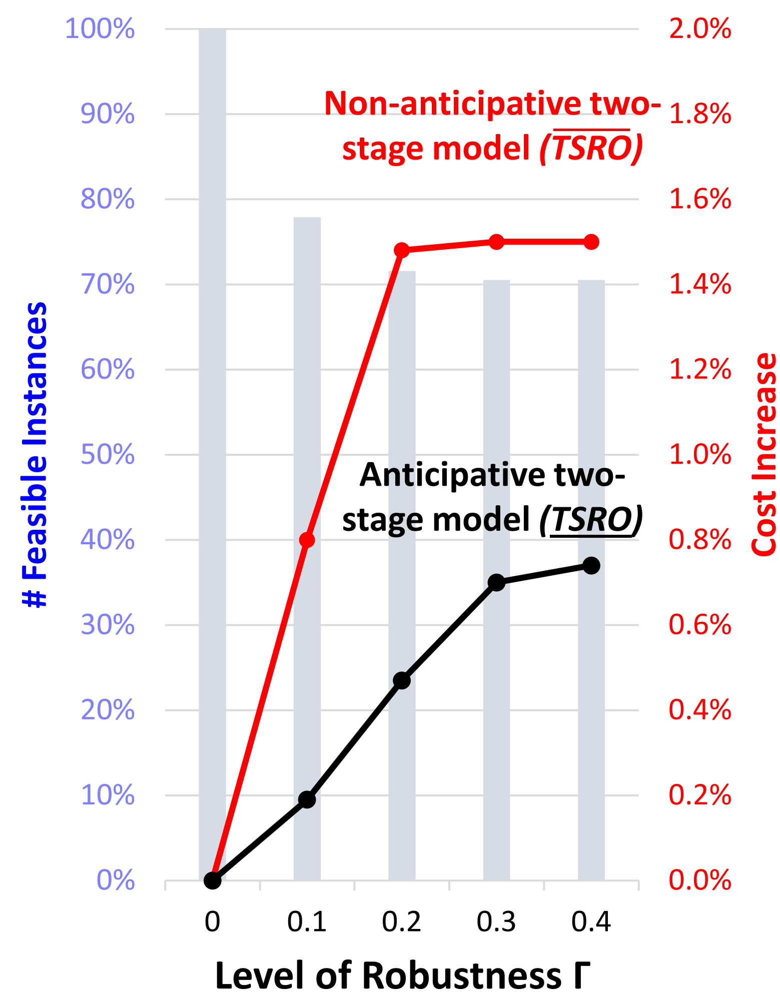
$\mathcal{O}(2^n)$
branch-and-cut

  – $\text{BPP}(S, \xi)$ = value of *Bin Packing Problem* with bin size = Q (vehicle capacity), and item size = $q_i \xi_i$ (demand quantity) for each $i \in S$

Computational Results

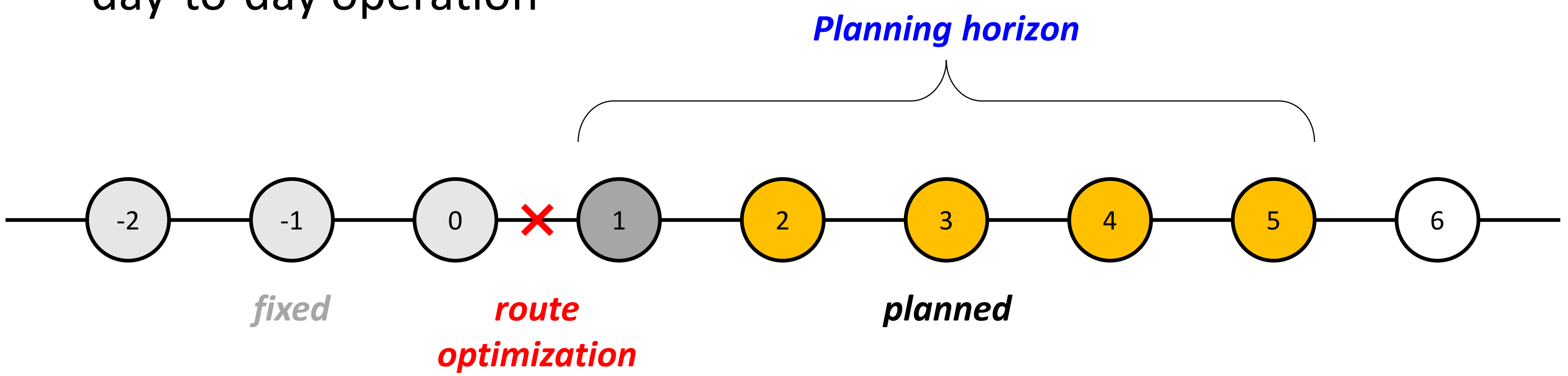
- 67 benchmark instances
- 5 periods
- 15–120 known customers
- 30–240 call-in customers
- 2–15 vehicles
- Budget Γ of calls made on each day of the week
- C++/CPLEX, 2hr. time limit

	Proven optimal (54 out of 67)		Reached time limit (13 out of 67)
Γ	Avg. nodes	Avg. time (sec)	Avg. gap (%)
	Deterministic setting		
0	2,050	232.8	2.73
	Robust setting		
0.1	4,886	534.2	3.66
0.2	2,705	314.9	3.60
0.3	2,424	302.6	3.26
0.4	2,666	335.3	3.21



In Progress: Rolling-Horizon Studies

- Develop a rolling-horizon simulation platform to mimic the actual day-to-day operation



- Computational studies were designed to
  - Compare the performance of various solution approaches (e.g., deterministic, robust, “greedy”)
  - Investigate the effect of length of planning horizon

- Tradeoff: numerical tractability vs. “closed loop” feasibility/performance

In Progress: Rolling-Horizon Studies

- Evaluation metrics
  - Frequency of infeasibility
  - Extent of infeasibility: incorporate penalty costs for unserved demand
  - Total costs: actual incurred costs + penalty costs

- Design of benchmark instances
  - Percentage of customers with 1-day and 2-day windows
  - Tightness of fleet capacity
  - Construction of uncertainty sets from historical data

Conclusions

- Ignoring uncertainty in future customer requests can lead to infeasible or highly expensive routing plans in multi-period VRPs

- We developed a robust optimization framework to account for the uncertainty of future call-in customers
  - The "price of robustness" is $\sim 1.6\%$ for $\Gamma \leq 40\%$
  - The gap from the true multi-stage adaptive solution is less than 1%

- *"Robust Multi-Period Vehicle Routing Problems under Customer Order Uncertainty"*, A. Subramanyam, F. Mufalli, J.M. Pinto, C.E., Gounaris, *submitted for publication*.

- Rolling-horizon simulation studies need to be completed to determine actual costs incurred and the best decision-making setup