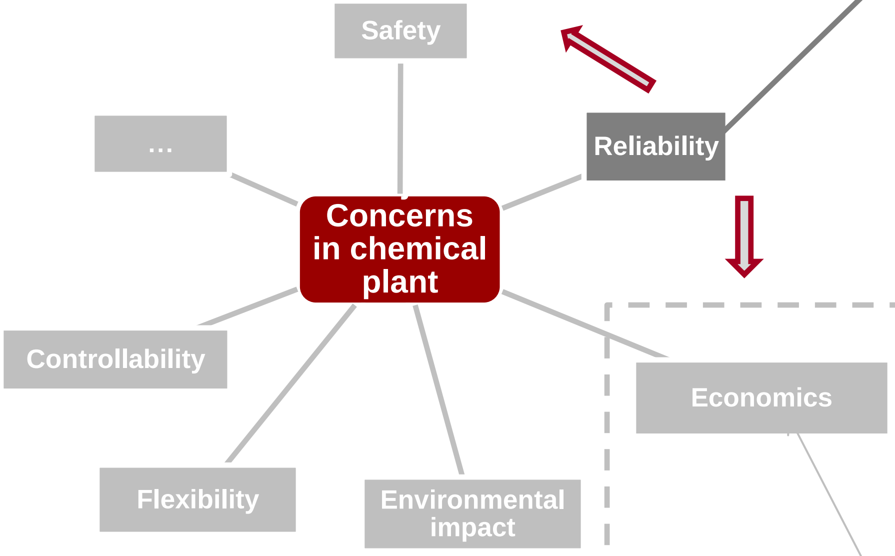
Mixed-integer programming models for optimal design of reliable chemical plants

Yixin Ye and Ignacio. E. Grossmann
Department of Chemical Engineering, Carnegie Mellon University

Sivaraman Ramaswamy and Jose M. Pinto
Business and Supply Chain Optimization R&D, Praxair

Design of chemical plants

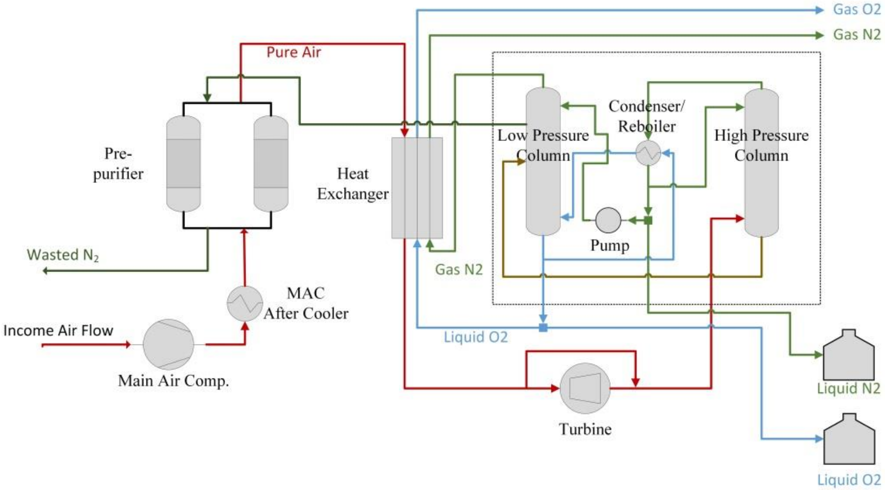


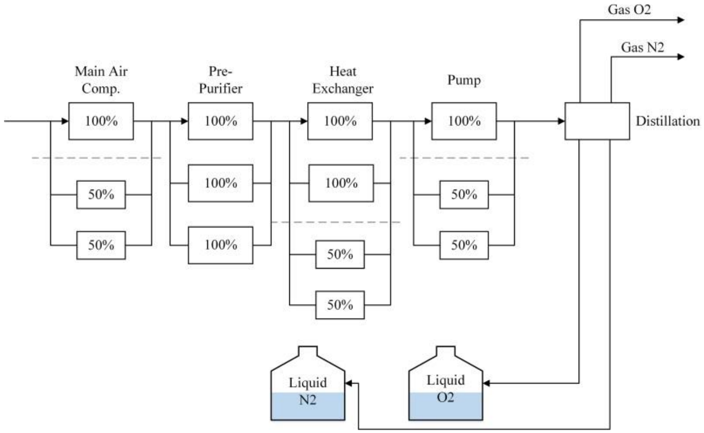
Reliability: The ability of a system or component to perform its required functions under stated conditions for a specified time.

A major factor of reliability that greatly impacts the economic efficiencies

Availability: The long-term probability that a system is capable of working properly

Motivating example





Goal: **General rigorous models for optimal design of a chemical process (**in terms of equipment redundancy, storage capacity and production planning**) regarding availability and economics**

Last progress

Continuous-time Markov Chain

Design decision +

Pseudo transition matrixes of stage ***k*** QM_k

↓

Pseudo transition matrix of system $\overline{QM}$

⇒ **Transition matrix of system** $\overline{Q}$

$$\pi \overline{Q} = 0$$

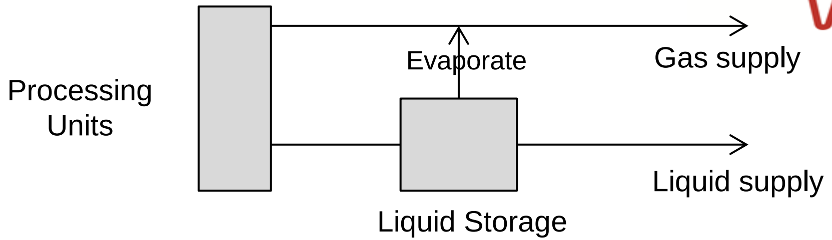
⇒ **Stationary probability distribution** π

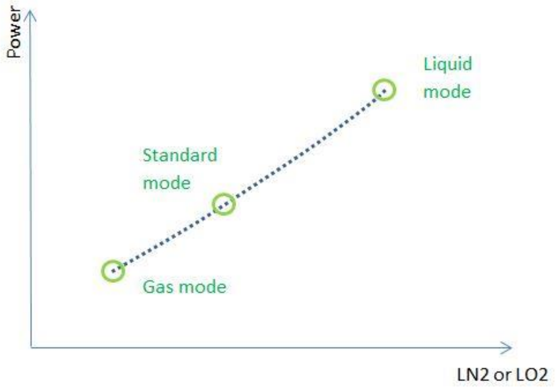
Availability = 1 - probability of failed states

New features

New method to increase availability:
Liquid storage

New trade-off:
Spending more money to install back-up unit
V. S.
Consuming more power to operate closer to liquid mode





More backup units ⟶ Higher availability ⟶ Lower storage to keep ⟶ Less liquid to produce

Model formulation

Material Balance

$$\delta_{n,\bar{s}} = f^{IN}_{n,\bar{s}} - f^{OUT}_{n,\bar{s}}, \quad \forall n \in N, \bar{s} \in \bar{S}$$

$$f^{IN}_{n,\bar{s}} = t^G flow^G_n + t^L flow^L_n, \quad \forall n \in N, \bar{s} \in \bar{S} \setminus (\bar{S}^f \cup \bar{S}^{half})$$

$$f^{IN}_{n,\bar{s}} = (t^G flow^G_n + t^L flow^L_n)/2, \quad \forall n \in N, \bar{s} \in \bar{S}^{half}$$

$$f^{IN}_{n,\bar{s}} = 0, \quad \bar{s} \in \bar{S}^f$$

$$f^{OUT}_{n,\bar{s}} = dm^L_n, \quad \forall n \in N, \bar{s} \in \bar{S} \setminus (\bar{S}^f \cup \bar{S}^{half})$$

$$f^{OUT}_{n,\bar{s}} = dm^G_n / 2 + dm^L_n, \quad \forall n \in N, \bar{s} \in \bar{S}^{half}$$

$$f^{OUT}_{n,\bar{s}} = dm^G_n + dm^L_n, \quad \forall n \in N, \bar{s} \in \bar{S}^f$$

Inventory calculation

$$\overline{inv} = inv_0 + \sum_{\bar{s} \in \bar{S}} \delta_{\bar{s}} mrt_{\bar{s}} fr_{\bar{s}} t$$

$$\sum_{\bar{s} \in \bar{S}} \delta_{n,\bar{s}} mrt_s fr_{\bar{s}} \le \delta^+, \quad \forall n \in N$$

$$\sum_{\bar{s} \in \bar{S}} \delta_{n,\bar{s}} mrt_s fr_{\bar{s}} \ge -\delta^-, \quad \forall n \in N$$

$$sd_n = \sum_{\bar{s} \in \bar{S}} \pi_{\bar{s}} t \, |\delta_{n,\bar{s}}|, \quad \forall n \in N$$

$$\overline{inv}_n + \tau sd_n \le \nu_n, \quad \forall n \in N$$

$$\overline{inv}_n - \tau sd_n \ge 0, \quad \forall n \in N$$

$$\overline{inv}_n + \delta_{n,\bar{s}} mrt_{\bar{s}} \le \nu_n, \quad \forall n \in N$$

$$\overline{inv}_n - \delta_{n,\bar{s}} mrt_{\bar{s}} \ge 0, \quad \forall n \in N$$

$$A = 2(1/(1+e^{-0.07056\tau^3 - 1.5976\tau}) - 0.5)$$

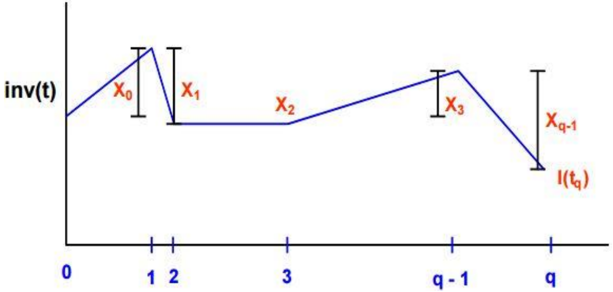
Additional costs

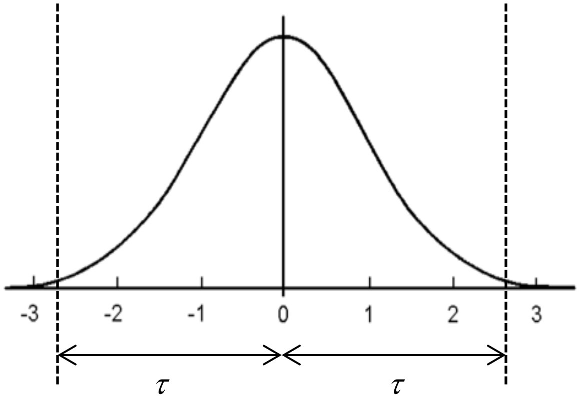
$$Op = (t^G c\_op^G + t^L c\_op^L) T$$

$$Inv = \sum_{n \in N} \nu_n c\_inv_n$$

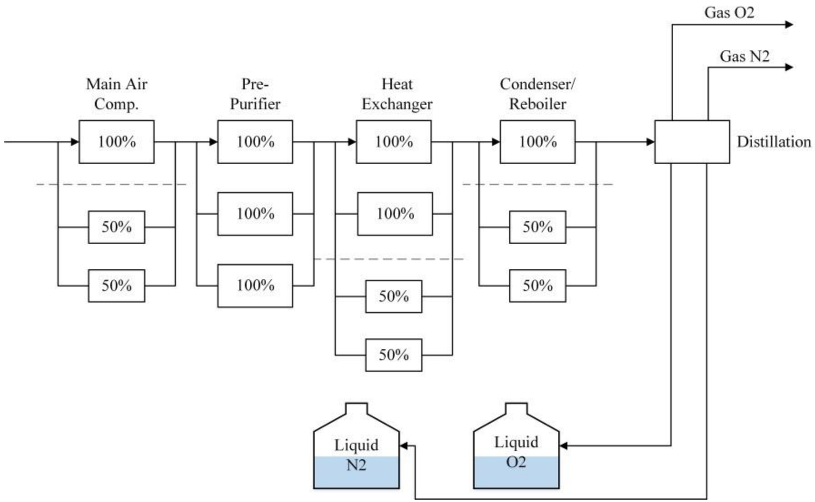
Objective function

$$\max \quad NP = rvA - C^{tot} - Op - Inv$$





Solving the motivating example



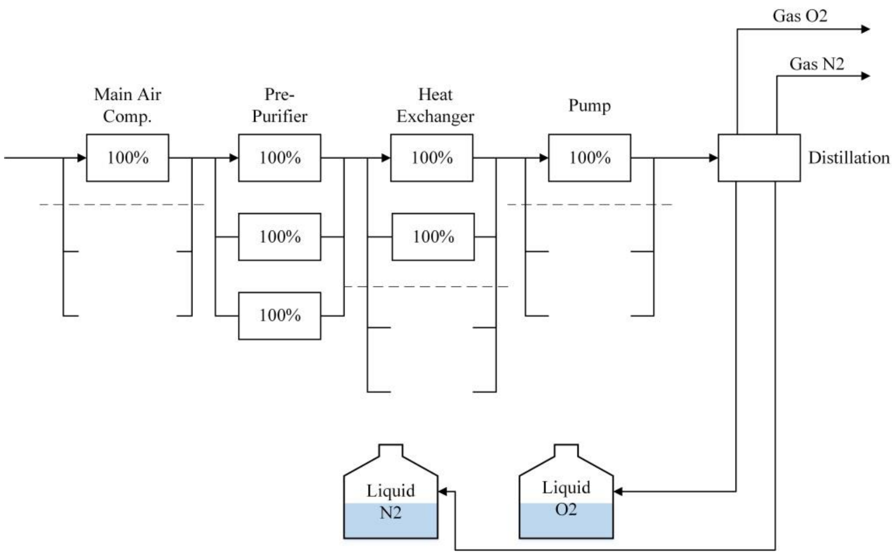
Parameters

Product	Standard Mode (t/h)	Liquid Mode (t/h)
Gas O2	2	2
Liquid O2	25	150
Gas N2	4	4
Liquid N2	70	250

	Cost rate of standard mode(k$/day)	Cost rate of liquid mode(k$/day)	Revenue rate(k$/day)	Time Horizon(day)
O2	50	300	200	100
N2	50	400	200	

Results

Net profit = 13421k$ **Availability =** 0.998



Results

Availability	Truncation(τ)	O2 Storage(kt)	N2 Storage(kt)	Net profit (k$)
0.998	3.091	17	47	13421

Computational data

No. Eqs	No. Vars	No. Discrete vars	CPU time(s)
98978	60544	27	102.55

Commercial solvers BARON 14.4.0 and DICOPT(based on CONOPT 3.16D and CPLEX 12.6.1.0) are used.

Conclusion

- Insights
  - Proposed an MINLP model based on Continuous-time Markov Chain
  - Solved the model that makes decisions including backup unit selection, storage capacity design and average operation point selection.
  - Devised general framework and adapted it for the specific case of air separation plants
- Future works
  - To reduce problem scale and solving time
  - To investigate more complex system structures
  - To incorporate preventive maintenance scheduling