Multiperiod Blend Scheduling Problem

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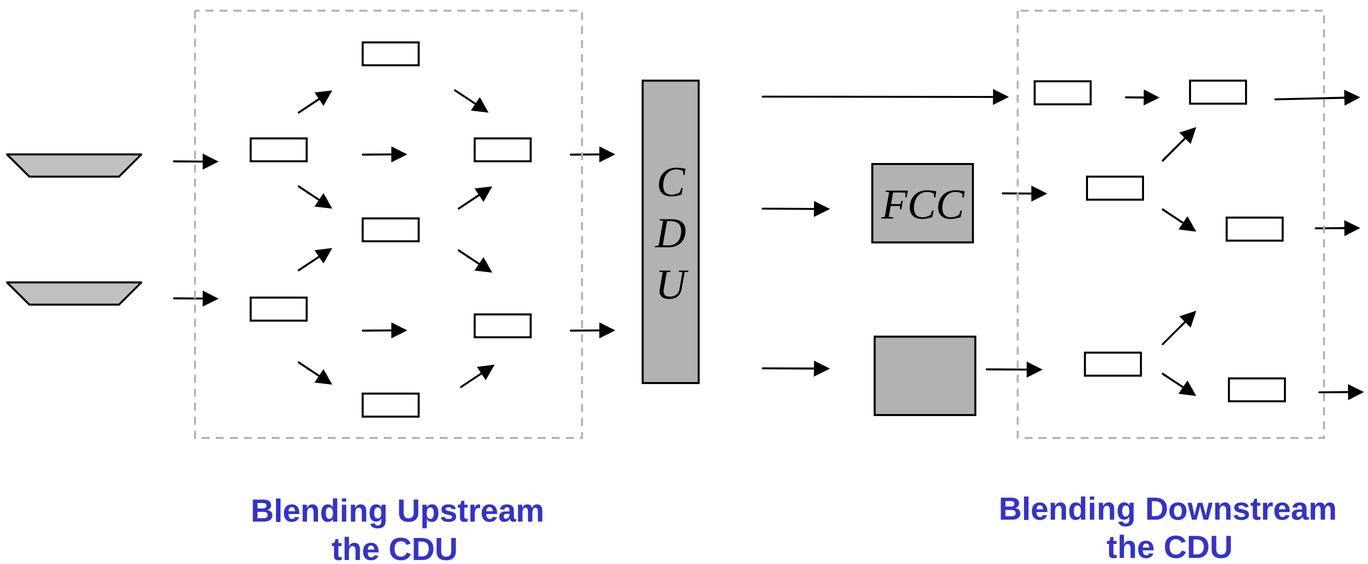
Motivation

-The **scheduling problem of blending operations** arises **frequently** in the petrochemical industry.

-**Large cost savings** can be achieved if the correct blending decisions are taken.

-Although simple to represent, these **models are highly nonconvex**, leading to the need of **global optimization** techniques to find the optimal solution.

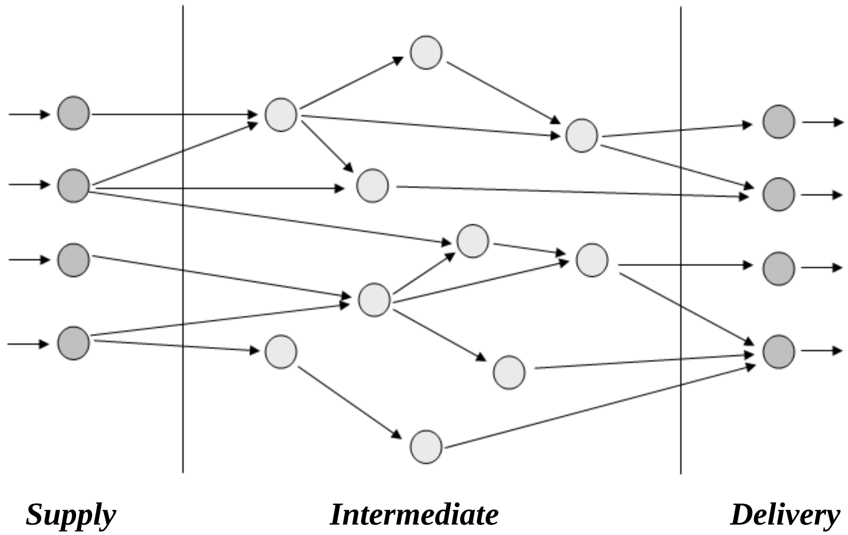
-The development of **efficient solution methods** that take care of the general case applied to large scale systems **remains as a challenge**…



Goal: *Develop* ***tools and strategies*** *aiming at* ***improving the efficiency*** *of the solution methods for the global optimization of the* ***multiperiod blend scheduling problem***

General Problem Topology

The **general case** of a blending problem can be represented schematically as follows



Remarks:

- Examples of the **supply nodes** are the tanks loaded by ships or the feedstocks downstream the CDU

- Examples of the **delivery nodes** are the tanks feeding the CDU or the tanks delivering to final customers

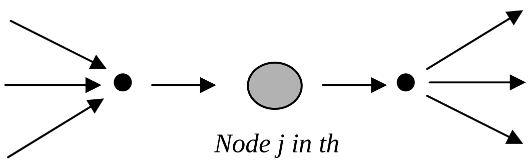
Important Assumptions:

- The **quality** of each stream/inventory is **constant** for a given **period**.
- A tank can **receive or deliver** in a given period of time but **not both**.
- **Supply** tanks keep a **constant quality**.
- **Delivery** tanks keep the **quality** within a given **range**.
- Streams **entering delivery** tanks should satisfy a **quality condition.**

GDP Formulation

The ***MINLP formulation*** *of the problem is easy to achieve, however, a more efficient representation can be obtained by using a* ***GDP framework***

MINLP Formulation
(Property Balance)



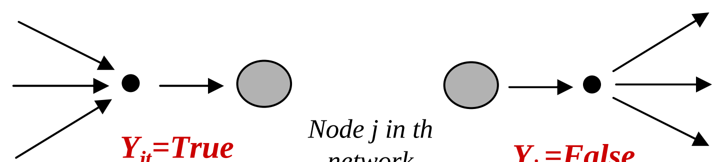
$$S^B_{qjt} I_{jt} = S^B_{qjt-1} I_{jt-1} + \sum_{\substack{j'\in J^P\\(j'j)\in E}} S^P_{qj'} F_{j'jt} + \sum_{\substack{j'\in J^B\\(j'j)\in E}} S^B_{qjt-1'} F_{j'jt} - \sum_{\substack{j'\\(j'j)\in E}} S^B_{qjt-1'} F_{j'jt} \quad \forall q\in Q, j\in J^B, t\in T$$

Significant number of bilinearities

$$x_{j''jt} + x_{jj't} \le 1 \quad \forall j\in J^B, (j''j),(jj')\in E, t\in T$$

$$F^L_{jj'} x_{jj't} \le F_{jj't} \le F^U_{jj'} x_{jj't} \qquad \forall (j'j)\in E, t\in T$$

GDP Formulation
(Property Balance)



$$\begin{bmatrix} Y_{jt} \\ S^B_{qjt} I_{jt} = S^B_{qjt-1} I_{jt-1} + \sum_{\substack{j'\in J^P\\(j'j)\in E}} S^P_{qj'} F_{j'jt} + \\ \sum_{\substack{j'\in J^B\\(j'j)\in E}} S^B_{qjt-1'} F_{j'jt} \quad \forall q\in Q, j\in J^B, t\in T \end{bmatrix} \vee \begin{bmatrix} \neg Y_{jt} \\ S^B_{qjt} = S^B_{qjt-1'} \quad \forall q\in Q, j\in J^B, t\in T \end{bmatrix}$$

No bilinear terms are necessary!

Either inlet streams or outlet streams per node are allowed (but not both)

$$\neg Y_{jt} \Rightarrow \bigwedge_{(j''j)\in E} \neg x_{j''jt} = 0 \quad \forall j\in J^B, t\in T$$

$$\bigvee_{(j''j)\in E} x_{j''jt} \Rightarrow Y_{jt} \quad \forall j\in J^B, t\in T$$

$$F^L_{jj'} x_{jj't} \le F_{jj't} \le F^U_{jj'} x_{jj't} \qquad \forall (j'j)\in E, t\in T$$

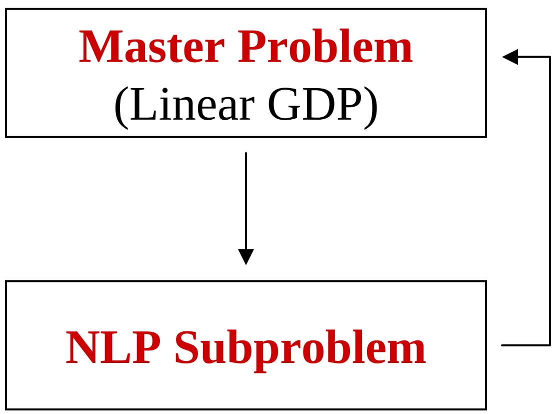
Remarks:

- GDP formulation has **reduced the number of bilinear terms**
- GDP formulation can be efficiently solved by **Logic Based OA methods**

Solution Methods

Logic Based Outer Approximation

Outline of the Logic Based OA



LGDP Master: This problem is obtained by relaxing the bilinear terms using the McCormick envelopes. It is solved as MIP using the Convex Hull reformulation.

NLP Subproblem: The set of boolean variables are fixed based on the result of the master problem. These are used to generate "on the fly" much smaller NLP formulations by getting rid of the disjuncts that are not active.

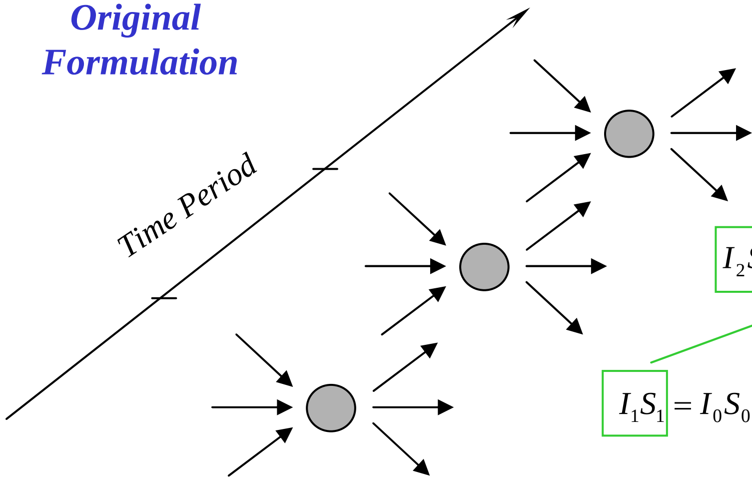
Remark:
This will not guarantee a global solution! (even if the NLP subproblem is solved with a global solver).

Iteration Step: Once a solution is obtained from the NLP subproblem a set of linear cuts are generated and passed to the master.

Solution Methods

Lagrangian decomposition (Review)

Original Formulation



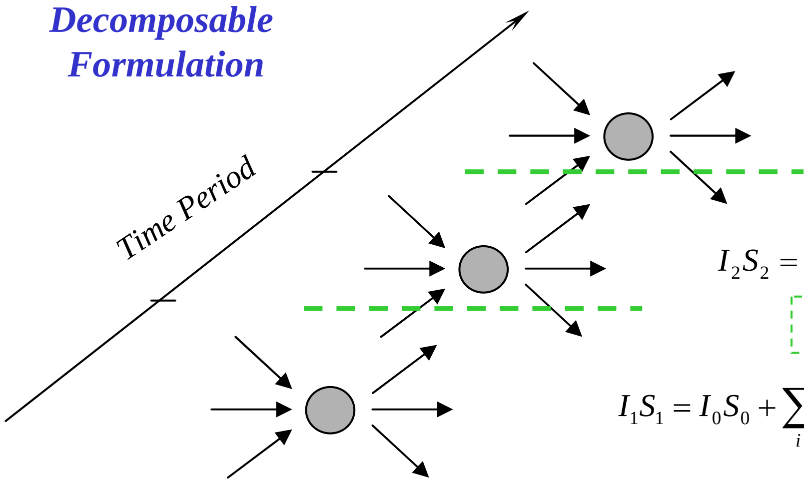
$$I_3S_3 = I_2S_2 + \sum_i Fin_{i3}S_{i3} - \sum_i Fout_{i3}S_3$$

$$I_2S_2 = I_1S_1 + \sum_i Fin_{i2}S_{i2} - \sum_i Fout_{i2}S_2$$

$$I_1S_1 = I_0S_0 + \sum_i Fin_{i1}S_{i1} - \sum_i Fout_{i1}S_1$$

*Constraints are **linked** together by the **inventory** and **composition** variables*

Decomposable Formulation



Dualize

$$I_3S_3 = I'_2\,S'_2 + \sum_i Fin_{i3}S_{i3} - \sum_i Fout_{i3}S_3$$

$$I'_2 = I_2 \;;\; S'_2 = S_2$$

$$I_2S_2 = I'_1\,S'_1 + \sum_i Fin_{i2}S_{i2} - \sum_i Fout_{i2}S_2$$

$$I'_1 = I_1 \;;\; S'_1 = S_1$$

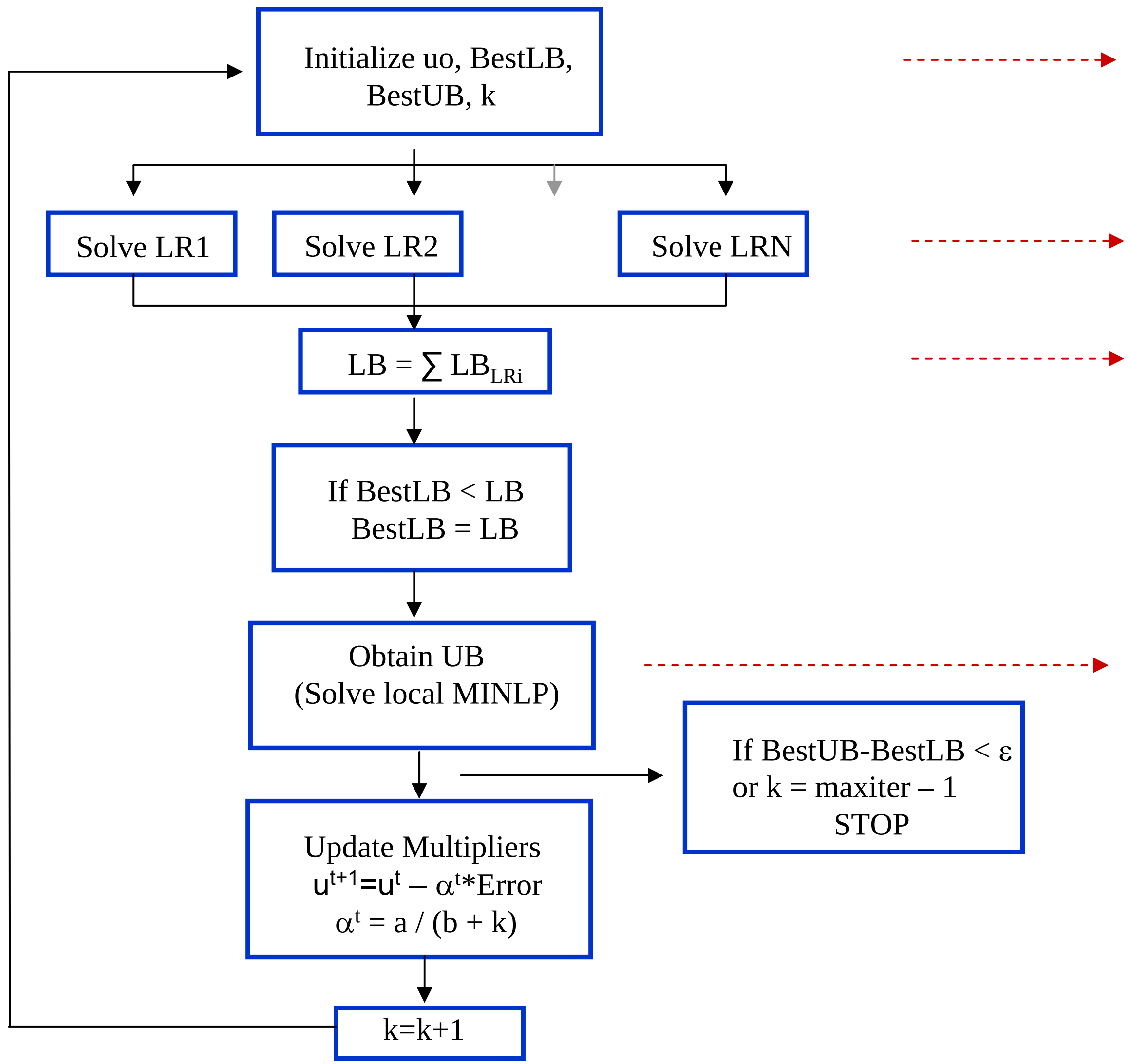
$$I_1S_1 = I_0S_0 + \sum_i Fin_{i1}S_{i1} - \sum_i Fout_{i1}S_1$$

***Duplicating inventory and composition** variables and **dualizing** the correspondent **equalities** lead to a temporal **decomposable** structure*

Solution Methods

Lagrangian decomposition (Algorithm)

Outline of the Lagrangian decomposition method



uo *represents the dual multipliers;* ***BestLB****, the best lower bound;* ***BestUB****, the best upper bound and* ***k****, the iter counter*

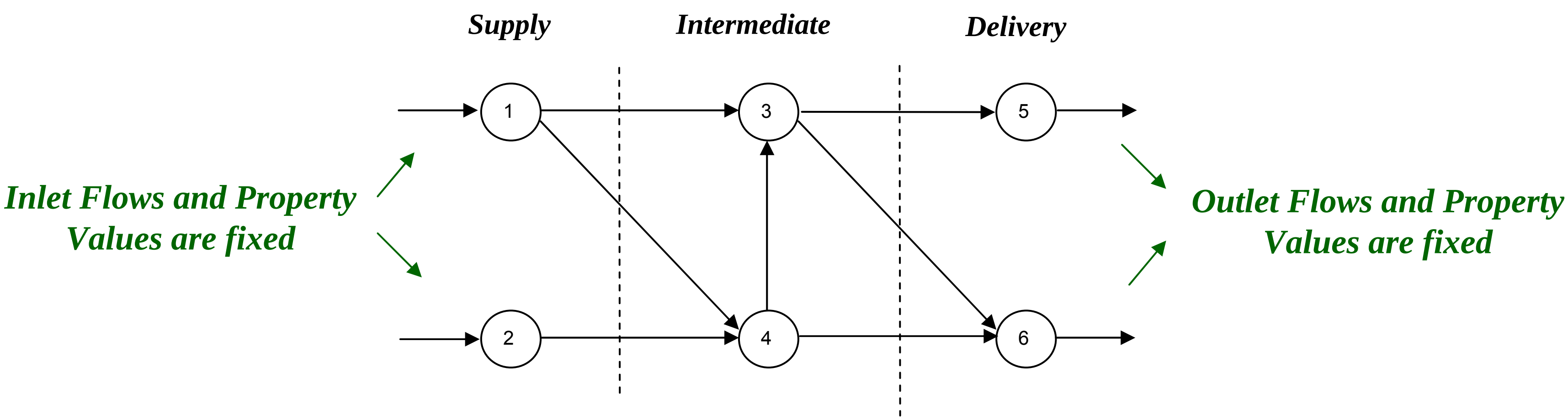
Each ***subproblem (LRi)*** *from the decomposition is solved*

A ***lower bound for the original nonconvex*** *problem can be obtained by adding up the solution of each (LRi) (i.e.* LB_{LRi}*)*

Any local optimization algorithm can be used to find an UB. (e.g. ***The logic based outer approximation*** *applied on the GDP formulation)*

Case Study

*The **implementation** of the **Lagrangian Decomposition** and **Logic Based OA** method has been tested in the following simple case*



Topology
(allowed connections)

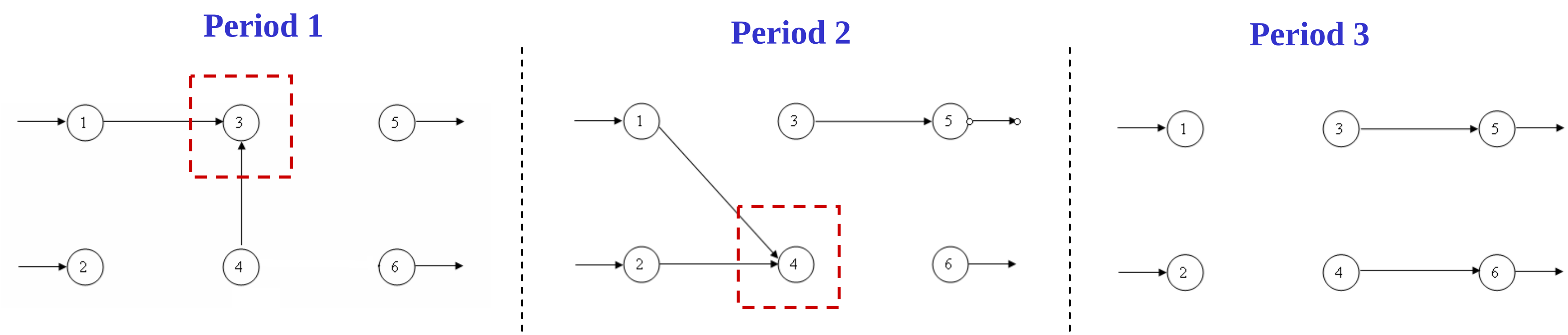
Network Description:

- **Two Supply, Intermediate and Delivery nodes**
- **Two properties transported**
- **Three time periods**

Numerical Results

Logic Based AO

Representation of the nonzero flow streams in the different periods for the optimal solution



Remarks

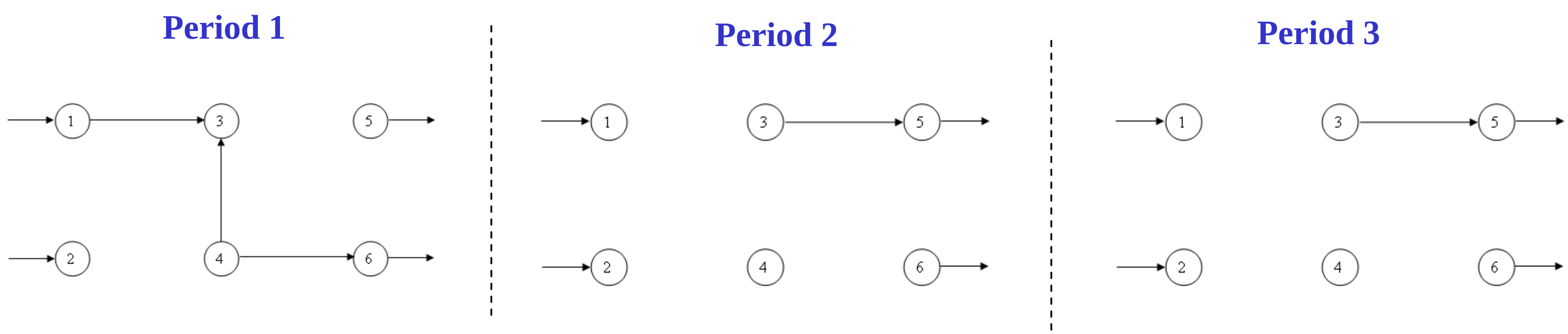
- It stops after **three iterations** in a local solution (**Z = 12.52**)

- The solution of the **NLP subproblem** which leads to the optimal solution only contains **six bilinear terms** as opposed to **thirty six bilinear terms** (the number of bilinear terms in the NLP subproblem using a **traditional MINLP formulation**)

Numerical Results

Lagrangian Decomposition

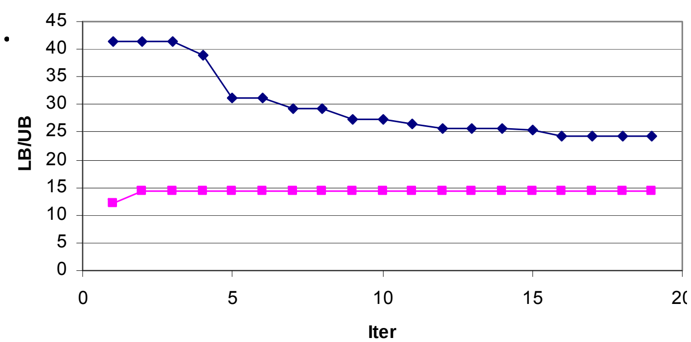
Representation of the nonzero flow streams in the different periods for the global optimal solution



Global Solution (Z = 14.22) (verified with BARON)

Remarks

- Forced to stop after **20 iterations** (no improvement observed).
- **Finds the global solution** (**Z = 14.22**)
- The existence of the **duality gap is due to the nonconvex** nature of the problem
- By experience, Lagrangian decomposition applied to **larger instances has shown smaller duality gaps**



Remarks

- Proposed a **GDP formulation** that aims at **reducing** the number of **nonconvex terms**
- Proposed a **Logic Based Outer Approximation** method that takes advantage of the logic structure of the problem
- Proposed a **Lagrangian decomposition** method to solve the problem to **global optimality**

Future Work

- Analyse the performance on **large scale problems**
- Combine LBOA with Lagrangian Decomposition. (i.e. Use **LBOA for upper bounding**)
- Analyse **best parameters** for LD approach