Progress Report – BP Case Study

Andrew Schaefer
Mehmet Demirci
Görkem Saka
March 15, 2006

Problem Description

- Medium-term operations planning model that produces a plan for monthly production, inventory targets and decisions on which demands should be satisfied from which inventory location.
- Forecasts of the future economic environment are used.
- BP’s model contains representation of the global production assets and distribution system for their PX and PTA businesses.

Problem Description

- Our Aim: Extend this model so that it will explicitly account for the uncertainty in the forecasts.

Products - PX

- Paraxylene (PX) is a colourless, flammable liquid that has a sweet odour. It is separated from a mixed xylene stream that results from the refining of petroleum.

- Can be used as a feedstock for the local manufacture of Purified Terephthalic Acid (PTA) or can be sold to customers.

Products - PTA

- BP is the largest PTA producer in the world.
- PTA is an aromatic acid, primarily applied in the production of polyester. The main raw material for PTA is PX.

Models

- Two models have been analyzed
- In both of the models
  - 5 scenarios are tested which represent 5 different economic views
  - Integrality restrictions are relaxed

Model 1

- Initial approach
- Only the shut-down decisions are in the first stage and all other decisions are in the second stage. First stage decisions are;
  - Number of days of operation of each unit running each valid feed for each break-point in a month
  - Total number of days running for a unit in a period summed over all feeds and break-point rates
  - Number of days spent shutdown

Model 2

- A more detailed model, enlarged first-stage decision space
- Operating policy for the first month as a whole constitutes the first-stage decision variables
  - production plan, days and rates running by unit, ending inventory levels etc.
- Almost twice the number of decision variables before

Schematic Comparison of Two Models

	First-stage decisions	**Second-stage decisions**
Model 1	Shut-down policy for the entire horizon (for all time periods) INTEGRALITY IN THE FIRST-STAGE ***Time Periods: 1,2,3…***	All remaining decisions for the entire horizon ***Time Periods: 1,2,3,…***
Model 2	Decisions corresponding to the operating policy for the first time period INTEGRALITY IN THE FIRST-STAGE ***Time Periods: 1***	Decisions corresponding to the operating policy for the remaining time period INTEGRALITY IN THE SECOND-STAGE ***Time Periods: 2,3,…***

Solving the Extensive Forms

	Model 1	Model 2
# of Constraints	9486	11086
# of Variables	16340	20444
# of Nonzeros	54736	84038
Time	~20 sec.	~30 sec.

Future Work

- Implement the L-shaped method for solving two-stage stochastic programs
  - This is not necessarily straightforward in a modeling language like AIMMS (although they say it is)
  - However, solving the extensive form will limit the number of scenarios and stages that can be considered

Multistage SMIP

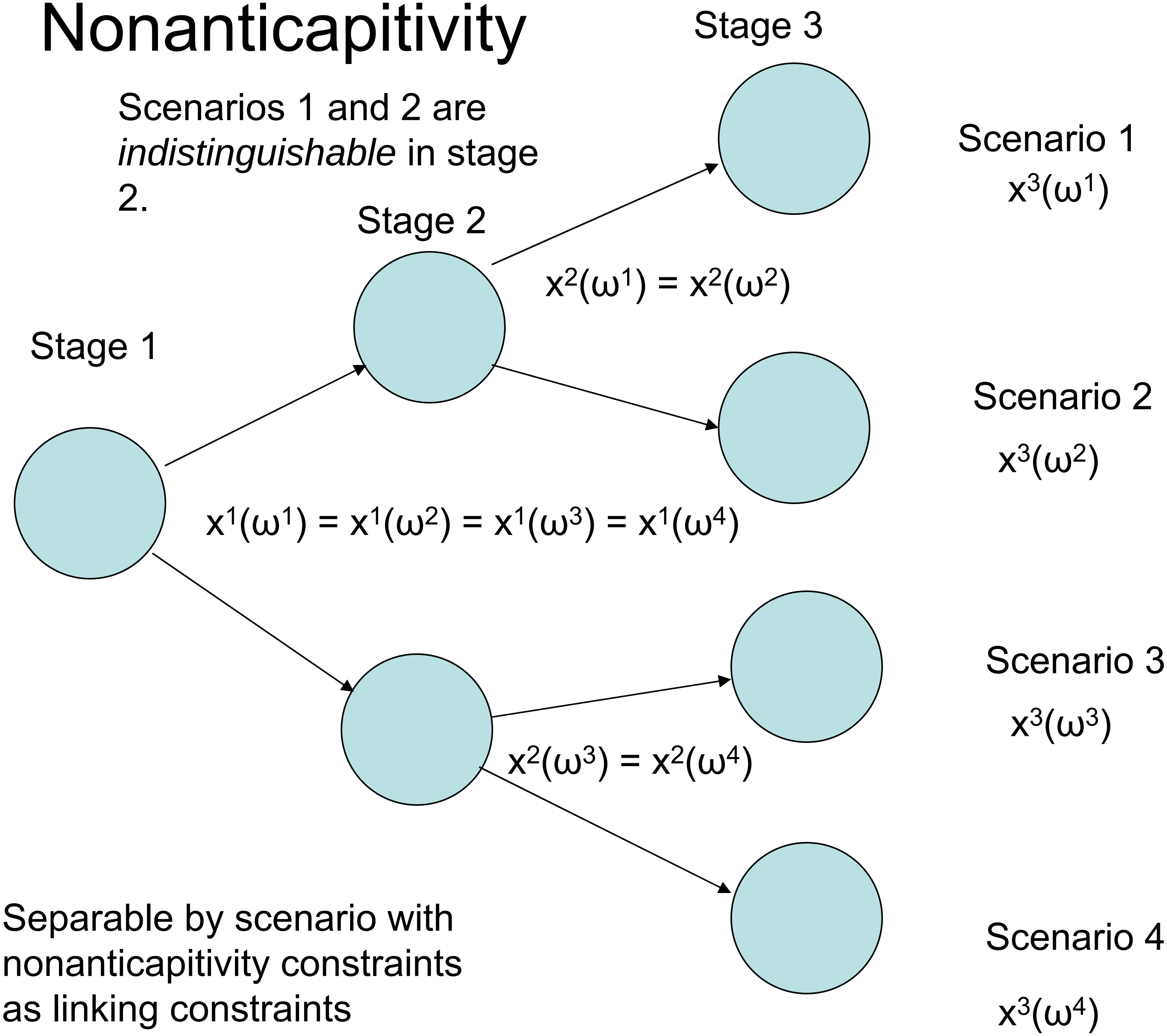
- One approach is to solve the multistage SMIP using Lagrangian relaxation of nonanticapivity constraints
  - There will be many such constraints, even with few scenarios/stages
  - There will (surely) be a duality gap
  - Finding an exact solution for such a large problem is well beyond the scope of the current state of the art

Solving SPs with Nonanticipativity

- Such a model is decomposable by scenario, where nonanticipativity constraints are linking constraints
- Lagrangian relaxation of linking constraints
- For reasonably large scenario trees, the number of possible nonanticipativity constraints is enormous

Nonanticapitivity

Scenarios 1 and 2 are *indistinguishable* in stage 2.



Separable by scenario with nonanticapitivity constraints as linking constraints

Nested Benders' for MSLP

- For a multi-stage SLP, much more is known
- While the nested Benders' procedure gives an optimal answer, many computational questions remain
- The downside is that all recourse decisions must be continuous

Solving Multistage SPs

Nested Decomposition

- Built on the two-stage L-shaped method
- Extended to the multistage case by Birge
- The idea is to place cuts on $\vartheta^{t+1}(x^t)$ and to add other cuts to achieve an x^t that has a feasible completion in all descendant scenarios
- Successive linear approximations of ϑ^{t+1}
- Due to the polyhedral structure of ϑ^{t+1}, the process converges finitely