Facility Location Models: An Overview

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EWO SEMINAR SERIES – APRIL 21, 2010

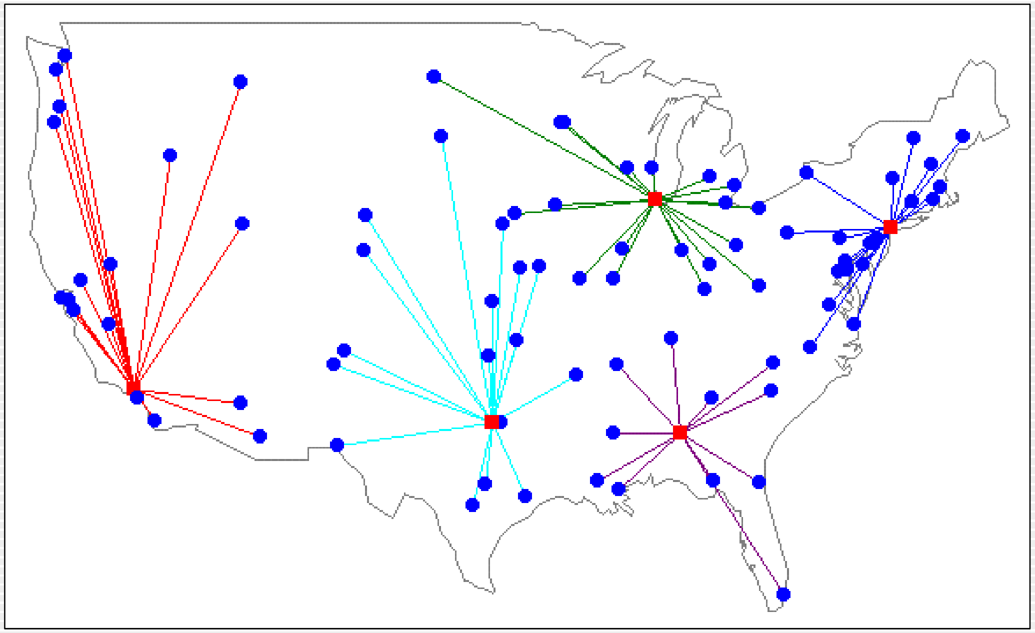
Outline

- Introduction
- Taxonomy of location models
- IP formulations for some classical models
- Algorithms
- Extensions
- Facility location / network design software

Introduction

Overview

- Decide where to locate facilities
  - (factories / warehouses / DCs / retail outlets / etc.)
- To serve customers
- In order to achieve some balance between
  - Cost
  - Service



Decisions

- Usually 2 decisions to make:
  - Where to locate?
  - Which customers are assigned/allocated to which facilities?
- Sometimes referred to as "location–allocation models"

Applications of Facility Location Models

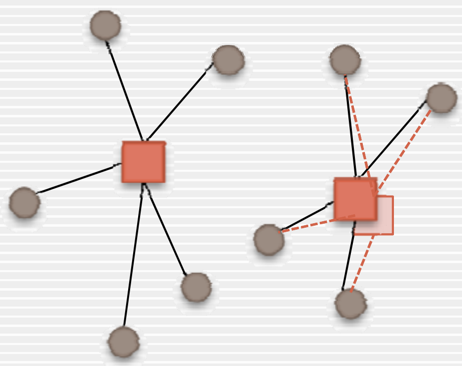
- Widely applied in public and private sectors:
  - Emergency medical services (EMS) / fire stations
  - Airline hubs
  - Blood banks
  - Hazardous waste disposal sites
  - Fast-food restaurants
  - Public swimming pools
  - Schools
  - Vehicle inspection stations
  - Bus stops
  - etc.

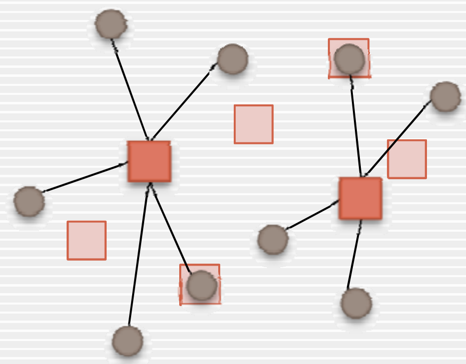
Uses for Facility Location Models

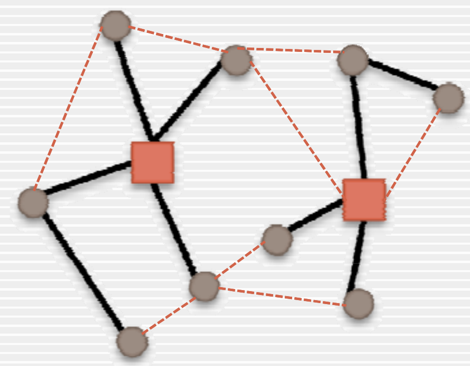
- Also applied to “virtual facilities”:
  - Wildlife reserves
  - Satellite orbits
  - Apparel sizes
  - Flexible manufacturing system tool selection
  - Location of bank accounts
  - Political party platforms
  - Product positioning
  - etc.
- Sometimes arise as subproblems for other OR problems
  - Vehicle routing

Taxonomy of Location Models

Topology







Continuous

- Locate anywhere on plane
- Continuous, non-linear optimization
- “Weber problem”

Discrete

- Locate at pre-defined points
- Integer programming

Network

- Locate anywhere on network
- Travel along arcs
- Integer programming
- **Hakimi property:** Optimal to locate at nodes
  - Holds for some (not all) problems

We’ll consider discrete problems

(Network problems are a special case)

Distance Metric

- Euclidean: $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$
- Rectilinear / Manhattan: $|x_1 - x_2| + |y_1 - y_2|$
  - (travel along streets)
- Great circle: accounts for Earth’s curvature
- Highway / network: shortest path within network
  - (e.g., U.S. highway network)
- Matrix: distance between each pair given explicitly

- For sake of generality, we’ll assume matrix distances
- Also, “distance” = “transportation cost”

Distance Objective

- Total distance: total distance between customers and their assigned facilities
  - (distance is usually demand-weighted)

$$\sum_{\text{customers}} \text{distance to assigned facility}$$

- Maximum distance: maximum distance between a customer and its assigned facility
  - (distance is usually unweighted)

$$\max_{\text{customers}} \{\text{distance to assigned facility}\}$$

- Coverage: cust. is “covered” if distance ≤ specified radius
- Can appear in objective function or constraints

Preventing Too Many Facilities

- Fixed cost: fixed (annual) cost to open/operate facility
  - Represents construction / leasing cost + overhead (lights, heat, security, etc.)
  - Independent of volume of demand served by facility
- Restriction on # facilities: require # of facilities $\leq P$ in constraints

Classical Models

- *P*-median problem: minimize demand-weighted distance
  s.t. locate ≤ *P* facilities
- Uncapacitated fixed-charge location problem (UFLP):
  minimize fixed cost + DWD
- *P*-center problem: minimize maximum distance
  s.t. locate ≤ *P* facilities
- Set covering location problem (SCLP):
  minimize # of facilities
  s.t. cover all customers
- Maximum covering location problem (MCLP):
  maximize covered demands
  s.t. locate ≤ *P* facilities

Capacity

- Most models also have a capacitated version
  - Facilities have fixed throughput capacity
- Capacity is usually an input
- But sometimes a decision variable
  - Discrete choices (50,000 sq ft / 100,000 sq ft / 200,000 sq ft)
  - Continuous variable (cost is a function of capacity)

IP Formulations for Some Classical Models

Notation

- Sets
  - I = {customers}
  - J = {potential facility sites}
- Parameters
  - h_i = annual demand of customer $i \in I$
  - c_{ij} = cost to transport one unit from $j \in J$ to $i \in I$
  - f_j = fixed (annual) cost to open a facility at site $j \in J$
- Decision variables
  - x_j = 1 if facility $j \in J$ is opened, 0 otherwise
  - y_{ij} = 1 if facility $j \in J$ serves customer $i \in I$, 0 otherwise

P-Median Formulation

$$\min \sum_{i \in I} \sum_{j \in J} h_i c_{ij} y_{ij}$$

Min demand-weighted distance (transportation cost)

$$\text{s.t.} \sum_{j \in J} y_{ij} = 1 \qquad \forall i$$

Satisfy all demands

$$y_{ij} \le x_j \qquad \forall i, j$$

Don’t assign cust to closed facility

$$\sum_{j \in J} x_j = P$$

Locate *P* facilities

$$x_j \in \{0,1\} \quad \forall j$$

$$y_{ij} \in \{0,1\} \quad \forall i, j$$

Integrality

UFLP Formulation

$$\min \sum_{j\in J} f_j x_j + \sum_{i\in I}\sum_{j\in J} h_i c_{ij} y_{ij}$$

Min fixed + transportation cost

$$\text{s.t.} \sum_{j\in J} y_{ij} = 1 \qquad \forall i$$

Satisfy all demands

$$y_{ij} \le x_j \qquad \forall i, j$$

Don't assign cust to closed facility

$$x_j \in \{0,1\} \quad \forall j$$

$$y_{ij} \in \{0,1\} \quad \forall i, j$$

Integrality

Maximal Covering Formulation

$$\max \sum_{i \in I} h_i z_i$$

$$\text{s.t.} \sum_{j \in J} x_j = P$$

$$z_i \le \sum_{j \in V_i} x_j \quad \forall i$$

$$x_j \in \{0,1\} \quad \forall j$$

$$z_i \in \{0,1\} \quad \forall i$$

Maximize covered demand

Locate P facilities

Definition of coverage

Integrality

where V_i = set of facilities that can cover customer i

z_i = 1 if customer i is covered, 0 otherwise

Algorithms

Algorithms

- Most facility location problems are NP-hard
- But many classical problems are “easy” computationally
  - LP relaxations are often extremely tight
  - Sometimes integer solutions “for free”
- Virtually every type of algorithm for discrete optimization has been applied to facility location

Algorithms

- Heuristics:
  - Greedy add/drop
  - Swap
  - Neighborhood search
  - Metaheuristics
    - Genetic algorithms, tabu search, variable neighborhood search, simulated annealing, ant algorithms, bee algorithms, …
- Exact Algorithms:
  - Branch and bound
  - Cutting planes
  - Benders decomposition
  - Column generation / Dantzig-Wolfe decomposition
  - Lagrangian relaxation

Lagrangian Relaxation for *P*-Median

$$\min \sum_{i \in I} \sum_{j \in J} h_i c_{ij} y_{ij}$$

$$\text{s.t.} \sum_{j \in J} y_{ij} = 1 \qquad \forall i$$ ← RELAX

$$y_{ij} \le x_j \qquad \forall i, j$$

$$\sum_{j \in J} x_j = P$$

$$x_j \in \{0,1\} \quad \forall j$$

$$y_{ij} \in \{0,1\} \quad \forall i, j$$

Lagrangian Subproblem

$$\min \sum_{i\in I}\sum_{j\in J} h_i c_{ij} y_{ij} + \sum_{i\in I}\lambda_i\left(1-\sum_{j\in J} y_{ij}\right)$$

$$= \sum_{i\in I}\sum_{j\in J}(h_i c_{ij} - \lambda_i) y_{ij} + \sum_{i\in I}\lambda_i$$

$$\text{s.t. } y_{ij} \le x_j \qquad \forall i, j$$

$$\sum_{j\in J} x_j = P$$

$$x_j \in \{0,1\} \ \forall j$$

$$y_{ij} \in \{0,1\} \ \forall i, j$$

Facility-j Subproblem

- Subproblem is separable by j
- Suppose we open j; need to solve

$$\min \sum_{i \in I} (h_i c_{ij} - \lambda_i) y_{ij}$$
$$\text{s.t.} \quad y_{ij} \in \{0,1\}$$

- Easy—solve by inspection:
  - Would set $y_{ij} = 1$ iff $h_i c_{ij} - \lambda_i < 0$
  - "Benefit" of opening j is

$$\beta_j = \sum_{i \in I} \min\{0, h_i c_{ij} - \lambda_i\}$$

  - Open P facilities with smallest β_j
- This gives lower bound
- Obtain upper bound from heuristic
- Update λ and repeat

Extensions

Other Flavors

- Obnoxious facilities
  - Obnoxious location: Maximize distance from facilities to customers
  - Dispersion: Maximize distance among facilities
- Competitive location
  - Multiple players try to capture demand by locating facilities
- Multi-objective models
  - Account for multiple stakeholders' objectives
- Hub location
  - Flows from facilities to customers but *also* among facilities
  - Quadratic objective
- Dynamic location
  - Facilities are located over time, or move over time

Uncertainty

- Types of randomness:
  - Demand-side: Randomness in demands, costs, etc.
  - Supply-side: Randomness in supply (e.g., disruptions)
- Approaches to uncertainty:
  - Stochastic programming: min expected cost
  - Robust optimization: minimax cost, minimax regret, CVaR, etc.
- Modeling approaches:
  - Scenario formulations
  - Interval uncertainty

Integrated Models

- Incorporate tactical / operational costs into strategic (facility location) decisions
  - Inventory
  - Routing
- Integrated location-inventory model
  - Daskin, Coullard, and Shen (*Ann OR*, 2002)
  - Objective function includes two concave terms:
    - Inventory economies of scale (EOQ)
    - Risk pooling (safety stock)
  - Constraints are same as UFLP
  - Solve via Lagrangian relaxation
    - Subproblem solved in $O(|I| \log |I|)$ time for each j
    - (UFLP: $O(|I|)$ time for each j)

Network Design

- Multi-echelon facility location models
  - Make open/close decisions for multiple tiers
  - Geoffrion and Graves (*MS*, 1974)
- Generalization: network design problems
  - Usually locate arcs in the network
    - But locating nodes is equivalent
- Rich literature on network design
  - e.g., Magnanti and Wong (*TS*, 1984)

Facility Location / Network Design Software

Software Packages

- LogicNet / LogicNet Plus
  - Originally LogicTools
  - Now ILOG Supply Chain Applications, part of IBM Consulting
- SAILS
  - INSIGHT
- SAP, Oracle modules

Capabilities

- Key decisions:
  - Facility locations, capacities, capabilities, volumes
  - Distribution lanes (yes/no, volumes)
  - Make vs. buy
- Key features:
  - Data import
  - Output report export
  - GUI, GIS
  - Optimization solver
  - Rating engine
  - What-if scenarios

Recent Developments

- New features:
  - "Green"
  - Risk management
  - Taxes
  - Seasonality
  - Cost vs. service level tradeoffs
- Things most current software can't do (very well):
  - Non-linearities (e.g., quantity discounts)
  - Open/close decisions on arcs for larger models
  - Close integration with inventory modeling
  - Uncertainty
    - Optimization under multiple scenarios
    - Risk-averse objectives

Further Reading

- Textbooks on facility location
  - Daskin (1995)
  - Drezner (1995)
  - Drezner and Hamacher (2001)
- Review articles / book chapters
  - Discrete location: Current, Daskin, and Schilling (D&H book, 2001)
  - Continuous location: Drezner, et al. (D&H book, 2001)
  - Stochastic location: Snyder (*IIE Trans*, 2006)
  - Location with disruptions: Snyder, et al. (INFORMS Tutorial, 2006)
  - Location-inventory models: Shen (*JIMO*, 2007)

Questions?

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