Automatic Strengthening of Crude-Oil Scheduling Operations Models

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Enterprise-Wide Optimization, 10 March 2010

Motivation

- Scheduling models based on discrete-time formulations are usually efficiently solved by commercial MILP solvers (CPLEX, Xpress)
  - Often solved at root node (after presolve) by pure cutting plane method
  - Also, they have small integrality gaps
- For continuous-time formulations, many authors have presented tightening constraints based on the time representation
  - However, commercial solvers are not able to generate cutting planes for most mixed-integer scheduling constraints
- Our main goal is to automatically derive strengthened formulations from the scheduling constraints

Scheduling approach

Time representations

- Mathematical models for 4 different scheduling formulations
- Common set of variables
- Operation and priority-slot based representations

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Set of priority-slots $T = \{1, \ldots, n\}$

Any operation assigned to priority-slot i is given scheduling priority i.

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Any operation assigned to priority-slot i is given scheduling priority i.

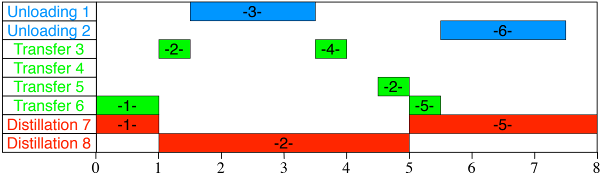
Type of scheduling problems

The proposed time representations can be used to model and solve scheduling problems in which operations can be sequenced as a whole.

- No sequencing of events such as start or end times
- Unsupported features: cumulative resource constraints, simultaneous inventory charging and discharging, ...

Multi-Operation Sequencing (MOS)

- 8 possible operations / 6 priority-slots
- Given 2 non-overlapping operations $v, w \in W$
  - v and w cannot be assigned to the same priority-slot
  - v and w are sequenced according to their scheduling priority
- Example: unloading operations 1 and 2 are assigned to slots 3 and 6

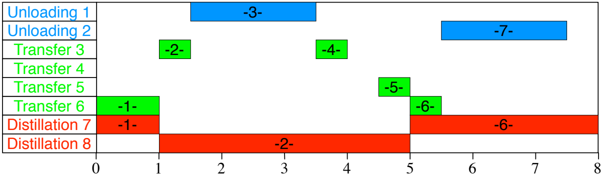


Priority-slot	-1-	-2-	-3-	-4-	-5-	-6-
Unloading 1			x			
Unloading 2						x
Transfer 3		x		x		
Transfer 4						
Transfer 5		x				
Transfer 6	x				x	
Distillation 7	x				x	
Distillation 8		x				

Figure: A solution schedule obtained using the MOS time representation

MOS with Synchronized Start Time (MOS-SST)

- 8 possible operations / 7 priority-slots
- Same features as the MOS representation
- Specific feature:
  - All operations assigned to priority-slot *i* must start at the same time
  - The scheduling horizon is divided into variable adjacent time intervals

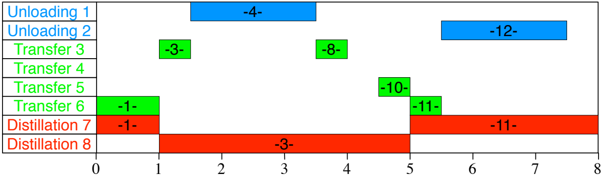


Priority-slot	-1-	-2-	-3-	-4-	-5-	-6-	-7-
Unloading 1			x				
Unloading 2							x
Transfer 3		x		x			
Transfer 4							
Transfer 5					x		
Transfer 6	x					x	
Distillation 7	x					x	
Distillation 8		x					

Figure: A solution schedule obtained using the MOS-SST time representation

MOS with Fixed Start Time (MOS-FST)

- 8 possible operations / 16 priority-slots
- Same features as the MOS-SST representation
- Specific feature:
  - All operations assigned to priority-slot *i* must start at fixed time point
  - The scheduling horizon is divided into fixed adjacent time intervals

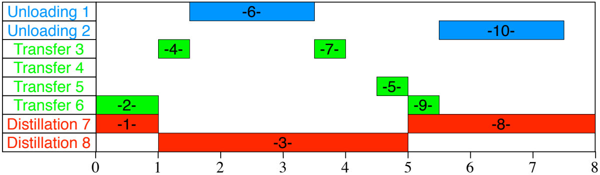


Priority-slot	-1-	-2-	-3-	-4-	-5-	-6-	-7-	-8-	-9-	-10-	-11-	-12-	-13-	-14-	-15-	-16-
Unloading 1				x												
Unloading 2												x				
Transfer 3			x					x								
Transfer 4																
Transfer 5										x						
Transfer 6	x										x					
Distillation 7	x										x					
Distillation 8			x													

Figure: A solution schedule obtained using the MOS-FST time representation

Single-Operation Sequencing (SOS)

- 8 possible operations / 10 priority-slots
- Same features as the MOS representation
- Specific feature:
  - At most one operation can be assigned to each priority-slot
  - The solution can be represented as a single sequence of operations



Priority-slot	-1-	-2-	-3-	-4-	-5-	-6-	-7-	-8-	-9-	-10-
Unloading 1						x				
Unloading 2										x
Transfer 3				x			x			
Transfer 4										
Transfer 5					x					
Transfer 6		x							x	
Distillation 7	x							x		
Distillation 8			x							

Figure: A solution schedule obtained using the SOS time representation

Time Representations

A few comments

For a given number of priority-slots n

- MOS-SST, MOS-FST and SOS models are special cases of the MOS model
- They can be obtained from the MOS model by adding new constraints
- The strengthened formulation for MOS models can also be applied to other time representations
- This is not usually the case for symmetry-breaking constraints which heavily depends on the time representation used

Mathematical Model: MOS

- Assignment variables

$$Z_{iv} \in \{0,1\} \qquad i \in T, v \in W$$

- Start, duration, end variables

$$S_{iv}, D_{iv}, E_{iv} \geq 0 \qquad i \in T, v \in W$$

- Assignment constraint

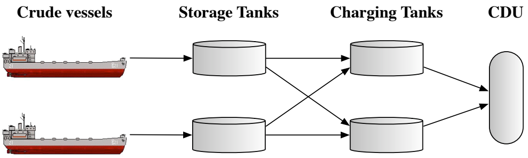
$$Z_{iv_1} + Z_{iv_2} \leq 1 \qquad i \in T, v_1, v_2 \in W, NO_{v_1 v_2} = 1$$

- Non-overlapping constraint

$$E_{i_1 v_1} \leq S_{i_2 v_2} + H \cdot (1 - Z_{i_2 v_2}) \qquad i_1, i_2 \in T, i_1 \leq i_2, v_1, v_2 \in W, NO_{v_1 v_2} = 1$$

Crude-oil operations scheduling problem

- Introduced in Lee et al. (1996)
- Scheduling horizon $[0, H]$
- 3 types of operations:
  - Unloading: Vessel unloading to storage tanks
  - Transfer: Transfer from storage tanks to charging tanks
  - Distillation: Distillation of charging tanks
- 4 types of resources:
  - Crude-oil marine vessels
  - Storage tanks
  - Charging tanks
  - Crude Distillation Units (CDUs)
- Each charging tank is dedicated to a specific type of crude-oil blends (e.g. sulfur content limitations)



Crude-oil operations schedule

Logistics constraints

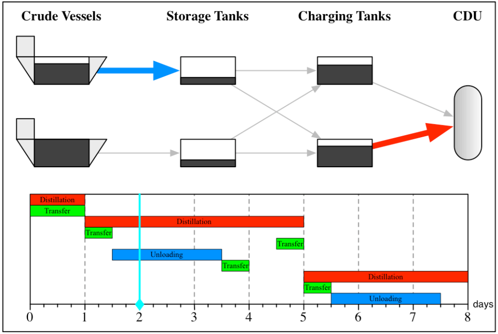
(i) Only one docking station available for vessel unloadings

(ii) A tank is either being filled, discharged, or idle

(iii) A tank can charge only one CDU at a time

(iv) A CDU can be charged by only one tank at a time

(v) Continuous operation of CDUs



Refinery operations

Gantt chart

Non-overlapping graph

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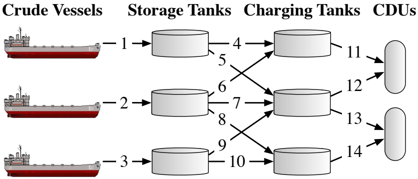
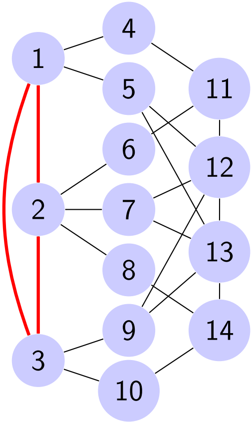


Non-overlapping graph $G_{NO} = (V, E)$:

- $V = W$ (set of operations)
- $E = \{\{v_1, v_2\}, NO_{v_1 v_2} = 1\}$ (set of non-overlapping relations)

Self-loops are not displayed !



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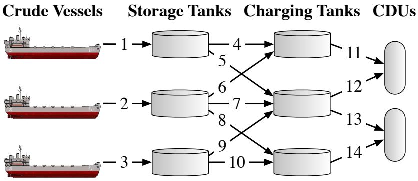

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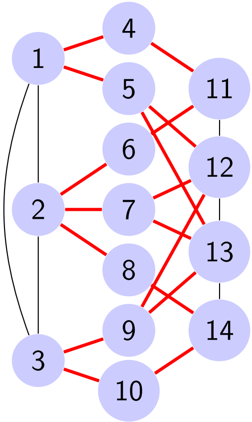

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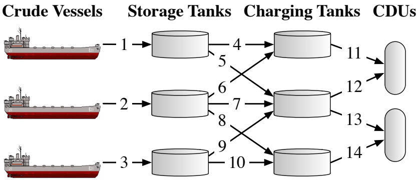

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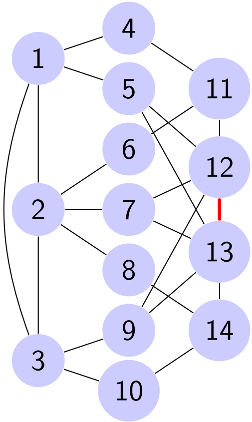

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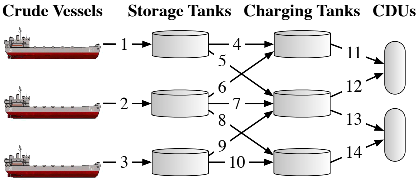

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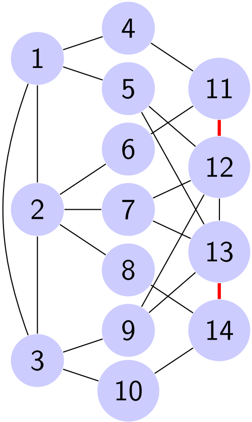

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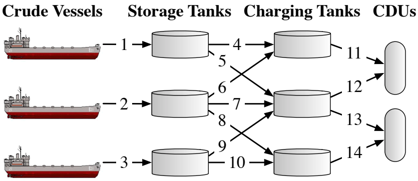

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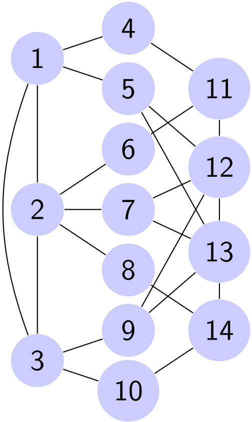
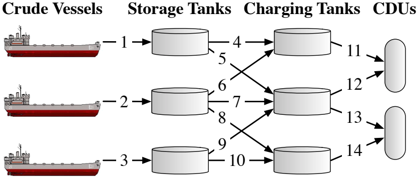


Cliques

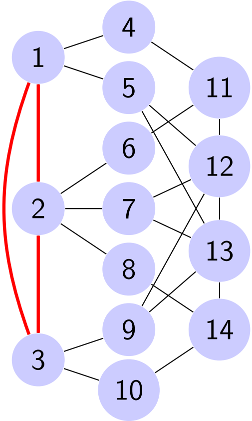
Graph cliques

- A clique of G_{NO} is a subset of the set of operations $W' \subset W$ such that any two operations in W' must not overlap
- A maximal clique is a clique that is not a subset of any other cliques

$\Rightarrow$ 4 maximal cliques of 3 vertices



- $\{1, 2, 3\}$ due to (i)
- $\{5, 12, 13\}$ due to (ii) and (iii)
- $\{7, 12, 13\}$ due to (ii) and (iii)
- $\{9, 12, 13\}$ due to (ii) and (iii)



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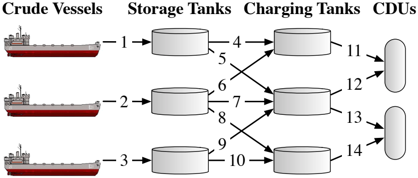

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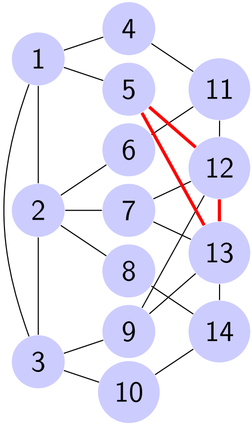

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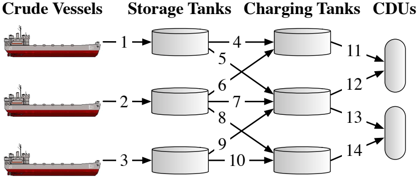

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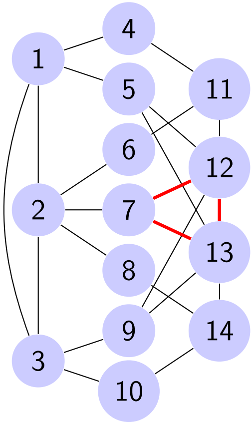

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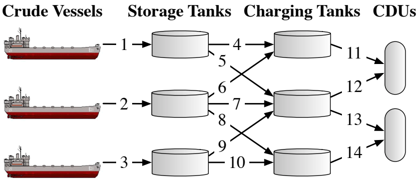

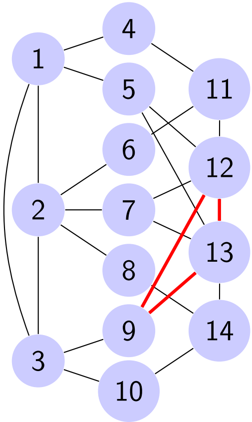

Strengthened reformulations

Strengthened non-overlapping constraints

- Assignment constraint using maximal cliques

$$\sum_{v \in W'} Z_{iv} \leq 1 \qquad i \in T, W' \in maxclique(G_{NO})$$

- Non-overlapping constraint using maximal cliques

$$\sum_{v \in W'} E_{i_1 v} \leq \sum_{v \in W'} S_{i_2 v} + H \cdot \left[1 - \sum_{v \in W'} Z_{i_2 v} \right] \qquad i_1, i_2 \in T, i_1 < i_2, W' \in maxclique(G_{NO})$$

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$\Rightarrow$ More compact and tighter formulation !

$$\begin{cases} E_{i_1 1} \leq S_{i_2 1} + H \cdot (1 - Z_{i_2 1}) \\ E_{i_1 1} \leq S_{i_2 2} + H \cdot (1 - Z_{i_2 2}) \\ E_{i_1 1} \leq S_{i_2 3} + H \cdot (1 - Z_{i_2 3}) \\ E_{i_1 2} \leq S_{i_2 1} + H \cdot (1 - Z_{i_2 1}) \\ E_{i_1 2} \leq S_{i_2 2} + H \cdot (1 - Z_{i_2 2}) \\ E_{i_1 2} \leq S_{i_2 3} + H \cdot (1 - Z_{i_2 3}) \\ E_{i_1 3} \leq S_{i_2 1} + H \cdot (1 - Z_{i_2 1}) \\ E_{i_1 3} \leq S_{i_2 2} + H \cdot (1 - Z_{i_2 2}) \\ E_{i_1 3} \leq S_{i_2 3} + H \cdot (1 - Z_{i_2 3}) \end{cases} \Rightarrow \sum_{v=1}^{3} E_{i_1 v} \leq \sum_{v=1}^{3} S_{i_2 v} + H \cdot (1 - \sum_{v=1}^{3} Z_{i_2 v})$$

$$\begin{cases} Z_{i1} + Z_{i2} \leq 1 \\ Z_{i1} + Z_{i3} \leq 1 \\ Z_{i2} + Z_{i3} \leq 1 \end{cases} \Rightarrow Z_{i1} + Z_{i2} + Z_{i3} \leq 1$$

For maximal clique $\{1, 2, 3\}$

Computational results

- Problem 2 from Lee et al. (1996) solved using the MOS representation
- Only the MILP relaxation is solved (nonlinear constraints are dropped)
- MILP solver : GAMS/CPLEX 12 (cut generation deactivated)
- LP relaxation is identical in all cases

	Original Formulation	Strengthened Assignment csts	Strengthened Non-overlap. csts	Full Strengthening
CPU time	95s	78s	88s	57s
Nb of nodes	2704	1652	2540	1708

- MILP solver : GAMS/CPLEX 12 (cut generation activated)

	Original Formulation	Strengthened Assignment csts	Strengthened Non-overlap. csts	Full Strengthening
CPU time	156s	143s	113s	34s
Nb of nodes	2624	1945	2209	570

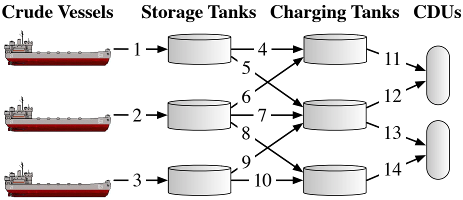
$\Rightarrow$ Strengthened formulations using maximal cliques are solved faster !

Bicliques

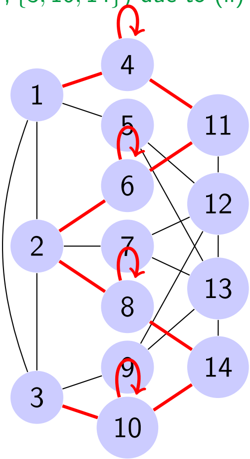
Graph bicliques

- A biclique of G_{NO} is a pair of sets of operations $(W_1; W_2) \in W^2$ such that for any pair of operations $(v_1; v_2) \in W_1 \times W_2$, v_1 and v_2 must not overlap
- A maximal biclique is a biclique that is not contained in any other bicliques

$\Rightarrow$ 4 maximal bicliques of 3 vertices



- $(\{4\}; \{1, 4, 11\})$ due to (ii)
- $(\{6\}; \{2, 6, 11\})$ due to (ii)
- $(\{8\}; \{2, 8, 14\})$ due to (ii)
- $(\{10\}; \{3, 10, 14\})$ due to (ii)



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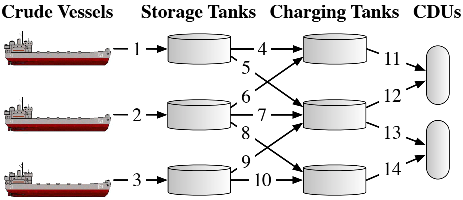
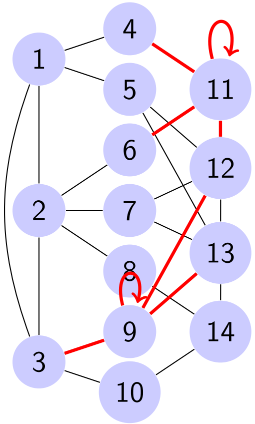
$\Rightarrow$ 5 maximal bicliques of 4 vertices



- $(\{9\}; \{3, 9, 12, 13\})$ due to (ii)
- + 2 similar based on 5 and 7
- $(\{11\}; \{4, 6, 11, 12\})$ due to (ii) and (iv)
- + 1 similar based on 14



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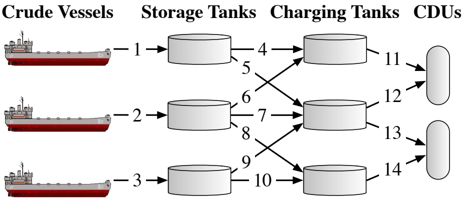
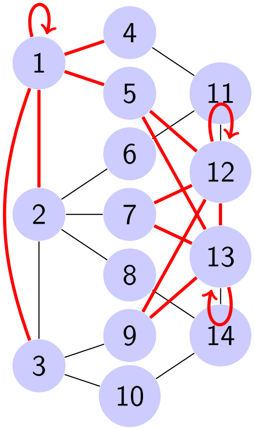
$\Rightarrow$ 3 maximal bicliques of 5 vertices



- $(\{1\}; \{1, 2, 3, 4, 5\})$ due to (i) and (ii)
- $(\{3\}; \{1, 2, 3, 9, 10\})$ due to (i) and (ii)
- $(\{5, 7, 9, 12, 13\}; \{12, 13\})$ due to (ii)
-



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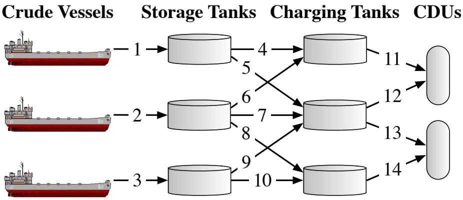
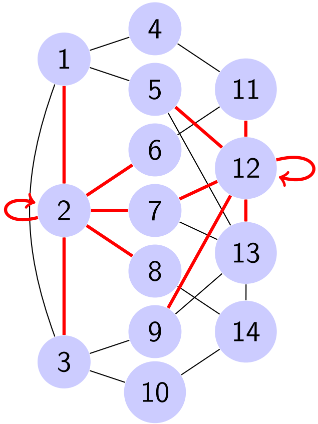
- A biclique of G_{NO} is a pair of sets of operations $(W_1; W_2) \in W^2$ such that for any pair of operations $(v_1; v_2) \in W_1 \times W_2$, v_1 and v_2 must not overlap
- A maximal biclique is a biclique that is not contained in any other bicliques

$\Rightarrow$ 3 maximal bicliques of 6 vertices



- $(\{2\}; \{1, 2, 3, 6, 7, 8\})$ due to (i) and (ii)
- $(\{12\}; \{5, 7, 9, 11, 12, 13\})$
- due to (ii),(iii) and (iv)
- $+$ 1 similar based on 13



Strengthened reformulations

Strengthened non-overlapping constraints

- Non-overlapping constraint using maximal bicliques

$$\sum_{v_1 \in W_1} E_{i_1 v_1} \leq \sum_{v_2 \in W_2} S_{i_2 v_2} + H \cdot \left[1 - \sum_{v_2 \in W_2} Z_{i_2 v_2}\right]$$

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⇒ More compact and tighter formulation !

$$\begin{cases} E_{i_1 1} \leq S_{i_2 4} + H \cdot (1 - Z_{i_2 4}) \\ E_{i_1 4} \leq S_{i_2 1} + H \cdot (1 - Z_{i_2 1}) \\ E_{i_1 4} \leq S_{i_2 4} + H \cdot (1 - Z_{i_2 4}) \\ E_{i_1 4} \leq S_{i_2 11} + H \cdot (1 - Z_{i_2 11}) \\ E_{i_1 11} \leq S_{i_2 4} + H \cdot (1 - Z_{i_2 4}) \end{cases} \Rightarrow \begin{cases} E_{i_1 4} \leq \sum_{v=1,4,11} S_{i_2 v} + H \cdot (1 - \sum_{v=1,4,11} Z_{i_2 v}) \\ \sum_{v=1,4,11} E_{i_2 v} \leq S_{i_2 4} + H \cdot (1 - Z_{i_2 4}) \end{cases}$$

For maximal biclique $(\{4\}; \{1, 4, 11\})$

Cliques and bicliques selection strategies

- Cliques can be used alone in the SOS time representation
- Bicliques can also help strengthening the model
- However, many redundant constraints would be generated by simultaneously using cliques and bicliques
- We propose three cliques/bicliques selection strategies:
  - Selection *a*: select all maximal cliques
  - Strategie *b*: select cliques and bicliques from constraint definitions
  - Selection *c*: improve selection *b* by making cliques and bicliques maximal and removing unnecessary elements

Constraint	No.	Selection *a*	Selection *b*	Selection *c*
(i)	1	$\{1,2,3\}$	$\{1,2,3\}$	implied by 2, 3, 4
(ii)	2	$\{1,4\}$, $\{1,5\}$	$(\{1\};\{4,5\})$	$(\{1\};\{1,2,3,4,5\})$
	3	$\{2,6\}$, $\{2,7\}$, $\{2,8\}$	$(\{2\};\{6,7,8\})$	$(\{2\};\{1,2,3,6,7,8\})$
	4	$\{3,9\}$, $\{3,10\}$	$(\{3\};\{9,10\})$	$(\{3\};\{1,2,3,9,10\})$
	5	$\{4,11\}$, $\{6,11\}$	$(\{4,6\};\{11\})$	$(\{4,6,11,12\};\{11\})$
	6	$\{5,12,13\}$, $\{7,12,13\}$, $\{9,12,13\}$	$(\{5,7,9\};\{12,13\})$	$(\{5,7,9,12,13\};\{12,13\})$
	7	$\{8,14\}$, $\{10,14\}$	$(\{8,10\};\{14\})$	$(\{8,10,13,14\};\{14\})$
(iii)	8	implied by 6	$\{12,13\}$	implied by 6
(iv)	9	$\{11,12\}$	$\{11,12\}$	implied by 5
	10	$\{13,14\}$	$\{13,14\}$	implied by 7
No. of non-overlap. csts		15	16	12

Computational results

- Problems 1 to 4 are solved using the SOS representation
- Only the MILP relaxation is solved (nonlinear constraints are dropped)
- MILP solver : GAMS/CPLEX 12 (cut generation deactivated)
  - Selection *a*: select all maximal cliques
  - Selection *c*: smart selection of maximal cliques and bicliques

Pb	Selection	n	LP	MILP	Nb of nodes	CPU
1	*a*	13	80.000	79.750	18	5.88s
	c	13	80.000	79.750	21	4.92s
2	*a*	21	103.000	101.175	36	120.42s
	c	21	103.000	101.175	25	60.50s
3	*a*	21	100.000	87.400	28	191.47s
	c	21	100.000	87.400	31	64.46s
4	*a*	26	132.585	132.548	16	606.86s
	c	26	132.585	132.548	32	308.43s

⇒ Maximal bicliques are more helpful than maximal cliques in SOS models

Conclusions

- Automatic strengthening of several scheduling formulations based on a global graph representation of the non-overlapping constraints
- The extraction of useful graph entities such as cliques and bicliques can be done automatically by well-known algorithmic tools:
  - very efficient for small and medium scale graphs
  - not polynomial
- The strengthened formulation is more compact and has a tighter LP relaxation:
  - 40% decrease in CPU time on problem 2 for MOS model (w/o cuts)
  - 78% decrease in CPU time on problem 2 for MOS model (w/ cuts)
  - 40% average decrease in CPU time for SOS models