

# Optimal Scheduling of Refinery Crude-Oil Operations

Sylvain Mouret, Ignacio E. Grossmann and Pierre Pestiaux

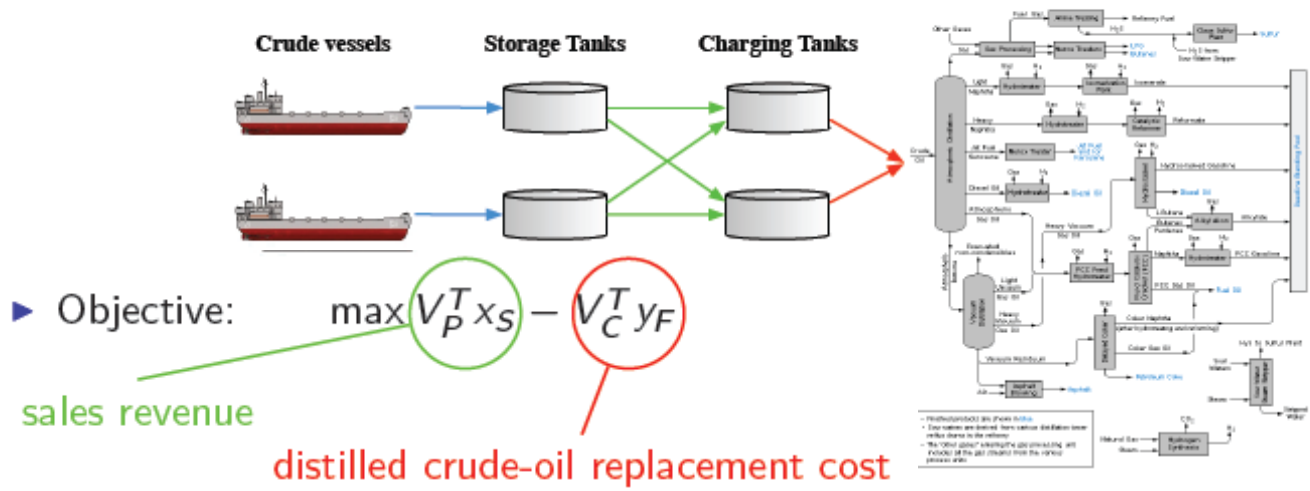
Entreprise-Wide Optimization

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# Integration of planning and scheduling in oil refineries

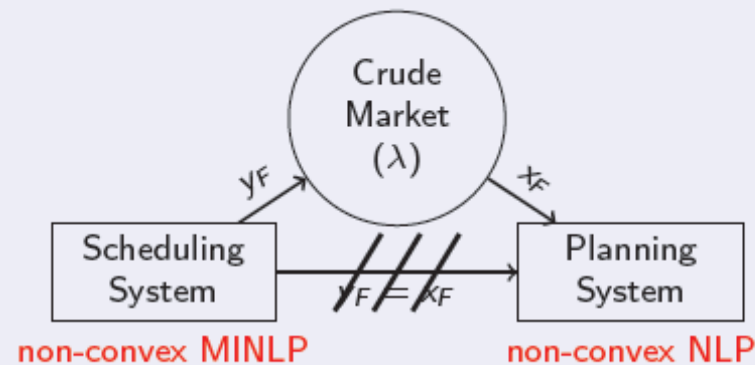
- ▶ Planning/Scheduling in oil refineries and in batch/continuous production plant are different !
- ▶ In oil refineries crude-oil scheduling and planning problems are 2 **distinct** problems connected by the CDUs (crude distillation targets)
- ▶ The **economic data** is only included in the planning level
- ▶ In practice, the crude-oil scheduling is mostly a **feasibility problem**



## Lagrangian decomposition approach

### Integration guidelines

- ▶ Form a **Lagrangian relaxation** ( $P_R(\lambda)$ ) of the full-space problem ( $P$ ):
  - ▶ Relax the complicating CDU feedstock constraint:  $y_F = x_F$
  - ▶ Penalize the violation of this constraint with Lagrange multiplier  $\lambda \in \mathbb{R}$

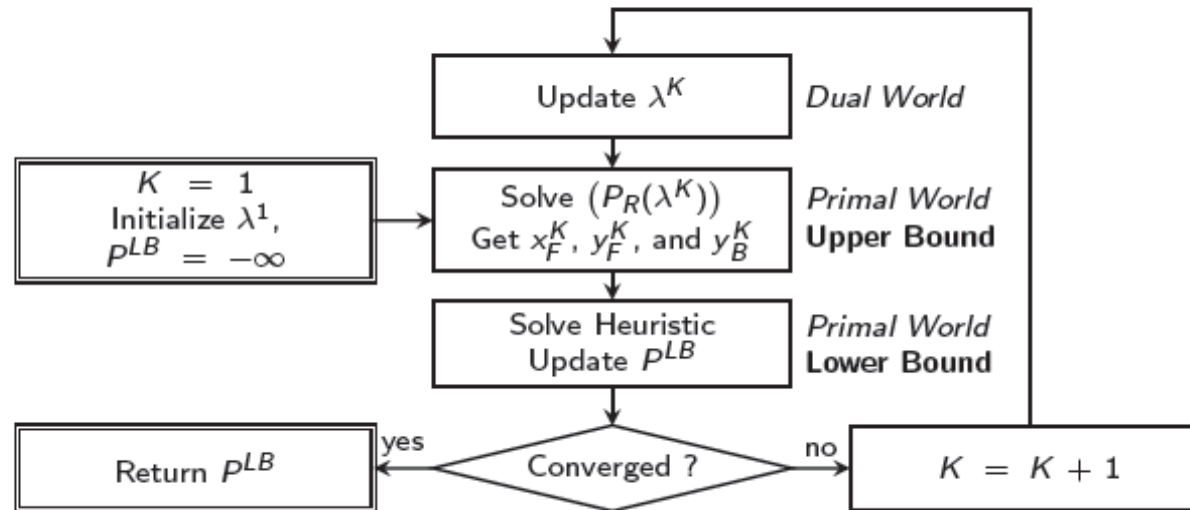


- ▶ Solve the **Lagrangian dual** problem ( $P_D$ ) (**upper bound**):

$$\min_{\lambda} v(P_R(\lambda))$$

- ▶ Meanwhile, generate **heuristic** feasible solutions (**lower bound**)

## Lagrangian algorithm



- ▶ The iterative primal-dual algorithm is composed of 3 main steps:
  - ▶ The Lagrange multipliers  $\lambda$  are updated by solving a **hybrid dual problem**
  - ▶ The **relaxed** full-space problem is solved
  - ▶ The **original** full-space problem is solved with fixed binary decisions from the solution of the relaxed problem (i.e. from the solution of the scheduling subproblem)
- ▶ Convergence based on the **Lagrangian gap** and **dual gap**

## Hybrid Lagrangian dual problem

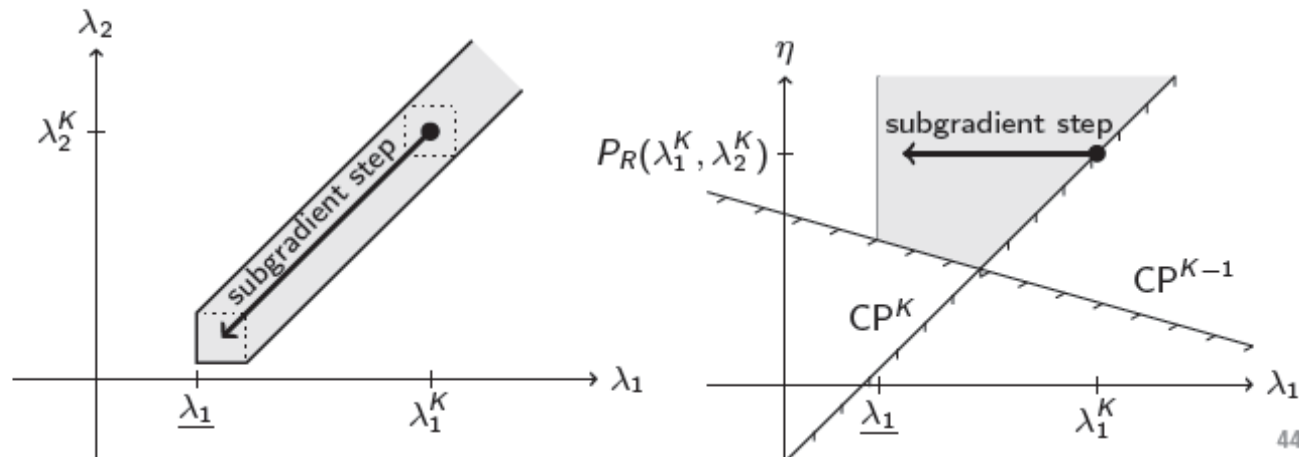
- ▶ The Lagrange multipliers  $\lambda$  are updated by solving the following hybrid LP dual problem ( $\hat{P}_D^{K+1}$ ):

$$\begin{aligned} \min \quad & \eta \\ \text{s.t.} \quad & \eta \geq V_P^T x_S^k - V_C^T y_F^k + \lambda^T (y_F^k - x_F^k) \quad \forall k = 1 \dots K \\ & \lambda = \lambda^K + \alpha \frac{v(P_D) - v(P_R(\lambda^K))}{\|y_F^K - x_F^K\|^2} (y_F^K - x_F^K) + \delta \\ & \eta \in \mathbb{R}, \lambda \in \mathbb{R}^{|F|} \\ & \alpha \in ]-\infty, \bar{\alpha}], \delta \in [-\bar{\delta}, \bar{\delta}]^{|F|} \end{aligned}$$

- ▶ It is based on the following concepts:
  - ▶ **Cutting planes** → objective value of current vertex solution for all  $\lambda$ s
  - ▶ **Subgradient step** → steepest descent direction from current solution
  - ▶ **Trust-region** → deviation from subgradient
    - ▶ solves convergence issues of pure subgradient method
    - ▶ helps with stability for nonlinear problems

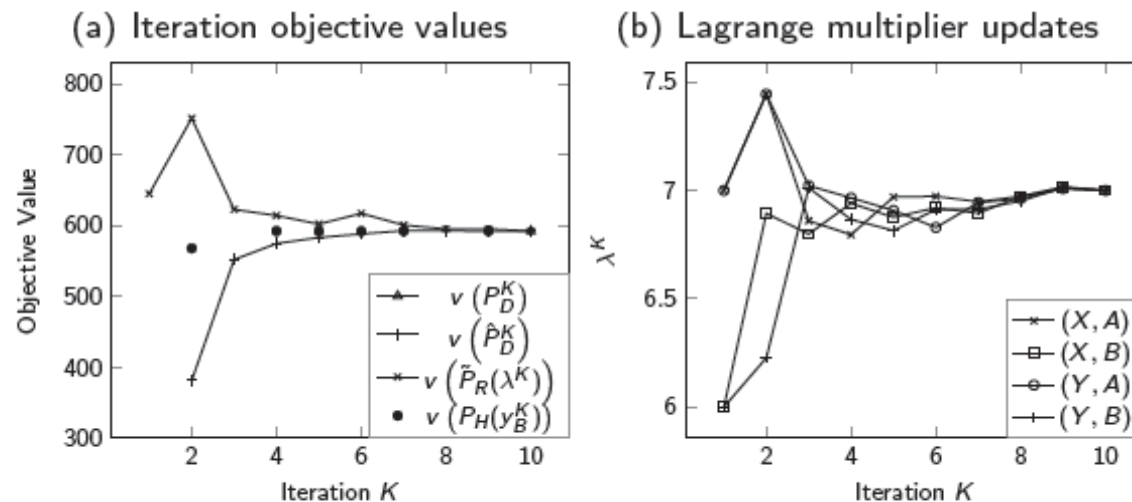
## Hybrid Lagrangian dual problem

$$\begin{aligned}
 & \min \quad \eta \\
 & \text{s.t.} \quad \eta \geq V_P^T x_S^k - V_C^T y_F^k + \lambda^T (y_F^k - x_F^k) \quad \forall k = 1 \dots K \\
 & \quad \lambda = \lambda^K + \alpha \frac{v(P_D) - v(P_R(\lambda^K))}{\|y_F^K - x_F^K\|^2} (y_F^K - x_F^K) + \delta \\
 & \quad \eta \in \mathbb{R}, \lambda \in \mathbb{R}^{|F|} \\
 & \quad \alpha \in ]-\infty, \bar{\alpha}], \delta \in [-\bar{\delta}, \bar{\delta}]^{|F|}
 \end{aligned}$$



## Numerical illustration: small case-study

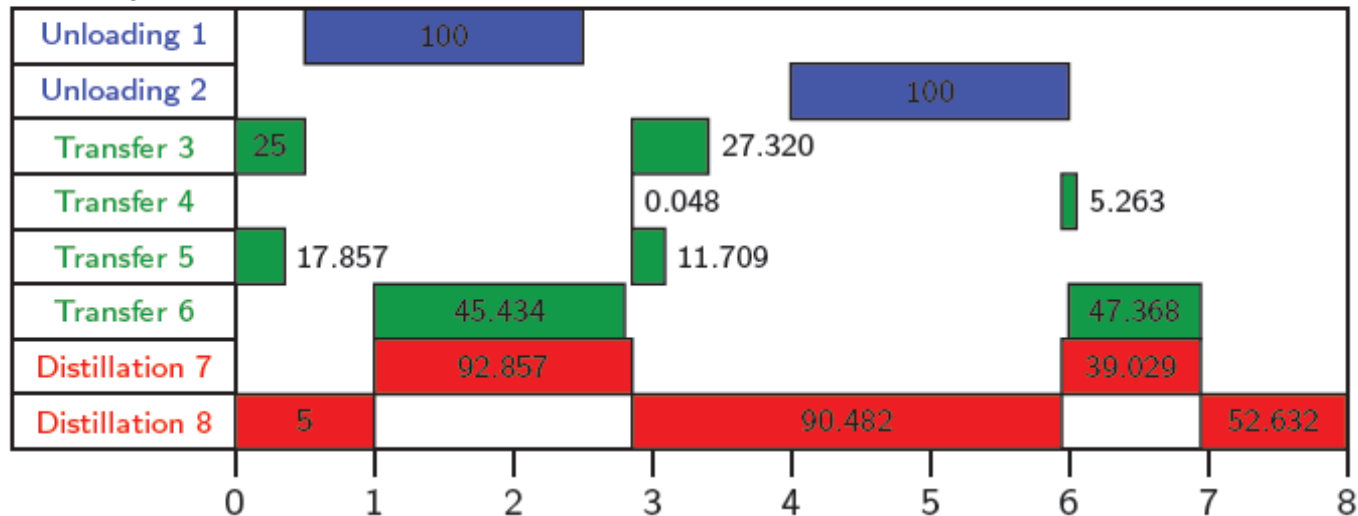
- ▶ Crude-oil operations scheduling problem COSP1
  - ▶ MOS time representation with 7 priority-slots
- ▶ Refinery planning problem expressed as a pooling problem
  - ▶ 1 CDU (2 operating modes), 4 intermediate pools, and 4 final products
  - ▶ Solved with BARON in order to provide rigorous upper bounds
- ▶ The number of Lagrange multipliers is 4 (2 crudes and 2 CDU modes)



- ▶ Fast and stable convergence in 100s (10 iteration) with 0% dual gap

## Numerical illustration: small case-study solution

► Optimal solution schedule:



► Optimal aggregated blend compositions:

	Blend 1 / Dist. 7	Blend 2 / Dist. 8
Crude A	92.320	15.311
Crude B	39.566	132.803



## Numerical illustration: larger refinery problem

- ▶ Crude-oil operations scheduling problem COSP3
  - ▶ MOS time representation with 6 priority-slots
- ▶ Refinery planning problem based on CDU fitting model expressed as an artificial neural network (Gueddar and Dua, 2010)
  - ▶ 1 CDU (3 operating modes), 5 final products
  - ▶ Solved to local optimality with CONOPT
- ▶ The number of Lagrange multipliers is 21 (7 crudes and 3 CDU modes)

(a) Comparative performance of several MINLP algorithms

MINLP Solver	Objective Value	CPU Time
Proposed	250.989	1,045s
Sequential	116.814	46s
DICOPT	—	+3,600s
AlphaECP	—	+3,600s
SBB	—	+3,600s

(b) Optimal blend composition

Crude	Mode 1	Mode 2	Mode 3
A	0.164	49.480	
B		33.293	16.707
C		50.000	
D	0.066	19.792	
E	3.500	55.943	2.982
F			50.000
G	9.480		

- ▶ Convergence in less than 20 iterations, more than 80% of CPU time is spent on solving the crude-oil scheduling problem

## Conclusions

- ▶ A general integration approach for refinery planning and crude-oil scheduling problems
  - ▶ Based on a **Lagrangian decomposition** of the full-space problem
- ▶ A new hybrid dual algorithm for optimizing the Lagrange multipliers
  - ▶ Based on **cutting planes** and a **subgradient-based trust-region**
  - ▶ Two parameters only:
    - ▶ maximum step size
    - ▶ maximum deviation from the subgradient step