Strategies for planning and long-term scheduling for PPG glass production

Ricardo Lima

Ignacio Grossmann

rlima@andrew.cmu.edu

Carnegie Mellon University

Yu Jiao

PPG Industries

Glass Business and Discovery Center

Introduction

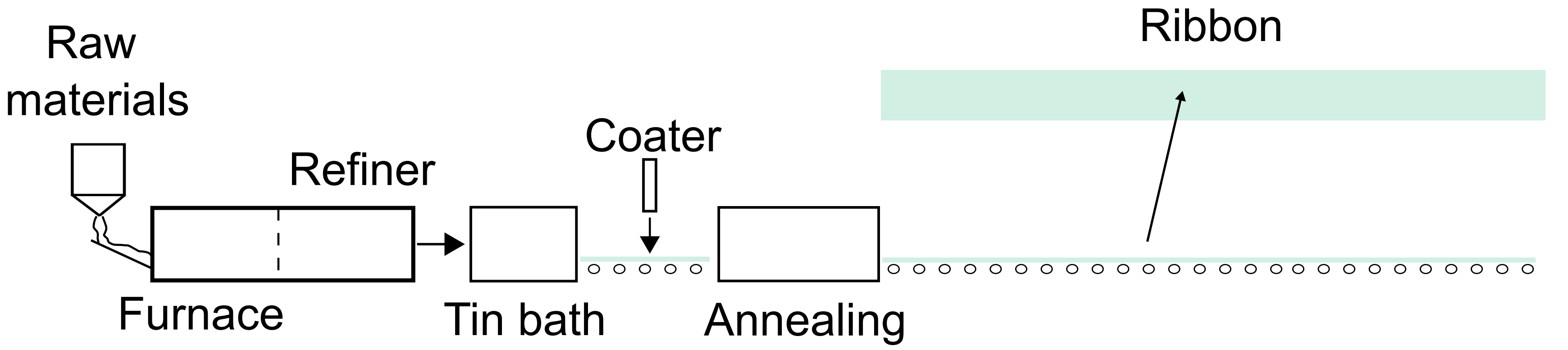
Aim of the project:

- Development of a Mixed Integer Linear Programming (MILP) model for the **planning and scheduling of PPG tinted glasses production**
  - introduce **economic factors** on the choice of **schedules**
  - **capture the essence** of the process that **is not considered** in the Master Production Schedule (MPS)
    - cullet management
    - changeovers driven by SKUs

    that may lead to infeasible schedules
- develop a decision making production tool to perform systematic choices of production schedules based on a performance criterion

Process description

Continuous process:



- **sequence dependent changeovers** + **no changeovers** between some products
- **long transition times** (order of days) with **high** transition **costs**
- constraints on the relation between the processing times of some products
- **minimum run length**
- process **does not stop**
- during changeovers a **waste product is produced**

- waste product produced during changeovers is recycled

Project complexity stages

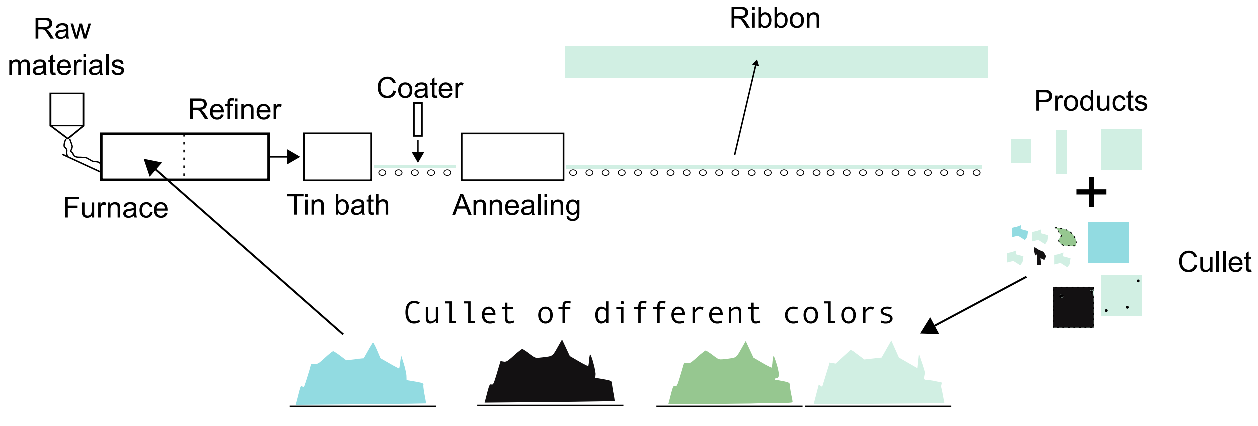
Stage I - Simplified model

Assume that the products are only defined by color, and characterized by

- production rates
- maximum and minimum inventory levels
- demand
- selling prices

Stage II - Detailed model

2 - Include cullet storage, recycling, and sales on the scheduling and planning



Optimize the profit considering **optimal cullet levels**

Problem statement

Given

- a time horizon
- transition, and inventory costs
- set of products
  - deterministic **demand**
  - initial, minimum, and maximum **inventory** levels
  - **production rates**
  - sequence dependent **transitions**
  - operating **costs**
  - **selling prices**

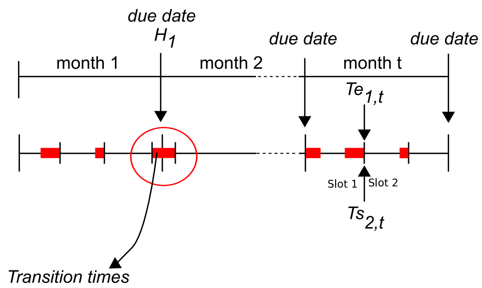
Determine

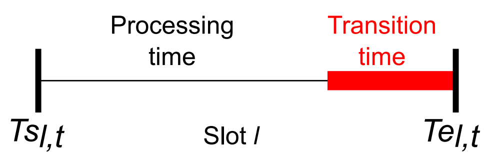
- amounts to be produced
- production times
- sequence of production
- inventory levels during and at the end of the time horizon
- economic terms: total operating, transition, inventory costs

That **maximize the profit**

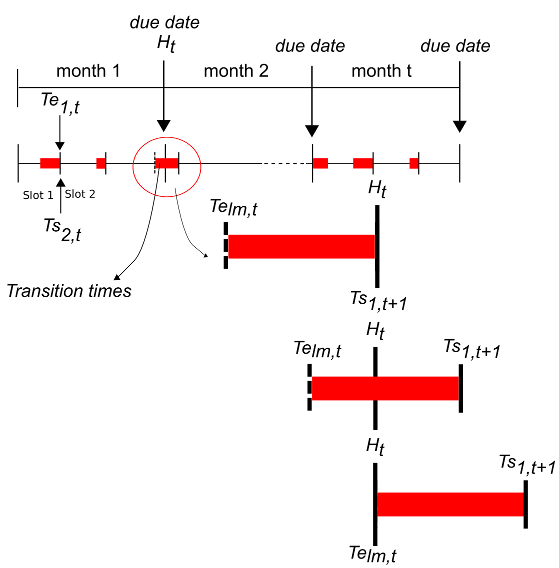
Model assumptions

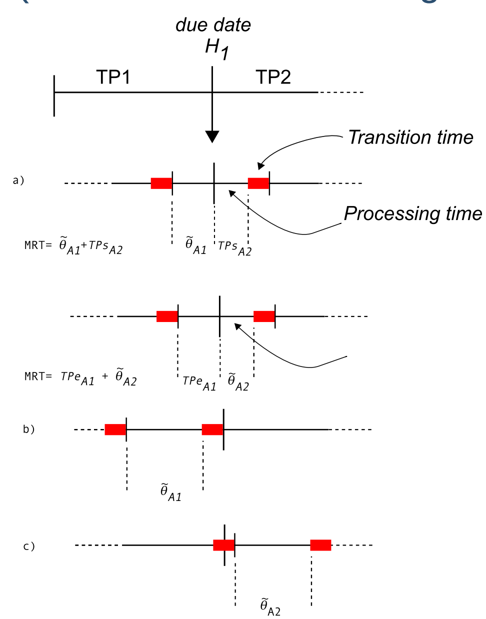
- **Continuous time slot-based formulation** (Erdirik-Dogan and Grossmann, 2008)





- **Transition times across due dates** (extension of Erdirik-Dogan and Grossmann, 2008)
- **Minimum run length across due dates** (extension of Erdirik-Dogan and Grossmann, 2008)

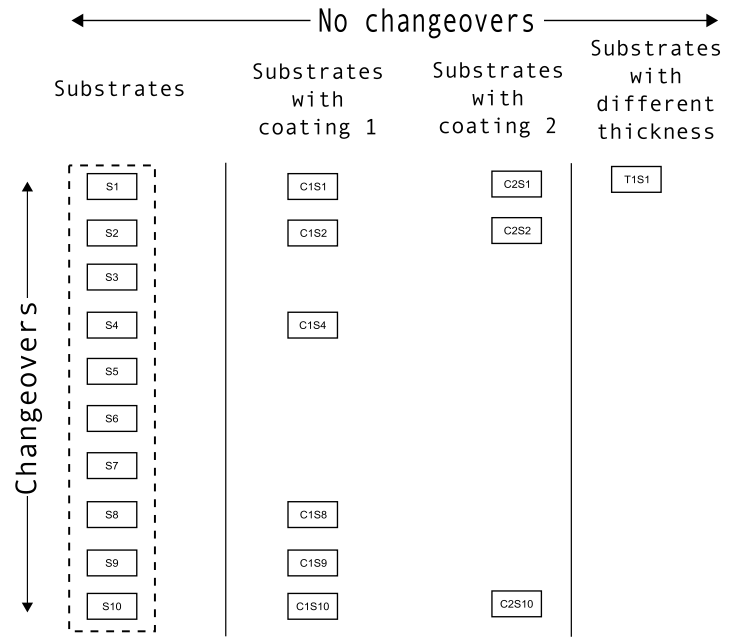




Motivated by long transition times and by minimum run lengths longer than the length of the time periods

Model assumptions

- Rigorous aggregation of the products



Inventory: all products are considered in inventory balances

Scheduling: only pseudo-products are considered, ie only the products with changeovers. The processing time of the pseudo-products is given by:

$$\tilde{\theta}_{i,t} = \tilde{\theta 2}_{i,t} + \sum_{k \in CT1} \tilde{\theta 2}_{k,t} + \sum_{k \in CT2} \tilde{\theta 2}_{k,t} + \sum_{k \in THK} \tilde{\theta 2}_{k,t}$$

Processing time = pseudo-product; processing time + substrate; processing time + coating 1; processing time + coating 2; Processing time $\neq$ thickness

Reduces the number of binary variables and equations

MILP model

Maximize the profit:

Profit = Sales - Inventory costs - Operating costs - Transition costs - - Backlog penalty - Inventory penalties

$$Z = \sum_{i \in IM} \sum_{t \in TSC} p_{i,t} \cdot S_{i,t} - \sum_{i \in IM} \sum_{t \in TSC} cinv_{i,t} \cdot Area_{i,t} - \sum_{t} \sum_{i \in I} coper_{i,t} \cdot r_i \cdot \tilde{\theta 2}_{i,t}$$
$$- \sum_{i \in IM} \sum_{k \in IM} \sum_{l \in LM \backslash LML} \sum_{t \in TSC} ctran_{i,k} \cdot Z_{i,k,l,t} - \sum_{i \in IM} \sum_{k \in IM} \sum_{t \in TSC \backslash TL} ctran_{i,k} \cdot TRT_{i,k,t}$$
$$- \sum_{i \in IM} \sum_{t \in TSC} PEN1_t \cdot q_t - \sum_{i \in IM} \sum_{t \in TSC} PEN2_{i,t} \cdot s_{i,t} - \sum_{i \in IM} \sum_{t \in TSC} PEN3_{i,t} \cdot BCKL_{i,t}$$

Subject to:

- Assignment and production constraints
  - Minimum run length across due dates
- Transitions
  - between pseudo-products in the time periods
  - between pseudo-products across the due dates
- Time relations
- Inventory, backlog and demand

Solution approaches

Model statistics:

- Time horizon of 18 months, time periods of 1 month
- 20 Products (10 pseudo-products, and 10 sub-products), using the rigorous aggregation of products that decreases number of products and slots.

	Subset of 0-1 var. as continuous	All 0-1 var. as binary
Equations	18,703	18,703
Variables	28,498	28,498
Binary variables	2,160	20,060

Solution strategies

- full space model
- bi-level decomposition
- forward rolling horizon algorithm
- inventory level at the end of the time horizon
  - extended time horizon of 18 months
  - obtain the inventory levels for the end of the 12th month
  - set 12 months of time horizon, and $INV_{i,TH} = \max\{INVmin_i, INV_{i,12}\}$

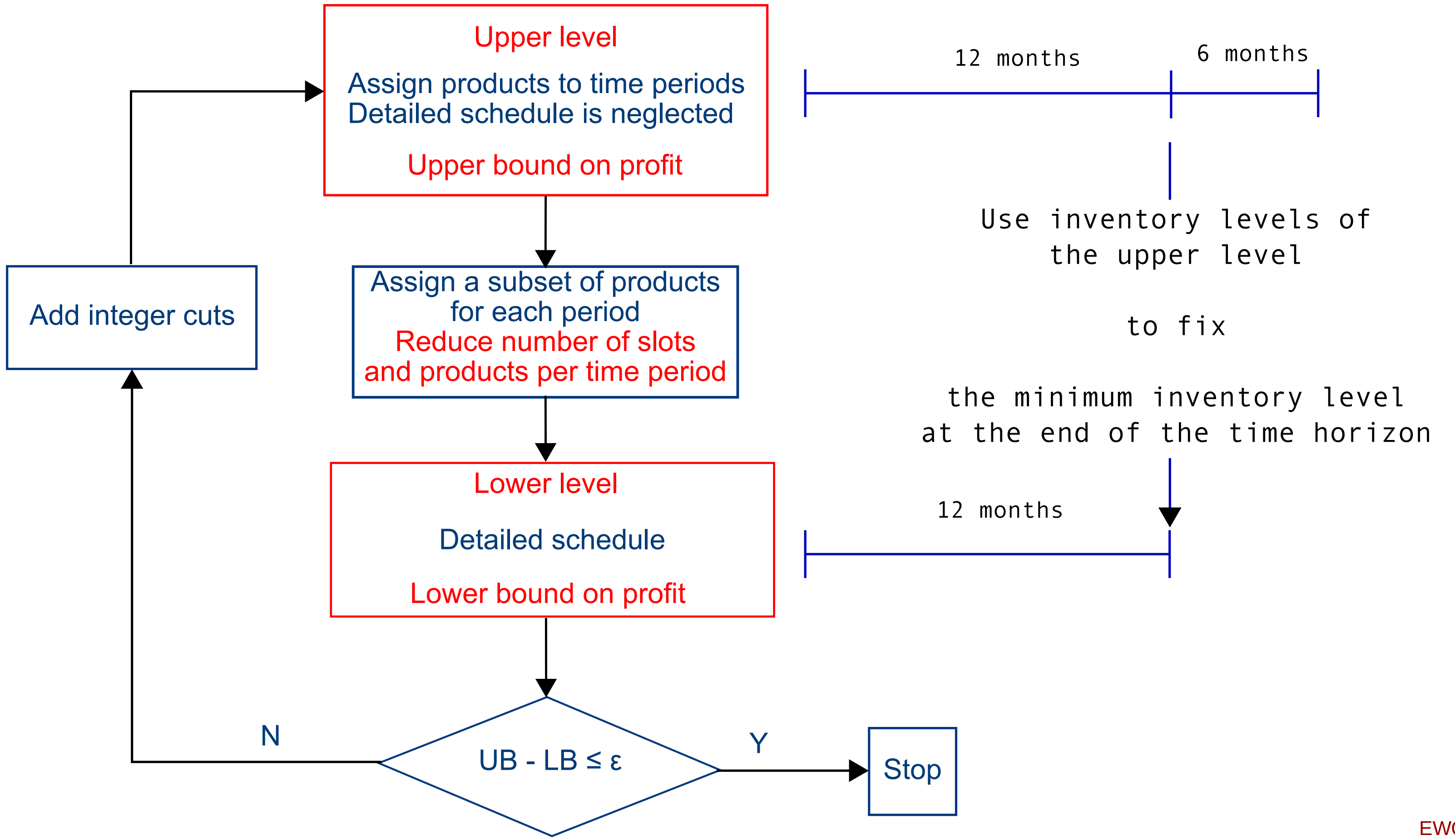
Bi-level decomposition

(Erdirik-Dogan and Grossmann, 2008)

Upper level: does not consider detailed scheduling, decides which products are produced in each time period

- advantage: reduction in the number of equations and continuous variables
- disadvantage: the number of binary variables increases

Lower level: Solve the full model for a subset of products in each time period

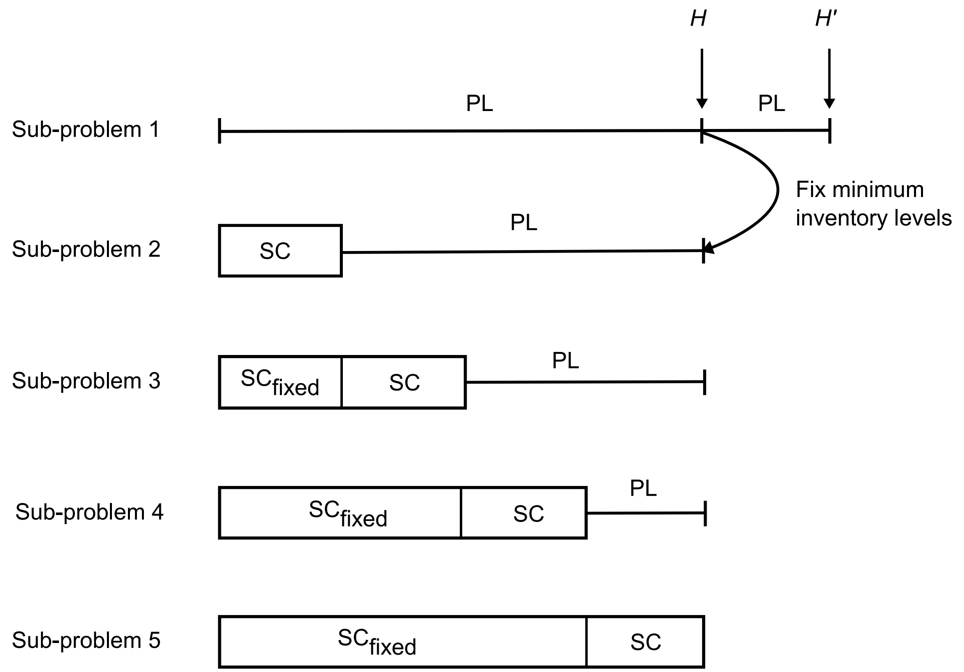


Rolling horizon algorithm

(Dimitriadis, A. D. et al,1997)

Main steps:

- solve a planning model for 18 months
- solve a sequence of subproblems with a time horizon of 12 months:
  - products assigned to each time period are defined by the previous solution
  - after the solution of a subproblem the binary variables of its scheduling model are fixed.



Remarks:

- Changeovers across due dates and minimum run lengths are considered in the interface of the detailed scheduling and planning model.

Results

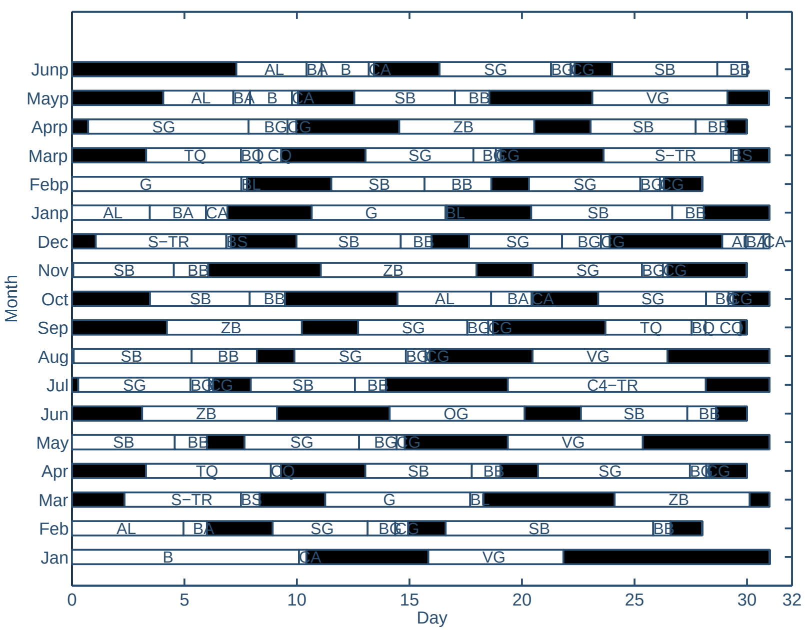
- **Case I** - Bi-level decomposition, without changeovers and minimum run lengths across time periods
- **Case II** - Bi-level decomposition
- **Case III** - Rolling horizon algorithm

Time horizon of 18 months in the upper level, and 12 months in the lower level. Time periods of 1 month.

Models implemented in GAMS and solved with Cplex 11.2.1. CPU used: 4 threads with Intel Xeon, 2.66GHz and 8 GB.

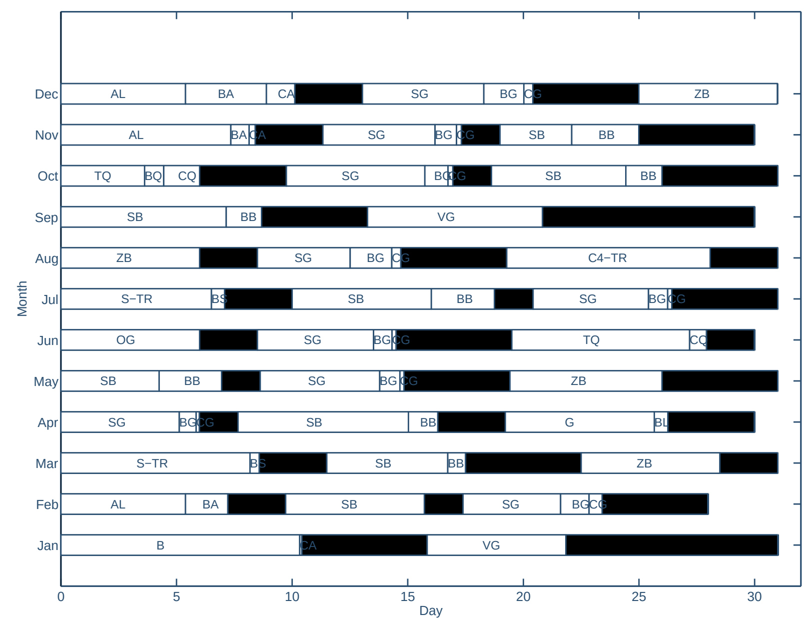
Full space

Profit = $557,945



Case I

Profit = $554,003

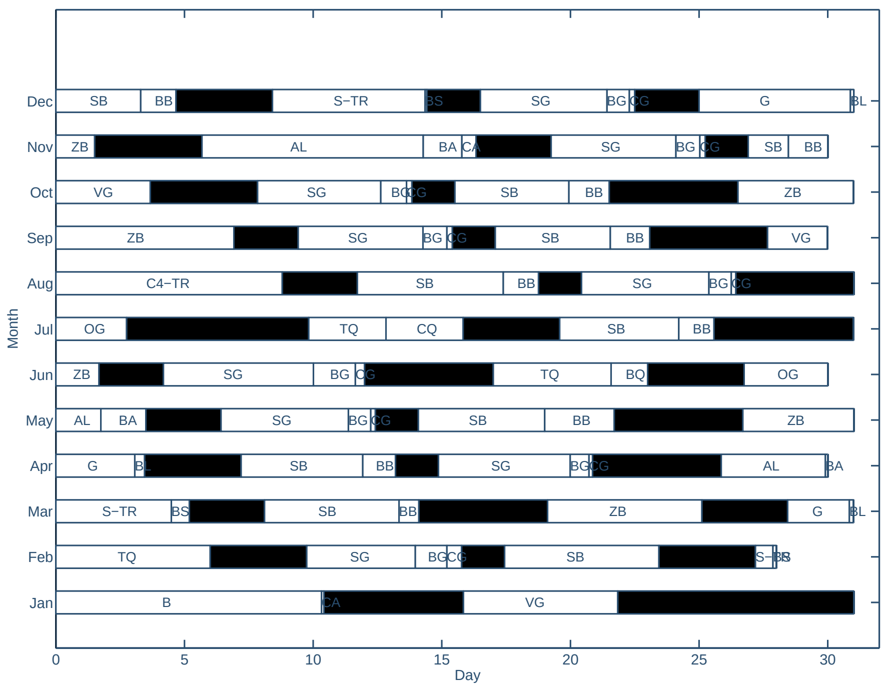


All $ values have been normalized.

Results

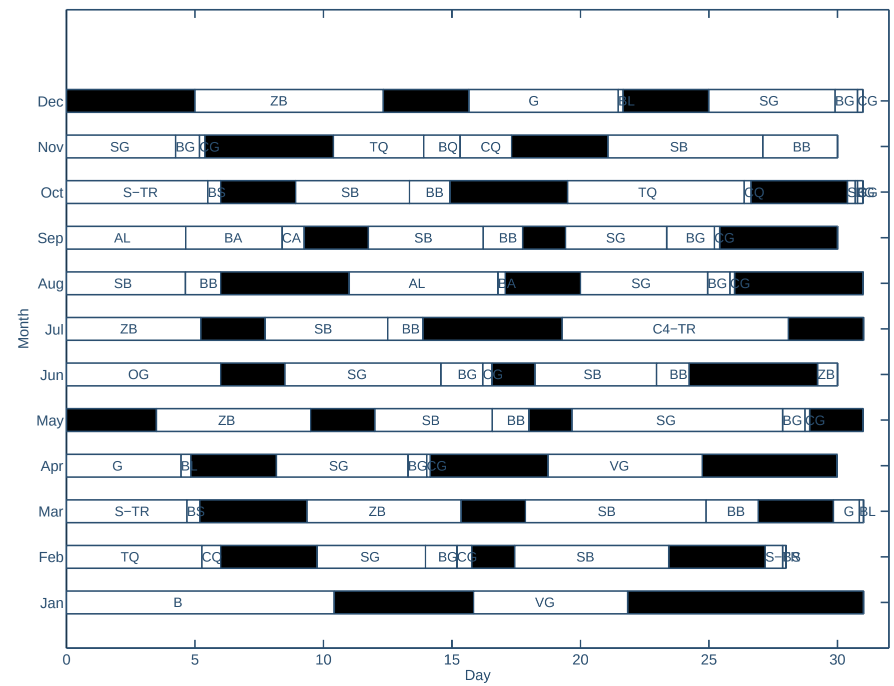
Case II

Profit = $554,284



Case III

Profit = $553,820



Case I does not allow changeovers and minimum run lengths across time periods. The last slot of the time period must include a transition. Case III shows minimum run lengths across time periods and changeovers crossing the due dates.

All $ values have been normalized.

Profit, sales, and costs

	Full space*	Full space$^+$	Case I	Case II	Case III
Profit ($)	557,945	558,409	554,003	554,284	553,820
Sales ($)	1,314,771	1,314,771	1,314,771	1,314,771	1,314,771
Inventory ($)	124,006	123,835	124,497	123,415	124,483
Operating ($)	479,011	478,242	488,175	490,406	488,373
Transitions ($)	153,810	154,286	148,095	146,667	148,095
# Transitions	36	36	33	35	34
# Transition days			130	128	130
Tons below min	325	30	350	305	305
Backlog (ton)	0	0	0	0	0
Above max Cap (ton)	1336	1895	1336	1336	1336

Full space*: All 0-1 variables declared as binary, Full space$^+$: some 0-1 variables declared as continuous.

All $ values have been normalized.

Computational results

		# Eq.	# Var.	#0-1 Var.	# Slots	CPU (s)	Gap (%)	Fobj
Full space*		18703	28498	20060	180	18000	20.1	-16519
Full space+		18703	28498	2160	180	18000	38.0	-19161
Case I	Upper level	10987	8217	3780		10800	23.32	-16769
Iter 1	Lower level	2307	2194	166	34	0.2	0	-16800
Case II	Upper level	16767	11957	5840		10800	11.23	-16319
Iter 1	Lower level	3453	3262	1053	45	12	0.00	-16350
Case III - Det								
Planning	18 P	16767	11957	5840		10800	11.22	-16319
Scheduling	6 S + 6 P	7335	5639	2555	23	54	0.01	-16351
Scheduling	12 S	3072	2758	531	46	1.1	0	-16350
Case III - Opp								
Planning	18 P	16767	11957	5840		2740	0.02	-16319
Scheduling	6 S + 6 P	7334	5639	2555	23	41	0.01	-16351
Scheduling	12 S	2815	2456	330	42	0.18	0.01	-16351

Full space*: all 0-1 variables declared as binary, Full space+: some 0-1 variables declared as continuous. Full space: maximum time of 18,000 s, tolerance of 0.05%, opportunistic parallel mode.Case I, II, and III: all 0-1 variables declared as binary. Case I: maximum time of 10,800 s, tolerance of 0.05%, deterministic parallel mode. Case II, III: maximum time of 10,800 s, tolerance of 0.05%, Det: deterministic, Opp: opportunistic parallel modes. Case III: 1st subproblem with scheduling has 6 months of scheduling and 6 months of planning. 18 P: 18 months of planing, 6 S: 6 months of scheduling, 12 S: 12 months of scheduling.

Concluding remarks

- **An improved model** was developed in order to cope with the features of the process
- The new features of the model (transitions and minimum run lengths across due dates) make the **model more flexible** to handle transitions and run lengths, and to cope with inventory constraints.
- The bi-level decomposition and the rolling horizon algorithm show better performance than the full space model with **lower CPU times**
  - better performance in terms of objective function and integrality gap
  - less violation of inventory constraints
- **Research challenges**
  - How to model the complex stages (cullet management and SKU)?
  - How to effectively solve industrial problems?