Planning and scheduling of spectrally selective tinted glasses production

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Aim of the project

- Development of an **MILP model** for the **integrated planning and scheduling** of the production line of spectrally selective glasses
  - **capture the essence** of the process that **is not considered** in the Master Production Schedule (MPS)
    - cullet management
    - changeovers driven by SKUs

    that may lead to infeasible schedules
  - develop a production tool to perform systematic choices of production schedules based on performance criteria

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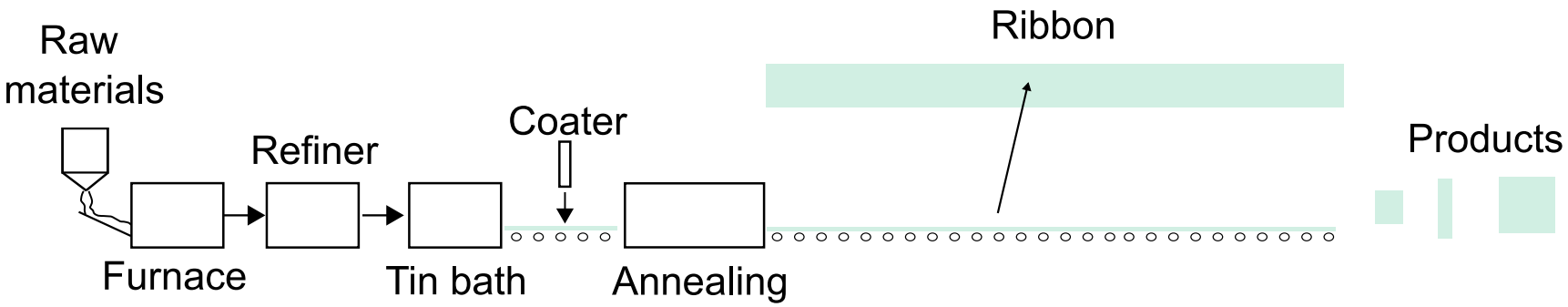
  - develop a production tool to perform systematic choices of production schedules based on performance criteria
- **Conduct economic studies**
  - Evaluate the **maximum profitability** of the production line
  - Analyze the possibility of producing **more products** in the same production line
  - Try to evaluate the **production cost** and **profit contribution** of each product

Process description

Continuous process



Characteristics

- sequence dependent transitions
- long transition times (order of days)
- high cost transitions
- no transition time between substrate and substrate with coating
- coater cleaning
- process does not stop

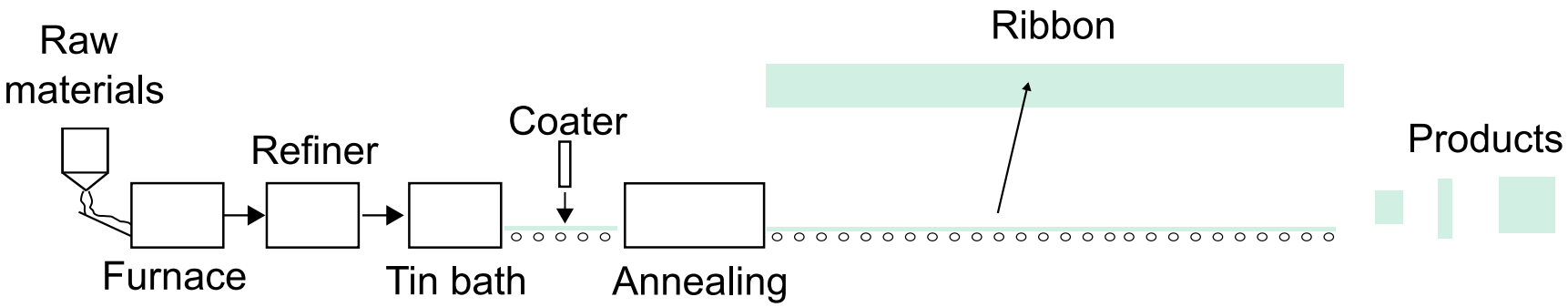
Spectrally selective tinted glasses

	Substrate	Coated*
Solexia		
Caribia		
Azuria		
Solarbronze		
Solargray		
Graylite		

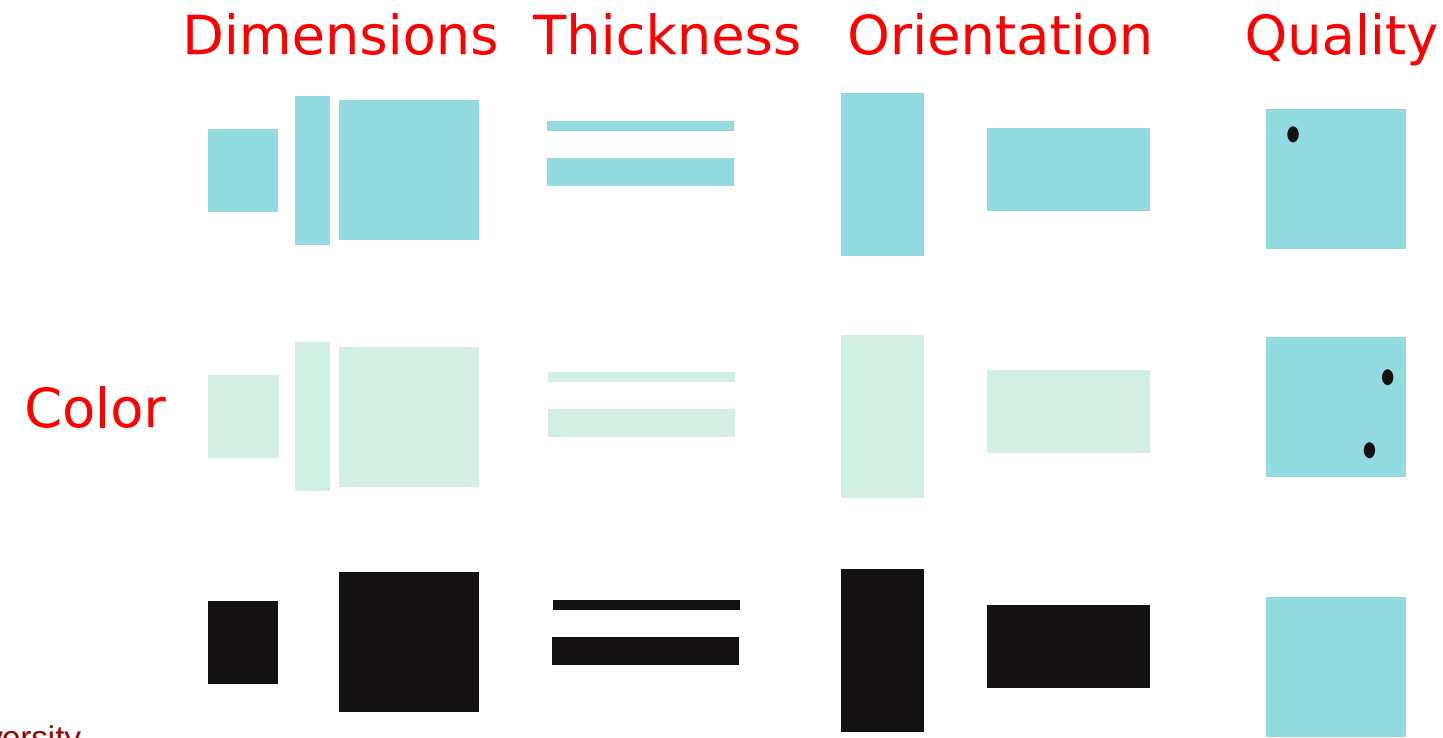
* Reflective coated glass

Process description

Continuous process



The products are defined by stock keeping unit (SKU), among others characterized by:



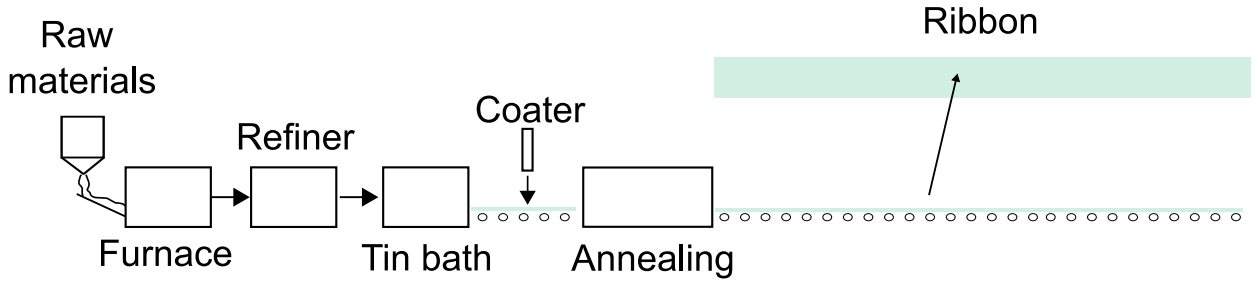
Project complexity stages

Stage I - Simplified model

Assume that the products are only defined by colors and coating, and

- production rates
- maximum and minimum inventory levels
- demand
- selling prices

are based on the amount (ton) of colors.



Define strategy for the integrated planning and scheduling, this may involve:

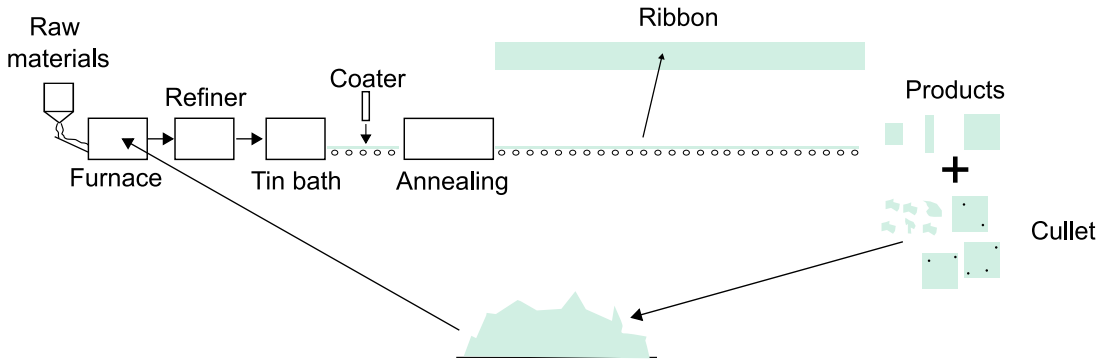
- decomposition approach
- forward rolling horizon algorithm
- Lagrangean decomposition

Project complexity stages

Stage I - Simplified model

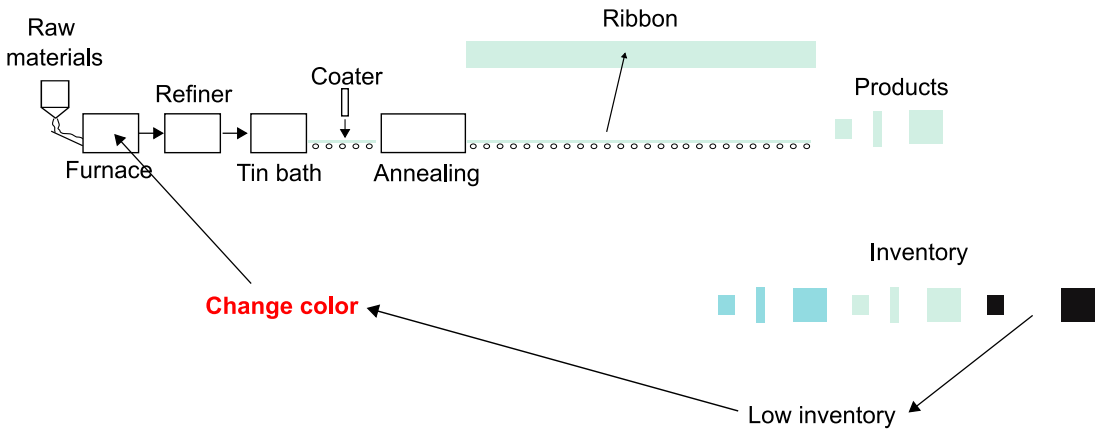
Stage II - Detailed model

1 - Include cullet storage, recycling, and sales



Cullet: glass produced during transitions, broken, defects

2 - Define the products by SKU, and consider SKUs inventory and sales levels



At present, both are not considered by the MPS

Problem statement

Given

- deterministic product **demands** over time
- initial, minimum, and maximum **inventory** levels of products
- **production rates**
- sequence dependent **transitions**
- operating, inventory, and transition **costs**
- **selling prices**

Determine

- amounts to be produced
- production times
- sequence of production
- inventory levels
- sales

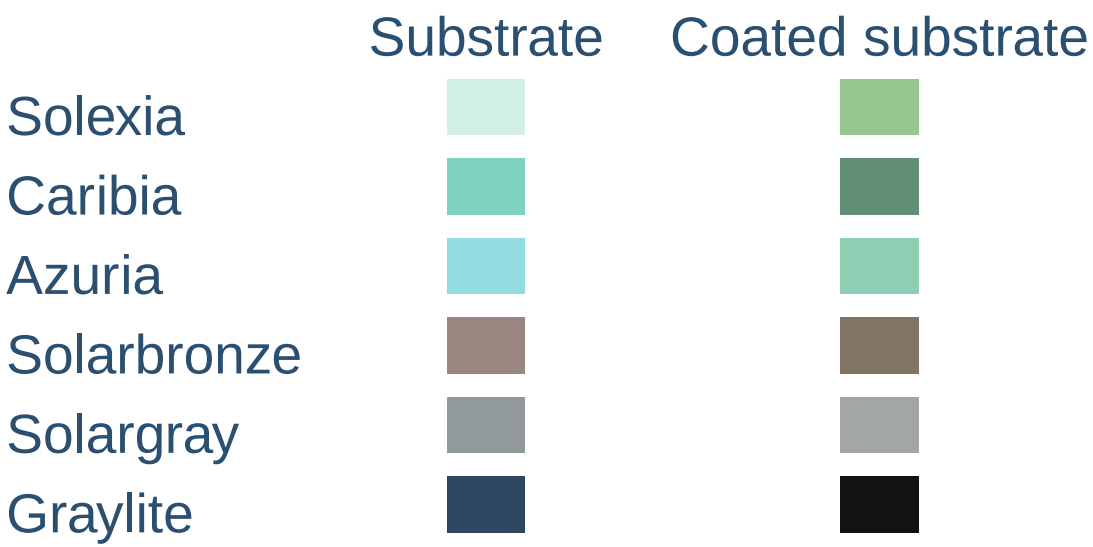
That **maximize profit**

Model

Assumptions

Process

- Neglect of transition times between substrate and substrate with coating
- Equal production times of substrate and substrate with coating in the same time period, to clean the coater
- Minimum run lengths for the total of substrate and substrate with coating
- Maximum inventory capacity

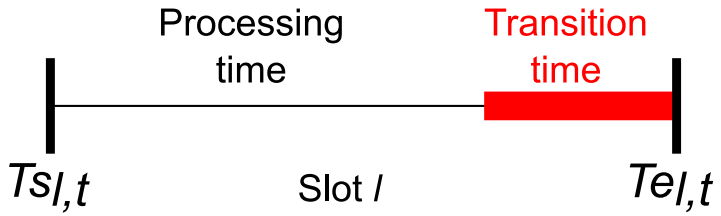


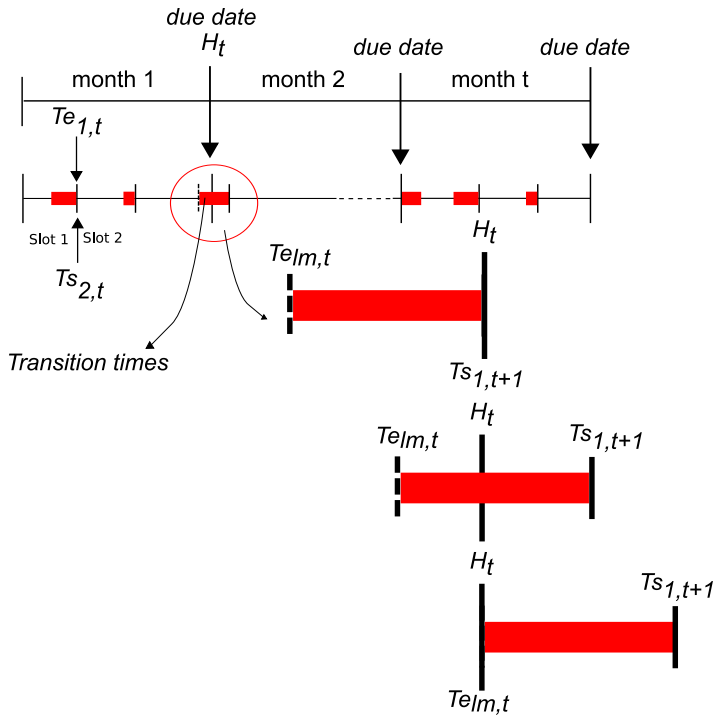
Model

Assumptions

Formulation

- slot based continuous time representation
- variable length slots
- one product per slot
- products may take more than one slot
- fixed number of slots per time period
- transition times across time periods (extension of Erdirik-Dogan and Grossmann (2007))





MILP Model

Transitions across time periods

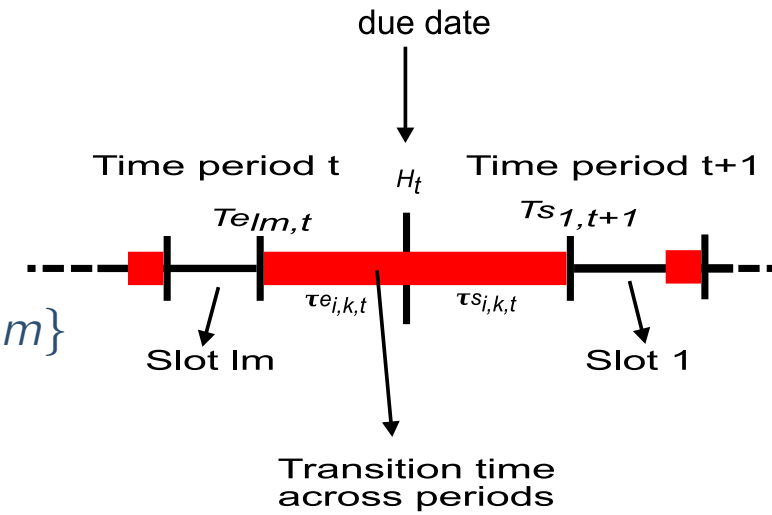
$$\tau_{i,k} TRT_{i,k,t} = \tau e_{i,k,t} + \tau s_{i,k,t} \qquad \forall i,k,\forall t \in T \setminus \{tm\}$$

$$Te_{lm,t} + \sum_i \sum_k \tau e_{i,k,t} = HT_t \qquad \forall l, t \in T \setminus \{tm\}$$

$$Ts_{l,t+1} = \sum_{i \in l} \sum_{i \in l} \tau s_{i,k,t-1} + HT_{t-1} \qquad l = 1, \forall t \in T \setminus \{tm\}$$

$$\tau e_{i,k,t} \leq \tau_{i,k} TRT_{i,k,t} \qquad \forall i,k,\forall t \in T \setminus \{tm\}$$

$$\tau s_{i,k,t} \leq \tau_{i,k} TRT_{i,k,t} \qquad \forall i,k,\forall t \in T \setminus \{tm\}$$



Coating application, and coater cleaning

$$YOP_{i,t} - YOP_{k,t} \geq 0 \qquad \forall i,k \in FAM$$

$$\sum_l \theta_{i,l,t} \geq \sum_l \theta_{k,l,t} - M(1 - YOP_{k,t}) \qquad \forall i,k \in FAM, \forall t$$

Minimum run lengths

$$\sum_{l \in L} \theta_{i,l,t} + \sum_{l \in L} \sum_{k \in FAM} \theta_{k,l,t} \geq MRT_i \cdot YOP_{i,t} \qquad \forall i \in COLOR, \forall t$$

Maximum inventory capacity

$$\sum_i INVO_{i,t} \leq INV_{max} \qquad \forall t$$

MILP Model

Maximize profit:

$$Z^P = \sum_i \sum_t MP_{i,t} S_{i,t} - \sum_i \sum_t CINV_{i,t} A_{i,t} - \sum_i \sum_t COP_{i,t} X_{i,t} - \sum_i \sum_t c^{trans}_{i,k} Z_{i,k,l,t} - \sum_i \sum_t c^{trans}_{i,k} Z_{i,k,l,t}$$

subject to:

$$\sum_i W_{i,l,t} = 1 \quad \forall l, t$$
$$0 \le \tilde{\theta}_{i,l,t} \le H_t W_{i,l,t} \quad \forall i, l, t$$
$$\theta_{i,t} = \sum_l \tilde{\theta}_{i,l,t} \quad \forall i, t$$
$$X_{i,l,t} = r_i \theta_{i,l,t} \quad \forall i, l, t$$
$$\tilde{X}_{i,l,t} = \sum_l X_{i,l,t} \quad \forall i, l, t$$

} Assignment and production

$$Te_{l,t} = Ts_{l,t} + \sum_i \theta_{i,l,t} + \sum_i \sum_k \tau_{i,k} Z_{i,k,l,t} \quad \forall l, t$$
$$Te_{l,t} = Ts_{l+1,t} \quad \forall l \neq N, t$$

} Time relations

$$\sum_k TRT_{i,k,t} = W_{i,l,t} \quad \forall i, l = \bar{l}, t \in T \setminus \bar{t}$$
$$\sum_i TRT_{i,k,t} = W_{i,l,t+1} \quad \forall i, l = \bar{l}, t \in T \setminus \bar{t}$$
$$\sum_k Z_{i,k,l,t} = W_{i,l,t} \quad \forall i, t, l \in L \setminus \bar{l}$$
$$\sum_i Z_{i,k,l,t} = W_{k,l+1,t} \quad \forall i, t, l \in L \setminus \bar{l}$$

} Transitions

$$S_{i,t} \ge d_{i,t} \quad \forall i, t$$

} Demand

$$INV_{i,t} = INVI_i + \sum_l r_i \theta_{i,l,t} \quad \forall i, t = 1$$
$$INV_{i,t} = INVO_{i,t-1} + \sum_l r_i \theta_{i,l,t} \quad \forall i, t \neq 1$$
$$INVO_{i,t} = INV_{i,t} - S_{i,t} \quad \forall i, t$$
$$A_{i,t} \ge INVO_{i,t} H_t + r_i \theta_{i,t} H_t \quad \forall i, t$$

} Inventory

- Transitions across time periods
- Coating application
- Minimum run lengths
- Maximum inventory capacity

Full model results

- 13 products
- time horizon of **6** and **10 months**
- **6 slots** per time period

Full model results

	Equations	Variables: Continuous	Variables: Binary	CPU (s)	Profit (m.u.)	Gap (%)
6 months	6,419	10,571	624	9,000	1.131	11.8
10 months	11,019	17,635	1,040	-	-	-

Intel Xeon CPU 1.86GHz, 8Gb, GAMS/Cplex 10.2

Remarks:

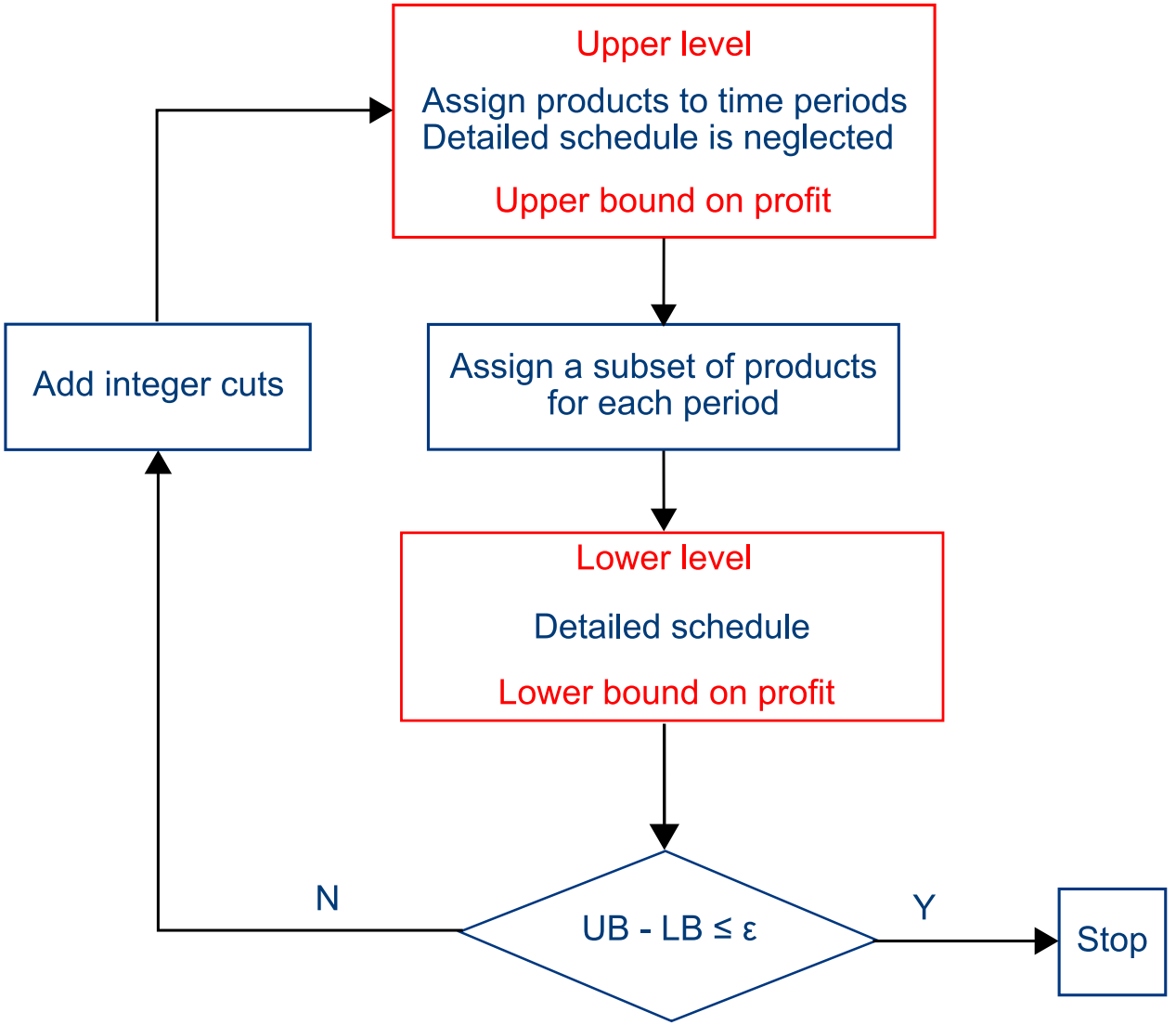
- **large scale model**, even though we are still using the simplified model
- **6 months** - Cplex stopped on the maximum time set, with a gap of 11.8%
- for **10 months** no solution was obtained
- problem **becomes intractable** for long time horizons

Bi-level decomposition

Bi-level decomposition proposed by Erdirik-Dogan and Grossmann I.E. (2007)

Upper level: Decide which products are produced in each time period

Lower level: Solve the full model for a subset of products in each time period



Case studies

Decomposition strategy

Case 1 - 6 months time horizon

- 13 products
- **6** and **13 slots** per time period

Case 2 - 10 months time horizon

- 13 products
- **6 slots** per time period

Computational results

Algorithm performance

Case 1 - 6 months

	6 slots				13 slots			
Iteration	**UB**	CPU (s)	**LB**	CPU (s)	**UB**	CPU (s)	**LB**	CPU (s)
1	1.196	125.2	1.109	65.5	1.196	129.5	1.109	1,882.2
2	1.196	101.6	1.111	126.7	1.196	134.1	1.111	1,178.3

Intel Xeon CPU 1.86GHz, 8Gb, GAMS/Cplex 10.2

Case 2 - 10 months

	6 slots			
Iteration	UB	CPU (s)	LB	CPU (s)
1	1.601	592.3	1.363	9,000*

* integrality gap 10.5%

Intel Xeon CPU 1.86GHz, 8Gb, GAMS/Cplex 10.2

Remarks:

- A number of slots equal to the number of products is not needed because of the large transition times.
- The same profit was obtained for 6 and 13 slots for Case 1.

Computational results

Comparison between full model solution and decomposition approach

	Equations	Variables: Continuous	Variables: Binary	CPU (s)	Profit (m.u.)	Gap (%)
Full model results						
6 months	6,419	10,571	624	**9,000**	**1.131**	11.8
10 months	11,019	17,635	1,040	**9,000**	-	-
Decomposition						
6 slots						
6 months	6,498	10,615	580	**126.7**	**1.111**	0.0
10 months	11,149	17,696	979	**9,000**	**1.363**	10.5

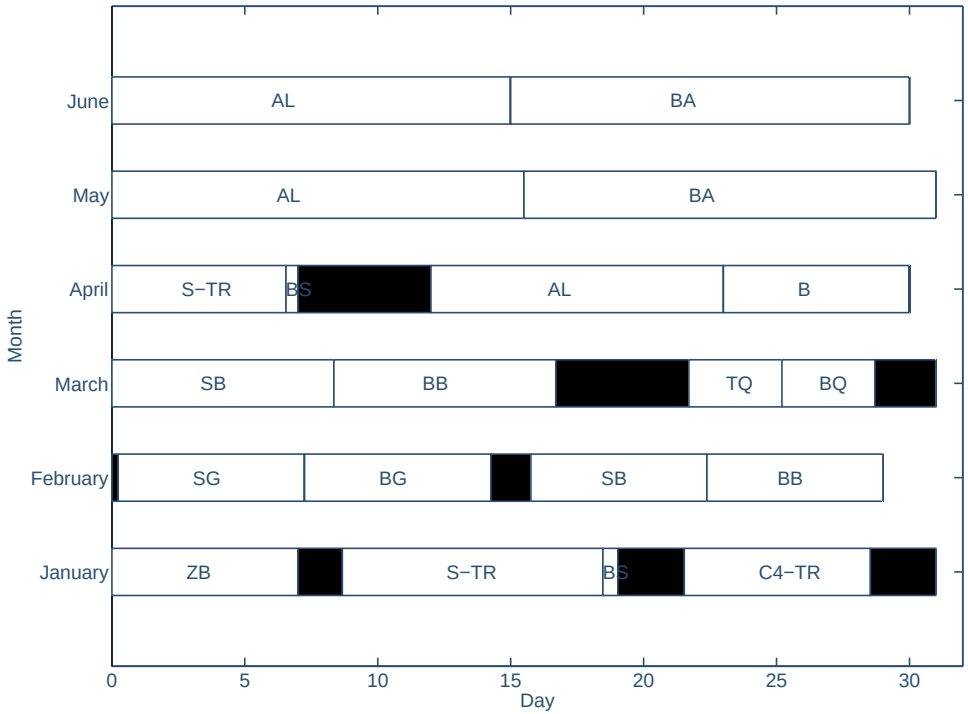
Intel Xeon CPU 1.86GHz, 8Gb, GAMS/Cplex 10.2

Remarks: 1.77% gap on the profit between full model and decomposition strategy.

Gantt charts

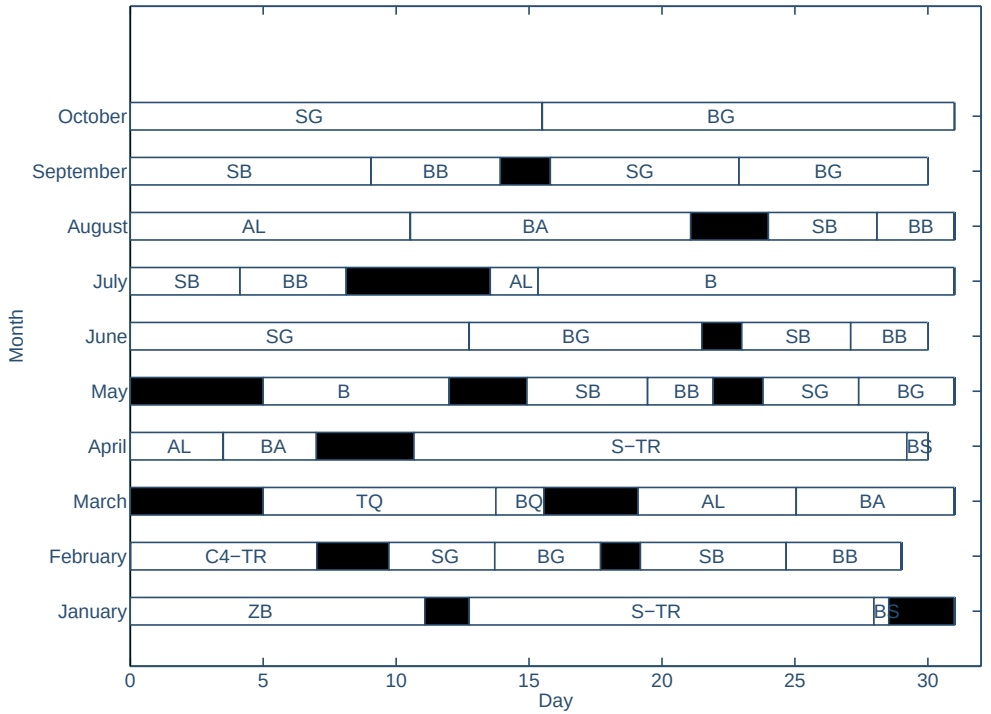
6 months, 6 slots

Profit = 1.111 m.u.



10 months, 6 slots

Profit = 1.363 m.u.



Remarks:

- We are considering sales $\geq$ demand
- Long transition times
- Transition times on the beginning of time periods

Summary and future work

- **New model** for the planning and scheduling of spectrally selective tinted glasses production
- **Complex model** that requires special decomposition strategies
- Solutions for 6 months time horizon can be **easily obtained**
- 10 months time horizon **challenges current decomposition strategy**

Future work

- Reduce computational burden of the simplified model
- **Improve or develop** a new solution strategy, that can cope with the current model complexity
- Extend the model for **cullet management and SKU**