Energy Procurement Portfolios and Production Planning
EWO Spring Meeting ’11

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An Update to the Old Project

Figure: Total Production to Total Demand Ratio
New Project

Production Setting
- consider a single product, single plant
- manufacturing requires only energy
- deterministic demand
- stockouts are not allowed
- the plant has an energy storage system (ESS)

Contracts
- fixed price (FP or $f$)
- quantity-based tiered contracts (QB or $q$)
- time-based tiered contracts (TB or $b$)
- spot market access (SP or $s$)
Objective

- Minimize production costs
- Find a portfolio of contracts
- Find the optimal power import schedule

Contract Attributes

- enabled demand charge for all contracts
- modified TB contract to use a generic subset of hours
- enabled energy storage right from the start
- all of the contracts can be active simultaneously
## Similar Models

### Chan et al., 2006

Time zone (TZ) contract and loading curve (LC) contract

### Conejo et al., 2006

- Spot market, bilateral contracts similar to TB contract and self-production. Uncertainty is related to electricity pool prices.
- Markovitz type
- Price volatility

### Carrion et al. 2007-2010

- Same setting as above.
- Stochastic programming (price scenarios)
- CVaR: the expected cost of the procurement in the worst (greater cost) $\alpha$% of the price scenarios
**Model Notation**

Sets

- \( A \) contract categories := \( \{f, q, b, s\} \)
- \( D \) set of days in planning horizon := \( \{1, \ldots, 7\} \)
- \( T \) set of hours per day := \( \{1, \ldots, 24\} \)
- \( T_1 \) set of peak hours := \( \{9, \ldots, 13\} \cup \{17, \ldots, 21\} \)
- \( T_2 \) set of off-peak hours := \( T \setminus T_1 \)

Hours

\( \mathcal{H} := D \times T \)

:= \( \{1, \ldots, 168\} \)

set of hours
Model Parameters

**Contract Cost Parameters**

- $c_f$: flat rate ($\mathcal{c}/\text{kWh}$)
- $c_b^1, c_b^2$: peak, off-peak rate ($\mathcal{c}/\text{kWh}$), requires $T_1, T_2$
- $c_q^1, c_q^2$: before-qlimit, after-qlimit rate ($\mathcal{c}/\text{kWh}$), requires $q\text{limit}$
- $c_s$: expected hourly spot market rate ($\mathcal{c}/\text{kWh}$), requires $[V^s]$
- $[V^s]$: sample price covariance matrix ($\mathcal{c}/\text{kWh})^2$
### Data

<table>
<thead>
<tr>
<th>Demand Charge</th>
<th>Fixed Price</th>
<th>Time Based</th>
<th>Quantity Based</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tier 1</td>
<td>20 (peak) 35</td>
<td>(≤ 500 kW) 15</td>
<td>1000 $</td>
</tr>
<tr>
<td>Tier 2</td>
<td>20 (off-peak) 15</td>
<td>(&gt; 500 kW) 35</td>
<td>1000 $</td>
</tr>
</tbody>
</table>

**Table: Pricing Schemes**
Data

Figure: Sample Covariance Matrix
Other Parameters

$\alpha$  risk aversion coefficient
$dc_a$  demand charge imposed by contract $a$ ($\$)
$ub_a$  max power that can be procured from contract $a$ (kW)
$M$    big M

Production Parameters

$\eta$  electricity to product conversion factor (unit/kWh)
$dem_d$  demand for end product faced on day $d$
$inv_0$  initial inventory for end product

ESS Parameters

$\xi_c, C_c$  ESS charging loss coefficient and capacity
$\xi_s, C_s$  ESS storage loss coefficient and capacity
$\xi_i, C_i$  ESS discharging loss coefficient and capacity
Variables

**Continuous Variables**

- $P_{h,a}$: power purchased from contract $a$ at hour $h$
- $P^u_h$: power used for production at hour $h$
- $P^e_h$: power stored at ESS at hour $h$
- $P^f_h$: power discharged from ESS at hour $h$
- $inv_d$: inventory at the end of day $d$
- $ess_h$: ESS charge level at hour $h$
- $K_h$: auxiliary variable for modeling QB cost component

**Binary Variables**

- $x_a$: 1 if contract $a$ is ever used in planning horizon
- $y_h$: auxiliary variable for modeling QB cost component
Model

\[
\begin{align*}
\text{min} & \quad \sum_{h \in \mathcal{H}} (c_f P_{h,f} + c_s P_{h,s} + K_h) + \\
& \quad \sum_{h \in \mathcal{H}_1} c_b^1 P_{h,b} + \sum_{h \in \mathcal{H}_2} c_b^2 P_{h,b} + \\
& \quad \alpha \sum_{h_1 \in \mathcal{H}} \sum_{h_2 \in \mathcal{H}} P_{h_1,s} [V^s]_{h_1,h_2} P_{h_2,s} + \sum_{a \in \mathcal{A}} x_{a d} c_a \\
\text{s.t.} & \quad \forall h \in \mathcal{H}, a \in \mathcal{A} \quad P_{h,a} \leq Mx_a \\
& \quad \forall h \in \mathcal{H} \quad \sum_{a \in \mathcal{A}} P_{h,a} = P^u_h + P^e_h \\
& \quad \forall d \in \mathcal{D} \quad inv_d = inv_{d-1} + \sum_{t \in \mathcal{T}} \eta(P^u_h + \xi_i P^f_h) - dem_d \\
& \quad \forall h \in \mathcal{H} \quad ess_h = \xi_s ess_{h-1} + \xi_c P^e_h - P^f_h
\end{align*}
\]
Model

\[ K_h = \begin{cases} 
  c_q^2 P^q_{h} & \text{if } y_h = 1 \\
  c_q^1 P^q_{h} & \text{o.w.}
\end{cases} \]

\[ \forall h \in \mathcal{H} \quad \sum_{\hat{h} \in \mathcal{H}, \hat{h} \leq h} P_{\hat{h}, q} - q\text{limit} \leq M y_h \tag{8} \]

\[ \forall h \in \mathcal{H} \quad M + K_h \geq c_q^2 P_{h, q} + M y_h \tag{9} \]

\[ \forall h \in \mathcal{H} \quad M y_h + K_h \leq c_q^2 P_{h, q} + M \tag{10} \]

\[ \forall h \in \mathcal{H} \quad M + K_h \geq c_q^1 P_{h, q} + M(1 - y_h) \tag{11} \]

\[ \forall h \in \mathcal{H} \quad M(1 - y_h) + K_h \leq c_q^1 P_{h, q} + M \tag{12} \]

\[ \forall h \in \mathcal{H} \quad y_h \geq y_{h-1} \tag{13} \]
Preliminary Results

![Graphs showing power imported spot market (kW), expected cost of importing power from spot market ($), and total cost of importing power ($).]
Preliminary Results

Figure: Objective Function Value for Different $\alpha$ Levels
Preliminary Results

Figure: Procurement Plan Avoids Peak Hours
Solution Techniques

Proposed Solution Techniques

- Improved Formulation by Symmetry Breaking Inequalities
- MIQP Branch and Bound with Warm Starting

Symmetry Breaking Inequalities

In some instances:

- Gurobi 4.0.0 and CPLEX 12.1 both reduced duality gap to 25% and stalled within the 2 hour limit
- Adding symmetry breaking constraints $y_h \geq y_{h-1} \quad \forall h \in \mathcal{H}$ reduced the solution time to 4 seconds
Proposed Solution Techniques

MIQP Branch and Bound with Warm Starting

- **A Good Initialization Heuristic**
  - Setting $x_s = 0$ and $x_b = 0$ reduces the problem to an LP.
  - Start from the optimal basis and the objective is an upper bound for B&B scheme.

- **A Relaxation Strategy**
  - Linear relaxation at each node is a QP. Efficiently solvable by using
    - Interior point methods (such as a predictor-corrector method)
    - Active-set methods
  - This provides us a lower bound for B&B scheme.

- **A Branching Rule**
  - First branch on all $x$ variables
  - Reduces the size of the search tree
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