Bi-level Optimization for Capacity Planning in Industrial Gas Markets


EWO Meeting
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Motivation

Industrial gas markets are dynamic:
• Suppliers must anticipate demand growth
• Most markets are served locally

Capacity expansion is a major strategic decision:
• Requires large investment cost
• Benefits are obtained during a long time-horizon

Benefits are sensitive to market behavior:
• Market preferences
• Economic environment

Sensitivity can be reduced by assuming rational behavior:
• Producers try to maximize their profit
• Markets try to minimize their cost

Need to model the conflicting interests of producer and markets
Problem Statement

Given:
- Set of capacitated plants from some of which can be expanded and some can not to be expanded
- Set of candidate locations for new plants
- Cost coefficients for investments, operation, and distribution
- Set of markets with deterministic demands during the time-horizon

Maximize net present value (NPV):
- Determine expansion plan
- While considering optimal distribution in each period
Standard Approach

Capacity expansion planning:

**Maximize:** Income – New plants – Maintenance – Expansion – Production – Transportation

\[
NPV = \sum_{t \in T} \frac{1}{(1 + r)^t} \left\{ \sum_{j \in J} \rho_{t,j} \sum_{i \in I} y_{t,i,j} - \sum_{i \in I} \left[ \alpha_{t,i} v_{t,i} + \beta_{t,i} w_{t,i} + \gamma_{t,i} x_{t,i} + \sum_{j \in J} (\phi_{t,i,j} y_{t,i,j} + \tau_{t,i,j,k} y_{t,i,j}) \right] \right\}
\]

**Subject to:**

\[
w_{t,i} = \sum_{t' = 1}^{t} v_{t,i} \quad (\forall t \in T, i \in I) \quad \text{Open plants must be maintained}
\]

\[
x_{t,i} \leq w_{t,i} \quad (\forall t \in T, i \in I) \quad \text{Expansion can only take place in open plants}
\]

\[
c_{t,i} = c_{t-1,i} + \delta x_{t-1,i} \quad (\forall t \in T, i \in I) \quad \text{Capacity in plants is incremental}
\]

\[
\sum_{j \in J} y_{t,i,j} \leq c_{t,i} \quad (\forall t \in T, i \in I) \quad \text{Demand satisfaction is constraint by capacities}
\]

\[
\sum_{i \in I} y_{t,i,j} = D_{t,j} \quad (\forall t \in T, j \in J) \quad \text{All markets are satisfied}
\]

\[
c_{j,k}, y_{s,j,i,k} \geq 0; \quad v_{t,i}, w_{t,i}, x_j \in \{0,1\} \quad (\forall t \in T, i \in I, j \in J) \quad \text{Bounds}
\]
Bilevel Optimization

Capacity expansion planning with rational market:

Two subsets: plants that can be expanded by leader ($I_1$) and that cannot be expanded by market ($I_2$)

\[
\max \ NPV = \sum_{t \in T} \frac{1}{(1 + r)^t} \left\{ \sum_{j \in J} \sum_{i \in I_1} \rho_{t,j} y_{t,i,j} - \sum_{i \in I_1} \left[ \alpha_{t,i} v_{t,i} + \beta_{t,i} w_{t,i} + \gamma_{t,i} x_{t,i} + \sum_{j \in J} (\varphi_{t,i,j} y_{t,i,j} + \tau_{t,i,j,k} y_{t,i,j}) \right] \right\}
\]

s.t.
\[
\begin{align*}
w_{t,i} &= \sum_{t' = 1}^{t} v_{t,i} \\
x_{t,i} &\leq w_{t,i} \\
c_{t,i} &= c_{t-1,i} + \delta x_{t-1,i} \\
\sum_{j \in J} y_{t,i,j} &\leq c_{t,i}
\end{align*}
\]

\[
\min \ \sum_{t \in T} \frac{1}{(1 + r)^t} \left[ \sum_{j \in J} \sum_{i \in I} \left( \rho_{t,j} y_{t,i,j} + \tau_{t,i,j,k} y_{t,i,j} \right) \right] \quad \text{Markets minimize their cost: price + transportation}
\]

s.t.
\[
\begin{align*}
\sum_{j \in J} y_{t,i,j} &\leq c_{t,i} \\
\sum_{i \in I} y_{t,i,j} &= D_{t,j} \\
c_{j,k}, y_{s,j,i,k} &\geq 0; \ v_{t,i}, w_{t,i}, x_{j} \in \{0, 1\} \\
\end{align*}
\]

\( \forall t \in T, i \in I_2 \) Capacity of plants excluded from expansion plan

\( \forall t \in T, j \in J \) All markets are satisfied
Solution Strategy

Transform to single-level by using KKT conditions of lower-level problem

The optimal solution for LP:

\[
\begin{align*}
\text{min} & \quad \sum_{k=1}^{|K|} c_k x_k \\
\text{s.t.} & \quad \sum_{k=1}^{|K|} a_{k,i} x_k \leq a_0 \quad (\mu_i) \quad i=1,\ldots,|I| \\
& \quad \sum_{k=1}^{|K|} b_{k,j} x_k = b_0 \quad (\lambda_j) \quad j=1,\ldots,|J| \\
& \quad x_k \in R \quad k=1,\ldots,|K| 
\end{align*}
\]

Can be obtained by solving:

\[
\begin{align*}
\text{Stationarity:} & \quad c_k + \sum_{i=1}^{|I|} a_{k,i} \mu_i + \sum_{j=1}^{|J|} b_{k,j} \lambda_j = 0 \quad k=1,\ldots,|K| \\
\text{Primal feasibility:} & \quad \sum_{k=1}^{|K|} a_{k,i} x_k \leq a_0 \quad i=1,\ldots,|I| \\
\text{Dual feasibility:} & \quad \sum_{k=1}^{K} b_{k,j} x_k = b_0 \quad j=1,\ldots,|J| \\
\text{Complementary slackness:} & \quad \mu_i \geq 0 \quad i=1,\ldots,|I| \\
& \quad \mu_i \left( \sum_{k=1}^{K} a_{k,i} x_k - a_0 \right) = 0 \quad i=1,\ldots,|I| \\
& \quad x_k, \lambda_j \in R \quad k=1,\ldots,|K|; j=1,\ldots,|J| 
\end{align*}
\]

Complementary slackness can be transformed to logic constraints:

\[
\begin{align*}
\sum_{k=1}^{K} a_{k,i} x_k - a_0 - s_i &= 0 \\
[s_i = 0] \lor [\mu_i = 0] \\
s_i &\geq 0
\end{align*}
\]

Use MILP reformulation
Problem structure:

- 3 existing plants which can be expanded
- 1 new candidate plant
- 3 existing plants which can not be expanded
- 15 markets with deterministic demand for 1 commodity
- 20 time-periods (quarters)

Formulations:

- Single-level \((SL)\): leader selects the markets to satisfy
- Bi-level \((BL)\): market minimizes cost in lower-level
- Single-level evaluation \((SL-eval)\): evaluation of single-level investment decisions in the market environment
Results

Computational statistics:

<table>
<thead>
<tr>
<th>Statistics</th>
<th>SL</th>
<th>BL</th>
<th>SL-eval</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of constraints:</td>
<td>783</td>
<td>9,220</td>
<td>481</td>
</tr>
<tr>
<td>No. of continuous variables:</td>
<td>3,012</td>
<td>6,183</td>
<td>2,701</td>
</tr>
<tr>
<td>No. of binary variables:</td>
<td>176</td>
<td>3,056</td>
<td>0</td>
</tr>
<tr>
<td>Solution time (CPLEX):</td>
<td>0.18 s</td>
<td>119 s</td>
<td>0.12 s</td>
</tr>
<tr>
<td>Optimality gap:</td>
<td>0.5%</td>
<td>0.5%</td>
<td>0.5%</td>
</tr>
</tbody>
</table>

Results:

<table>
<thead>
<tr>
<th>Element of objective function</th>
<th>SL</th>
<th>BL</th>
<th>SL-eval</th>
</tr>
</thead>
<tbody>
<tr>
<td>Income from sales [MM$]</td>
<td>1,601</td>
<td>960</td>
<td>1,247</td>
</tr>
<tr>
<td>Investment in new plants [MM$]</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Capacity expansion cost [MM$]</td>
<td>242</td>
<td>30</td>
<td>242</td>
</tr>
<tr>
<td>Maintenance cost [MM$]</td>
<td>127</td>
<td>127</td>
<td>127</td>
</tr>
<tr>
<td>Production cost [MM$]</td>
<td>580</td>
<td>357</td>
<td>454</td>
</tr>
<tr>
<td>Transportation cost [MM$]</td>
<td>22</td>
<td>9</td>
<td>11</td>
</tr>
<tr>
<td>Total NPV [MM$]</td>
<td>630</td>
<td>438</td>
<td>413</td>
</tr>
<tr>
<td>Market cost [MM$]</td>
<td>1,691</td>
<td>1,681</td>
<td>1,681</td>
</tr>
</tbody>
</table>

Bilevel optimization **NPV higher by 25 million (438 vs 413)** when compared to single-level expansion strategy.
Conclusions

Novelty:

• MILP bi-level optimization model for capacity expansion
• Considers the conflicting interest of producers and markets
• Models market behavior according to their interests
• Includes market preferences in capacity expansion planning

Impact for industrial applications:

• Allows developing capacity expansion plans that are less sensitive to market preferences and economic variability
• Avoids overestimating expansion
Future Steps

• Include uncertainty in market demands
• Model markets that can induce expansion and location of other plants
• Allow to reduce capacity and shut-down of plants